Monetary policy, leverage premium and loan default probabilities

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PRELIMINARY AND INCOMPLETE

Abstract

This paper uses the "costly state verification" model to argue that monetary expansions a) decrease firms' default probabilities on outstanding loans by unexpectedly pushing up their revenues and b) increase firms' default probabilities on new loans because it induces them to take more leverage. The model is consistent with the empirical finding by Jimenez *et al.* (2007) and formalizes the risk taking channel of monetary policy commented by Rajan (2006) and Borio and Zhu (2008). In the model, a 1 % decrease in the annual policy rate delivers an impact decrease in defaults of 87 annual basis points below steady state and a following increase in defaults of 282 annual basis points above steady state.

JEL Classification: E44, E52

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1 Introduction

This paper studies the effect of monetary policy on the probability that firms default on loans.

It would be sensible to guess that a monetary expansion reduces this default probability because it mitigates the debt burden of firms by pushing down the borrowing rate charged by banks. Despite being arguably in place, this effect might not dominate. By estimating duration models on loan applications in Spain between 1985 and 2006, Jimenez, Ongena, Peydro and Saurina (2007) find that, other things equal, a reduction in the policy rate reduces the default probability of *outstanding* loans, but it also increases the default probability of *new* loans, i.e. loans originated within the first quarter after the policy shock.[1] This empirical finding suggests that new loans are subject to some additional effect that dominates on the lower cost of borrowing.[2]

Jimenez *et al.* (2007) interpret this additional effect in terms of an *indirect risk-shifting effect* by banks across heterogeneous borrowers. When the policy rate decreases, borrowing rates decrease. Outstanding loans have already been issued when the monetary shock occurs, but new loans have not, implying that the bank has now the option of shifting towards borrowers with different risk profiles. The authors interpret their result as evidence that banks react to a monetary expansion by shifting towards more risky borrowers, a reaction that offsets the lower cost of

[1] An excellent introduction to duration models is offered by Kleinbaum and Klein (2005) and by Keifer (1988). In short, time to default on loans is taken as the realization of a random variable with some given probability density function. Rewrite this pdf as an hazard function, which approximately detects the conditional probability of default in time t given that default has not occured yet. Using maximum likelihood estimation one can use the ex post information on defaults to infer the parameter values of the hazard rate function. If one assumes that external variables like the policy rate controlled by the central bank affects the hazard rate proportionally, then one can study if covariates shift the hazard rate upwards or downwards.

[2] This empirical regularity was also found by Ioannidou, Ongena and Peydro (2007) for Bolivia and by Lopez, Tenjo and Zarate (2011) for Colombia. Ioannidou *et al.* (2007) also study the effect of monetary policy on the pricing of risk.

borrowing and ultimately increases defaults on new loans.[3]

This paper argues that the result by Jimenez *et al.* (2007) on new loans could also be driven by an *indirect leverage effect* in a framework in which borrowers are homogeneous. The intuition goes as follows. A monetary expansion decreases borrowing rates and increases the discounted return to capital. Outstanding loans benefit from this because the shock generates an unexpected increase in aggregate demand and in revenues. New loans, instead, are also subject to an additional *indirect* effect because new borrowers react to the lower cost of borrowing by increasing leverage to expand investments above steady state. Under certain conditions, the increase in borrowers' leverage *increases* their default probability because net worth provides now a smaller buffer to their risky loan. If the hike in the borrower's leverage is strong enough, the indirect effect might prevail and push up the equilibrium default probability.[4]

This paper uses the costly state verification model by Townsend (1979) to formalize the above intuition. The model is well known in the macroeconomic literature, but it has been mainly used to generate hump-shaped impulse reponse functions for output and to develop a financial accelerator (Carlstrom and Fuerst (1997) and Bernanke, Gertler and Gilchrist (1999)).[5] The prediction of the model with regard

[3]To argue that this shift in risk profiles is intentionally taken by banks, Jimenez *et al.* (2007) take the ex post realizations of defaults as a good proxy of the predictions that loan officers form regarding the risk profiles of loan applications. Ioannidou, Ongena and Peydro (2007) and by Lopez, Tenjo and Zarate (2011) take the same assumption. In this paper I do not go this far and interpret their result more as suggestive evidence of an equilibrium phenomenon rather than as an effect fully and intentionally driven by banks' behaviour.

[4]This is of course only one possible story. Monetary policy might also affect risk aversions (Bekaert, Hoerova and Lo Duca (2010)), induce banks to unintentionally take risk (FIND REFERENCES) or affect the heterogeneity of borrowers that demand for a loan (Stiglitz and Weiss (1988)). While the first two arguments probably reinforce the result derived in this paper, the third one works in the opposite direction, since Stiglitz and Weiss (1988) show that lower interest rates bring safer borrowers to demand for credit and lead borrowers to finance safer projects.

[5]The model generates this result because firms smooth investment decisions across time in order to take advantage of the lower agency costs brought about by an increase in net worth. EXPLAIN ALSO FINANCIAL ACCELERATOR AND DAMPENING OF THE SHOCK. In this model the effect that increases defaults following a monetary expansion is the same effect that increases defaults following a positive productivity shock. While the result by Jimenez *et al.* (2007) suggest that this increase in defaults might be realistic for monetary shocks, there is evidence that

to defaults has received less attention, except for commenting and fixing the procyclicality of default probabilities, which is predicted by the baseline model but rejected by the data (Dorofeenko, Lee and Salyer (2006) and Covas and Den Haan (2011)). This paper contributes to the literature by arguing that, in light of the empirical result by Jimenez *et al.* (2007), the costly state verification model might be a useful model to study the effect of monetary policy on loans default probabilities.[6]

As it will be explained, the model features homogeneous entrepreneurs who have limited net worth and borrow using risky debt contracts to finance a project which gives stochastic revenues. Whether ex post revenues are low enough to lead the entrepreneur to default on the loan depends on the borrowing conditions, because, other things equal, a higher borrowing cost increases the entrepreneur's probability of default. Knowing this, the entrepreneur chooses the size of the loan anticipating that, given his initial net worth, investing more implies an increase in his leverage ratio which pushes up the borrowing cost and the default probability due to asymmetric information. The equilibrium is pinned down by the trade off between running an investment of bigger size relative to paying a higher borrowing cost to the lender in order to obtain more credit. The decrease in the policy rate increase the discounted return to capital and leads the borrower to increase his leverage ratio, which pushes up the default probability.

The paper is divided in two parts. The first part, developed in section 2, uses the costly state verification model by Townsend (1979) in partial equilibrium to argue

defaults are countercyclical, not procyclical. Dorofeenko, