Deviating from the Friedman Rule: A Good Idea with Illegal Immigration?

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Abstract

This paper studies the optimal inflation rate in a transactions costs model with illegal immigration. Although unauthorized immigrants use domestic money for making transactions and consume in the host country, their welfare does not enter the objective function of a benevolent Ramsey planner because of their unofficial status. In this environment, the Friedman rule is nonoptimal, when an income tax is available, as the inflation tax allows to collect revenues from illegal immigrants, who are difficult to subdue to taxation. When a consumption tax is available, the zero inflation tax prescription becomes efficient provided that the consumption-money ratio of domestic consumers is not greater than the illegal immigrants' one.

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1 Introduction

Avoiding to tax real money balances —i.e., setting the opportunity cost of holding money to zero—is optimal in many standard intertemporal optimizing monetary models with infinite-lived consumers. This policy prescription, known as the Friedman rule, since initially advocated by Friedman (1969) from a first-best perspective, maximizes in general the consumer surplus also when distortionary taxes, used to finance exogenous government spending, are available. The validity of the zero inflation tax result in a second-best setting with full price flexibility has received support, among others, from Kimbrough (1986), Chari, Christiano and Kehoe (1996), Correia and Teles (1996 and 1999), Chari and Kehoe (1999), and De Fiore and Teles (2003).[1]

The consideration of tax limitations (associated with factors of productions, monopoly profits, pure rents and informal sectors), exogenous government transfers and costly tax collection, instead, imply that setting a positive nominal interest rate is efficient from a second-best ground. Such cases —that give support to the Phelps (1973) idea of optimally using the inflation tax along with all other available taxes to collect a given amount of revenue— are investigated, for example, by Ainzenman (1983), Faig (1988), Vegh (1989), Woodford (1990), Guidotti and Vegh (1993), Mulligan and Sala-i-Martin (1997), Nicolini (1998), Schmitt-Grohè and Uribe (2004a and 2010), Cunha (2008), and Petrucci (2011).

Schmitt-Grohè and Uribe (2010 and 2012) find another reason that justifies the optimality of collecting revenues from seignorage: the existence of a foreign demand for domestic currency.[2] In such a circumstance, people that hold domestic currency and hence are hit by the inflation tax can be divided into two types: nationals, whose welfare does matter for a benevolent social planner, and foreigners, whose welfare does not enter the planner's objective function. This discrepancy implies that a Ramsey planner faces two options when deciding on the optimal inflation rate.

[1] The way in which money is introduced into the economy is basically irrelevant for the optimality of the Friedman rule with distortionary taxation. See Christiano, Chari and Kehoe (1995), and Chari and Kehoe (1999).

[2] This case is particularly relevant, for example, for the U.S. dollar and the euro.

One option is to adopt the Friedman rule. In this case, there are no transaction costs for domestic consumers, while foreign residents holding domestic money receive an indirect transfer of resources by the domestic economy since the inflation rate is negative. The second option is to set a positive nominal interest rate. In this case, while domestic households suffer from the inflation tax because of costly transactions and the implied distortion in the resource allocation, the domestic economy is also collecting seignorage revenue from the rest of the world that hold domestic currency.

The solution of this normative trade-off implies the optimalityof extracting resources from foreigners by means of the inflation tax. This result is satisfied both from a first-best perspective and a second-best one.[3] The imposition of the inflation on foreign holders of domestic money is to be ascribed to the government objective to increase the amount of resources available for consumption of domestic consumers.

In this paper, we study the issue of optimal monetary policy in an intertemporal optimizing transactions costs model with illegal immigration when prices are fully flexible. The analytical mechanism employed here for studying the optimal inflation rate is quite similar, although with different macroeconomic underpinnings, to the one based on the foreign demand for domestic currency. But differently from Schmitt-Grohè and Uribe (2010 and 2012), in our case there exists the possibility of taxing agents holding domestic money, whose welfare is not taken into account by the social planner, by using a fiscal instrument other than the inflation tax. This additional tax instrument may lead to the optimality of the zero inflation tax rule.

In addition to the comparison with the analysis of Schmitt-Grohè and Uribe (2010 and 2012), the study of the optimal inflation tax with illegal immigration is interesting for the following reasons. First, the phenomenon of illegal immigration deeply affects the economic life of many advanced and

[3] While in general the second-best invalidity of the Friedman rule hinges on the fact that real money balances (through a derivative of the transaction cost technology) enter either the implementability constraint (that is, the planner's pseudo-welfare function) or additional constraints faced by the planner, which private agents do not face, in the Schmitt-Grohè and Uribe (2010 and 2012) analysis, instead, the nonoptimality of the Friedman rule is due to the fact that money (through a derivative of the transaction cost function) appears in the feasibility constraint.

emerging countries.[4] Although different macroeconomic aspects of such a phenomenon have been analyzed,[5] there are no studies that investigate the issue of optimal monetary policy in a world with illegal immigration.

Second, in an economy with illegal immigration, a benevolent planner considers welfare of domestic residents and illegal immigrants, who both demand domestic money (thus suffering from the erosion of purchasing power due to inflation) and consume in the domestic country, differently. In fact, while illegal immigrants are not considered in terms of welfare by the Ramsey planner because of their unofficial status, the social planner's objective function coincides with the domestic consumers' utility function.

Third, illegal immigration gives rise to a monetary policy externality as domestic output is also absorbed by unauthorized foreign workers for consumption and transaction costs. This implies that illegal immigrants' money demand, that enters the feasibility constraint, is indirectly taken into account by the Ramsey planner when setting the efficient inflation rate.

Fourth, illegal immigration implies *de facto* tax restrictions (i.e., incompleteness of the tax system). In fact, unauthorized aliens who work illegally are difficult to tax, while domestic residents who work legally can be taxed without restrictions. As illegal immigration consume in the host country, a consumption tax (alongside the inflation tax) represents an instrument to tax also illegal workers; therefore, such an additional fiscal instrument makes the tax system complete.[6]

We discover that in an environment with illegal immigrants setting a positive inflation tax may be optimal in a second-best sense, depending on

[4]The U.S., India, China and the European Union, among others, are characterized in a relevant way by illegal immigration. In the U.S., for example, illegal immigrants are estimated around 11.5 millions in 2011, which amounts to more than 6% of the labor force.

[5]These are, for example, the implications for domestic residents' welfare, the domestic job displacement impact, the effects on national resource allocation, the consequences for capital accumulation and economic growth, etc. See, among others, Ethier (1986), Bond and Chen (1987), Djajic (1987), Hazari and Sgro (2003), Palivos (2009) and Liu (2010).

[6]A tax system is complete (incomplete) when the number of tax instruments is equal to (lower than) the number of tax wedges. See Chari and Kehoe (1999). The completeness of the tax system plays a fundamental role for the optimality of the Friedman rule. See, for example, Cunha (2008) and Petrucci (2011).

whether the tax system is complete or not. In the setup of this paper the first-best monetary policy is the Friedman rule.

When only income taxation is available, it is optimal to depart from the Friedman rule. This is because illegal immigrants that use domestic money for making transactions and cannot be taxed through income taxation (because they are hardly detected) may be taxed through inflation.[7]

But while unauthorized immigrants do not pay income taxes, as they are officially non-existent, they cannot avoid to pay consumption taxes when buying consumption goods in the host country. When consumption taxation is available, the gap between the consumption-money ratio of domestic consumers and the one of illegal workers determines whether the optimal monetary policy should obey to the Friedman rule or not.

When the consumption-money ratio of illegal immigrants is equal to the corresponding ratio of domestic workers, the inflation tax should be zero as a consumption tax is normatively equivalent to a tax on real money balances since they both make transactions more costly and the tax burden is balanced across all the agents. In this case, consumption may be taxed more efficiently by using an explict consumption tax rather than using an implicit consumption tax, like the inflation tax.

When instead the consumption-money ratio of domestic consumers is higher than the one of illegal workers, it is Ramsey optimal to set a positive nominal interest rate. In this case, as consumption taxation would be relatively heavier for nationals than for undocumented workers (which would be relatively favored under the Friedman rule, being subsidized on their holdings of real money balances), the inflation tax plays the role of balancing the tax burden across agents. Therefore, the inflation tax is a socially welfare-improving mean to lessen consumption tax load on nationals and to increase the tax burden on illegal immigrants.

The paper is structured as follows. Section 2 builds an intertemporal

[7]A similar idea is provided by Nicolini (1998), Cavalcanti and Villamil (2003), and Schmitt-Grohè and Uribe (2010) for economies with informal sectors. Also in the Schmitt-Grohè and Uribe (2010 and 2012) contribution, like in the other second-best cases mentioned before, the deviation from the zero seignorage result depends on a form of tax restriction, given by the impossibility of collecting resources from foreigners that hold domestic money through fiscal instruments other than the inflation tax.

optimizing monetary model with transactions costs and illegal immigration. Section 3 analyzes the optimal inflation tax when only income taxation is available. Section 4 investigates the implications of introducing a consumption tax on efficient monetary policy. Section 5 concludes.

2 The model

Consider a monetary economy, peopled by infinite-lived domestic consumers, who do not emigrate, and foreign workers, who illegally enter and work in the domestic country. Immigration is only of the illegal type. Illegal aliens are expatriated to the country of origin if they are detected. All domestic consumers and illegal immigrants are employed.

Perfectly competitive firms produce domestic output y by using labor supplied by native households, l, along with labor provided by illegal migrants, l_I. l and l_I are both expressed in terms of hours worked. The production technology is given by $y = F(l + l_I)$, where $F(\cdot)$ is continuosuly twice differentiable, strictly increasing and concave. Without any loss of generality, it is assumed that domestic labor and labor of illegal immigrants are perfectly substitutable in production.

Labor of domestic residents is paid at the competitive wage w, while labor of illegal immigrants is paid at the exploitative wage $w_I < w$, as hiring illegal immigrants is risky for firms.[8]

Maximum profit of firms requires that

$$F'(l + l_I) = w, \tag{1a}$$

$$w_I = \beta w, \tag{1b}$$

where $\beta \in (0, 1]$ is the fixed immigrant wage to native wage ratio. β measures of the effectiveness of policies against illegal immigrants.

[8] Firms that employ illegal workers, once discovered, have to pay a fine, while immigrants are expatriated. See, for example, Hazari and Sgro (2003), Moy and Yip (2006), and Palivos (2009) for the same hypothesis. As shown by Palivos (2009), the condition $w_I < w$ can be obtained from a model in which firms that employ unauthorized immigrants have to pay a penalty when they are discovered operating illegally.

By using (1), firms' profits Π, which are positive, can be expressed as

$$\Pi = F(l + l_I) - (l + \beta l_I)F'(l + l_I) > 0. \tag{2}$$

Domestic consumers, whose number is constant over time, maximize the integral utility

$$\int_0^\infty U(c, x)\mathrm{e}^{-\rho t} dt, \tag{3}$$

where c is consumption, x leisure, and ρ the fixed rate of time preference. The instantaneous utility function $U(\,\cdot\,)$ is strictly increasing and concave in its arguments.

Native consumers accumulate wealth by holding government bonds, d, and real money balances, m. The real rate of return earned by holding government bonds is r. The opportunity cost of holding money —i.e., the nominal interest rate— is $i = r + \pi$, where π is the inflation rate. Domestic households have to pay transaction costs $s(v)$ to consume one unit of the consumption good, where v denotes the consumption to money ratio; that is,

$$v = \frac{c}{m}. \tag{4}$$

The transaction cost function $s(\,\cdot\,)$, which is continuous and twice differentiable, satisfies the following properties:

i) $s(v) \geqslant 0$ for $v \geqslant 0$;

ii) a critical level $\widetilde{v}$ of v exists, which corresponds to the satiation level of real money balances, for which $s(\widetilde{v}) = s'(\widetilde{v}) = 0$;

iii) $(v - \widetilde{v})s'(v) > 0$ for $v \neq \widetilde{v}$;

iv) $s'' > 0$ for $v \geqslant 0$.

The domestic residents' flow budget constraint is described by

$$c[1 + s(v)] + \dot{d} + \dot{m} = r(d + m) + (1 - \tau)(wl + \Pi) - im, \tag{5}$$

where τ is a proportional income tax rate.[9]

[9] The assumption that domestic consumers do not hold foreign bonds in their portfolios has no implications for the analysis.

The time endowment of domestic residents, which is normalized to one, being fixed, is used for either working or consuming leisure; that is,

$$1 = x + l. \tag{6}$$

The maximization of (3) subject to (4), (5) and (6) yields the following first-order conditions

$$U_c = \lambda(1 + s + vs'), \tag{7a}$$

$$U_x = (1 - \tau)w\lambda, \tag{7b}$$

$$v^2 s' = i, \tag{7c}$$

$$-\dot{\lambda} + \lambda\rho = \lambda r, \tag{7d}$$

where λ is the shadow value of domestic resident wealth.[10]

Foreign agents, that enter and work illegally in the host country accumulate wealth by holding real money balances m_I, necessary for purchasing domestic consumption goods, and foreign bonds (denominated in domestic currency) b in their portfolios. Because of perfect capital mobility, the real rate of return earned by holding foreign bonds, expressed in terms of the domestic numeraire, is r.

The illegal immigrants' flow budget constraint in aggregate terms is[11]

[10] Equations (7) are fairly standard. After using (7a), equation (7b) can be written as $\dfrac{U_x}{U_c} = \dfrac{(1-\tau_l)w}{(1+sv+s')}$. Such an equation states that the marginal rate of substitution of consumption for leisure must equal the opportunity cost of leisure in terms of consumption; this opportunity cost is equal to the after-tax wage divided by the unit price of consumption (i.e., one plus the marginal cost of consumption). Equation (7c) is the implicit demand for real money balances. The money demand in explicit terms is: $m = cL(i)$, with $L' = -\frac{(2vs'+v^2s'')}{v^2} < 0$.

[11] Total hours worked by foreign workers l_I is proportional to the number of immigrants if each immigrant works the same fixed number of hours.

$$c_I[1+q(v_I)] + \dot{m}_I + \dot{b} = r(m_I + b) + w_I l_I - i m_I, \tag{8}$$

where c_I denotes illegal immigrants' consumption, m_I their money holdings and $q(\,\cdot\,)$, which satisfies the same qualitative properties of $s(\,\cdot\,)$, their transaction costs function; v_I is the consumption-based money velocity of illegal immigrants, defined as[12]

$$v_I = \frac{c_I}{m_I}. \tag{9}$$

It is assumed that unauthorized immigrants do not pay income taxes because of their illegal conditions.

Illegal immigrants maximize their utility function subject to (8) and (9). We are only interested in their money demand function, because this is an element that the domestic planner has to take into account when choosing the optimal monetary policy —as v_I enters the feasibility constraint through $q(\,\cdot\,)$. Therefore, only the first-order condition of the illegal immigrant utility maximization with respect to m_I, can be considered; this is given by

$$v_I^2 q' = i. \tag{10}$$

Since illegal immigrants may use domestic money more intensively than native consumers because they operate in the illegal sphere of the economy and aim at remaining concealed, we assume that $q'(v_0) \gtrless s'(v_0)$.[13] This implies that $v \gtrless v_I$ for every i.[14]

[12] By keeping the function $q(\,\cdot\,)$ potentially separated from the function $s(\,\cdot\,)$, we want to pay attention to the differential role played by v_I with respect to v for the optimal monetary policy.

[13] As illegal immigration are associated with a sort of informal sector, such an hypothesis is consistent with the assumption adopted by Nicolini (1998) that the informal economy is more currency intensive than the official one.

[14] Assuming the same functional form for the transaction costs technology as Schmitt-Grohè and Uribe (2010 and 2012), we have that

$$s(v) = Av + B/v - 2\sqrt{AB},$$

and

The government finances its budget deficit by issuing government debt and money, whose total stock in real terms is $m + m_I$. The government budget constraint is given by

$$\dot{d} + \dot{m} + \dot{m}_I = r(d + m + m_I) + g - \tau(wl + \Pi) - i(m + m_I), \qquad (11)$$

where g denotes government spending, assumed to be exogenous.

The resource constraint states that domestic output plus interest income earned by illegal immigrants by holding foreign bonds is equal to total consumption costs of domestic consumers and foreign workers, plus government spending and plus the rate of accumulation of foreign bonds; that is,

$$F(l + l_I) + rb = c[1 + s(v)] + c_I[1 + q(v_I)] + g + \dot{b}. \qquad (12)$$

3 Ramsey monetary policy with income taxation

Only second-best monetary policy is analyzed here. The first-best monetary policy, obtained when distortionary taxation is absent and there are only lump-sum taxes, is the Friedman rule if each agent is compensated on a lump-sum basis for the inflation tax.[15]

The second-best monetary policy is obtained by maximizing the utility of the representative domestic consumer subject to the competitive equilibrium with distortionary taxes and the constraint that a given flow of government spending has to be financed.

As illegal immigrants use domestic money for making transactions in the host country, the Ramsey planner, who will not consider their welfare, be-

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$$q(v_I) = A_I v_I + B/v_I - 2\sqrt{A_I B},$$

where $A > 0$, $B > 0$ and $A_I \geqslant A$ are parameters. The satiation levels of the consumption-money ratio for nationals and illegal immigrants are $\widetilde{v} = \sqrt{B/A}$ and $\widetilde{v}_I = \sqrt{B/A_I}$ (with $\widetilde{v} \geqslant \widetilde{v}_I$), respectively.

[15] The analysis of the first-best monetary policy is provided in the Appendix.

cause of their unauthorized status, takes indirectly into account, when choosing the optimal inflation rate, their money demand through the feasibility constraint.

The analysis of the efficient policy employs the implementability constraint of nationals as a constraint of the planner's problem. This is obtained from the domestic consumers' intertemporal budget constraint after expressing prices and taxes in terms of quantities through the marginal efficiency conditions (7).

The implementability constraint of the domestic consumers is derived as follows. By integrating the flow budget constraint (5) forward and incorporating the condition preventing 'Ponzi games' —that is, $\lim_{t\to\infty}$ $(d+m)$ $\mathrm{e}^{-\int_0^t r du}$=0— yields the residents' intertemporal budget constraint

$$\int_0^\infty \{c[1+s(v)]+im-(1-\tau)(wl+\Pi)\}\,\mathrm{e}^{-\int_0^t r du}dt=0, \tag{5'}$$

where we have set $d_0+m_0=0$ (d_0 and m_0 are government debt and real money balances at $t=0$).

By integrating (7d), we get

$$e^{-\int_0^t r du}=\frac{\lambda}{\lambda_0}e^{-\rho t}, \tag{7d'}$$

where λ_0 is λ at time 0.

Substituting (2), (7a), (7b), (7c) and (7d') into (5'), one obtains, after rearranging, the implementability constraint; that is,

$$\int_0^\infty \{cU_c-U_x\frac{[F(l+l_I)-\beta F' l_I]}{F'}\}\mathrm{e}^{-\rho t}dt=0. \tag{13}$$

A second constraint employed for the analysis of the optimal monetary policy is the feasibility constraint expressed in terms of v, which is obtained as follows. After using (1), the combination of the flow budget constraint of illegal immigrants (8) with (12) allows us to express the feasibility constraint as

$$\dot{m}_I=c[1+s(v)]+g+\beta F' l_I-F(l+l_I)-(i-r)m_I. \tag{14}$$

By equating (7c) and (10) for i, and solving for v_I, yields

$$v_I = V(v), \tag{15}$$

where $V' = \dfrac{(2vs' + v^2 s'')}{(2v_I q' + v_I^2 q'')} > 0$.

Using (9) and (15), the money demand of illegal immigrants can be expressed as $m_I = \dfrac{c_I}{V(v)}$. The total differentiation of this equation yields $\dot{m}_I = \dfrac{\dot{c}_I}{v_I} - \dfrac{c_I V'}{v_I^2}\dot{v}$. Plugging $\dot{m}_I$ from such a relationship into (14), making use of (7c) and (7d) to eliminate i and r, respectively, and rearranging, we obtain

$$\dot{v} = \frac{v_I^2}{c_I V'} \left\{ \frac{\dot{c}_I}{v_I} - c[1 + s(v)] - g - \beta F' l_I + F(l + l_I) + \frac{c_I}{v_I}(v^2 s' + \frac{z}{\lambda} - \rho) \right\}, \tag{16}$$

where $z = \dot{\lambda}$. Equation (16) represents the feasibility constraint expressed in terms of v.

The Ramsey planner chooses the second-best allocation by maximizing the social welfare function, given by the domestic consumer utility integral (3), subject to the implementability constraint (13) and the feasibility constraint (16), once the relationships $z = \dot{\lambda}$ and $\lambda(1 + s + vs') = U_c$ are taken into account. Moreover, as the nominal interest rate cannot be negative, also the inequality $v \geqslant \widetilde{v}$ has to be imposed. The variables chosen by the social planner are: c, l, v, z and λ.

The optimal tax structure can be summarized as follows

Proposition 1 *In an infinitely lived monetary model with transaction costs and illegal immigrants, that demand domestic money for making transactions in the host country, second-best efficiency prescribes to tax real money balances in addition to domestic consumers' income when consumption taxation is not available.*

Proof. The first-order conditions of the Ramsey problem with respect to v, z and λ, and the Kuhn-Tucker conditions regarding the inequality constraint

$v \geqslant \widetilde{v}$, are given by

$$-\dot{\Gamma} + \rho\Gamma = \Gamma \frac{v_I^2}{c_I V'} \left[-cs' + \frac{c_I}{v_I}(2vs' + v^2 s'') - (v^2 s' + \frac{z}{\lambda} - \rho)\frac{c_I}{v_I^2} V' \right] +$$

$$+\Sigma\lambda(2s' + vs'') + \Xi, \tag{17a}$$

$$\frac{\Gamma v_I}{\lambda V'} = -\Delta, \tag{17b}$$

$$-\dot{\Delta} + \rho\Delta = -\frac{\Gamma v_I z}{\lambda^2 V'} + \Sigma(1 + s + vs'), \tag{17c}$$

$$\Xi(v - \widetilde{v}) = 0,\ \Xi \geqslant 0,\ v - \widetilde{v} \geqslant 0, \tag{17d}$$

where Γ, Δ and Σ denote the Lagrange multipliers on the constraints (16), $z = \dot{\lambda}$ and $\lambda(1 + s + vs') = U_c$, respectively. Ξ is the Kuhn-Tucker multiplier on the inequality constraint $v \geqslant \widetilde{v}$.

In the long-run, plugging (17b) into (17c) for Δ and rearranging, we get

$$\Sigma\lambda = -\frac{\Gamma\rho v_I}{(1 + s + vs')V'}. \tag{18}$$

Using such an expression into the steady state version of (17a) yields

$$\Gamma \left(\frac{v_I^2 c}{c_I V'} + v^2 \right) s' = \frac{\Gamma v_I}{V'}(2s' + vs'') \left[v - \frac{\rho}{(1 + s + vs')} \right] + \Xi. \tag{19}$$

The nonoptimality of the Friedman rule can be demonstrated by showing that (19) is contradicted when the nominal interest rate is zero. In fact, if $v = \widetilde{v}$ —and hence $s = s' = 0$— equation (19) becomes

$$\frac{\Gamma \widetilde{v}_I \widetilde{v} s''}{V'}(\widetilde{v} - \rho) + \Xi = 0. \tag{19'}$$

As plausibly $\widetilde{v} > \rho$,[16] (19') implies that $\Xi < 0$ because $\Gamma > 0$.[17] But since the Kuhn-Tucker condition (17d) is violated, this fact implies that v cannot be equal to $\widetilde{v}$; that is, the Friedman rule cannot be optimal.

[16] Also Schmitt-Grohè and Uribe (2010 and 2012) assume that $\widetilde{v} > \rho$.

[17] The fact that $\Gamma > 0$ can be demostrated as follows. The first-order condition of the Ramsey problem with respect to c is

Therefore, it must alternatively be that $v > \widetilde{v}$ and $\Xi = 0$. After using $\Xi = 0$ and plugging (7c) into (19), we obtain that

$$i^* = \frac{v^2 v_I s''[v(1+s+vs') - \rho]}{(1+s+vs')\left[\frac{v_I^2 c}{v^2 c_I} + V' - \frac{2v_I}{v} + \frac{2\rho v_I}{v^2(1+s+vs')}\right]} > 0. \qquad (20)$$

□

When only income taxes are available, the presence of illegal immigrants implies that it is Ramsey efficient to collect seignorage revenue even if this is costly (in terms of transactions and resource allocation) for domestic consumers.

As there are restrictions of the capacity of the planner to tax illegal immigrants, the inflation tax represents an indirect way of taxing those who cannot be taxed. This is a well-established principle, in a context in which the tax code is not sufficiently rich. With no illegal immigration, the optimal tax on real money balances would be zero.

Analytically speaking, in an environment with illegal immigration, the first derivative of the transactions technology enters, because of the inflation tax, the feasibility constraint that the planner has to consider in the second-best problem. Similarly to Schmitt-Grohè and Uribe (2010 and 2012), this fact undermines the optimality of the Friedman rule.

$$U_c[1 + \Phi(1+\eta_c)] = \frac{\Gamma v_I^2(1+s)}{c_I V'} + \Sigma U_{cc},$$

where $\eta_c = \frac{cU_{cc}}{U_c} - \frac{U_{xc}(F - \beta F' l_I)}{F'}$. After using (18), such an equation can be written as

$$\Gamma = \frac{U_c[1 + \Phi(1+\eta_c)]V'}{v_I[\frac{v_I(1+s)}{c_I} - \frac{\rho U_{cc}}{U_c}]} > 0.$$

Therefore, $\Gamma > 0$ as the marginal pseudo-utility of consumption is positive since $\Phi > 0$, because of positive income taxation, and $1 + \Phi(1+\eta_c) > 0$.

4 Ramsey monetary policy with income and consumption taxes

Suppose that a proportional tax on consumption (whose rate is τ_c), paid by both domestic consumers and illegal immigrants when they acquire consumption goods, is introduced. The domestic consumers' cost of consumption, which has to be considered in their flow budget constraint (5), is $c[1 + \tau_c + s(v)]$, while the consumption cost for unauthorized immigrants, that has to be included in their budget constraint (8), is $c_I[1 + \tau_c + q(v_I)]$.

The only change that the presence of a consumption tax implies for the first-order conditions (7) is the following

$$U_c = \lambda(1 + \tau_c + s + vs'), \tag{7a'}$$

which replaces (7a).

In the government budget constraint (11) the term $\tau_c(c + c_I)$ has to be included on the revenue side.

The second-best tax problem is the same as in the case in which only income taxation is available with three differences. One is that (7a') instead of (7a) has to be considered within the constraints of the Ramsey problem, one is that the feasibility constraint (16) has to include the term $+\tau_c c_I$ inside the curly brackets of the right-hand side, and one is that now τ_c is endogenosuly chosen.

The conceptual characterization of the efficient tax policy is

Proposition 2 *In an immortal monetary economy with costly transactions and illegal immigration, the optimal inflation tax strictly depends on the consumption-money ratios of nationals and unauthorized immigrants when income and consumption taxes are available. If both agents have the same consumption-money ratio, the Friedman rule is second-best optimal. If instead the consumption-money ratio of illegal immigrants is lower than the consumption-money ratio of nationals, it is efficient to deviate from the Friedman rule.*

Proof. The relevant first-order conditions for the second-best optimal monetary policy are given by (17a), (17b), (17d) and

$$-\dot{\Delta}+\rho\Delta=-\frac{\Gamma v_I z}{\lambda^2 V'}+\Sigma(1+\tau_c+s+vs'), \quad \text{(17c')}$$

$$\frac{\Gamma v_I^2}{V'}=-\Sigma\lambda. \quad \text{(17e)}$$

Plugging (17e) into the long-run version of (17a) yields

$$\Gamma\left(\frac{v_I^2 c}{c_I V'}+v^2\right)s'=\frac{\Gamma v_I}{V'}(2s'+vs'')(v-v_I)+\Xi. \quad \text{(21)}$$

Two cases are possible: i) $v=v_I$; and ii) $v>v_I$. Consider each case in turn.

i) $v=v_I$[18]

Suppose that $v>\widetilde{v}$. In such a case, given that $\Gamma>0$, equation (21) implies that $\Xi>0$ as $s'(v)>0$. But this is impossible since the Kuhn-Tucker condition (17d) requires that $\Xi=0$.

Therefore, the Friedman rule is optimal.

ii) $v>v_I$[19]

Suppose that $v=\widetilde{v}$ and hence $v_I=V(\widetilde{v})=\widetilde{v}_I$. In this case equation (21) becomes

$$\frac{\Gamma\widetilde{v}\widetilde{v}_I}{V'}s''(\widetilde{v}-\widetilde{v}_I)+\Xi=0. \quad \text{(21')}$$

This implies that $\Xi<0$. But then, as the Kuhn-Tucker condition (17d) is contradicted, v (v_I) cannot be equal to $\widetilde{v}$ ($\widetilde{v}_I$).

Therefore, we must instead have $v>\widetilde{v}$ (hence $v_I>\widetilde{v}_I$) and $\Xi=0$. From (21), we get

$$i^*=\frac{v^2 c_I v_I s''(v-v_I)}{[v_I^2 c+V' v^2 c_I-2v_I c_I(v-v_I)]}>0. \quad \text{(22)}$$

Thus, when $v>v_I$, deviating from the Friedman rule is optimal. □

The consideration of consumption taxation makes the tax system complete. When $v=v_I$, it is optimal to set the nominal interest rate equal

[18] In this case, $A=A_I$ if the functional form of transaction costs of footnote 14 were employed.

[19] Now $A>A_I$ and $\widetilde{v}>\widetilde{v}_I$ in the transaction cost function of footnote 14.

to zero as illegal immigrants contribute to the revenue collection through consumption taxation in the same relative amount as domestic consumers. In this case, the consumption tax and the inflation tax work similarly in terms of revenues; the inflation tax is, however, a less efficient way to obtain seignorage.

When $v > v_I$, consumption taxation and the inflation tax are no longer equivalent in revenue collection terms. In fact, the consumption tax hits relatively more domestic consumers, while the inflation tax hits relatively more illegal immigrants. Therefore, it is optimal to set a positive nominal interest rate alongside a positive consumption tax for balancing the tax burden across different agents.

5 Concluding remarks

This paper investigates the issue of optimal monetary policy in a transaction costs model with illegal immigration. The presence of illegal immigrants — who demand domestic currency, consume in the host country, are difficult to tax and are not considered in the planner's objective function— may support the optimality of collecting revenues from seignorage.

The mechanism for deviating from the Friedman (1969) rule offered here is similar to that one analyzed by Schmitt-Grohè and Uribe (2010 and 2012), which is based on the demand for domestic currency by foreigners, whose welfare is not considered by the benevolent Ramsey planner.

The element invalidating the zero inflation tax result here is the impossibility of taxing illegal immigrants that hold domestic money and consume in the domestic country. Therefore, the incompleteness of the tax system is once again at the basis of the invalidity of the Friedman rule.

The fundamental results of the paper are the following. If only income taxation is available, the optimal inflation tax should be positive, as it represents a way to tax illegal immigrants.

When instead a consumption tax is at disposal of the planner, optimality calls for either the Friedman rule — when domestic residents and illegal aliens have the same consumption-money ratio— or the Phelps rule —when the consumption-money ratio of domestic residents is higher than the one of illegal immigrants.

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Appendix

In this Appendix, the first-best monetary policy is studied.

We assume that there is no distortionary taxation and only lump-sum taxes on domestic consumers and illegal immigrants are levied.

Now the optimal monetary policy is obtained by maximizing the utility functional (3) subject to the proper feasibility constraint, to be derived, and the inequality constraint $v \geqslant \widetilde{v}$.

The feasibility constraint for the first-best monetary policy, expressed in terms of v, is obtained as follows. By using the flow budget constraint of illegal immigrants together with (12) —under the hypothesis that such agents are lump-sum compensated for the inflation — the feasibility constraint can be written as

$$\dot{m}_I = c[1 + s(v)] + g + \beta F' l_I - F(l + l_I). \tag{22}$$

After differentiating the illegal immigrants' money demand and using (15), from (22), we obtain

$$\dot{v} = \frac{v_I^2}{c_I V'} \left\{ \frac{\dot{c}_I}{v_I} - c[1 + s(v)] - g - \beta F' l_I + F(l + l_I) \right\}, \tag{23}$$

Equation (23) is the feasibility constraint expressed in terms of v under the assumption that illegal immigrants are compensated for the inflation tax.

The social planner chooses the first-best allocation by maximizing the social welfare function, given by (3), subject to (23) and the restriction $v \geqslant \widetilde{v}$. The variables chosen by the social planner are: c, l, and v.

The relevant first-order conditions for the optimal monetary policy are

$$-\dot{\Gamma} + \rho\Gamma = -\Gamma \frac{v_I^2 c s'}{c_I V'} + \Xi, \tag{24a}$$

$$\Xi(v - \widetilde{v}) = 0, \; \Xi \geqslant 0, \; v - \widetilde{v} \geqslant 0. \tag{24b}$$

In the long-run, (24a) becomes

$$\rho\Gamma = -\Gamma \frac{v_I^2 c s'}{c_I V'} + \Xi. \tag{24a'}$$

If $v > \widetilde{v}$ and, hence, $\Xi = 0$, (24a') would imply that $s' = i/v^2 < 0$, thus contradicting the restriction of a nonnegative nominal interest rate.

If instead $v = \widetilde{v}$, (24a') implies, consistently with the Kuhn-Tucker condition, that $\Xi = \rho\Gamma > 0$. Therefore, the Friedman rule is first-best optimal.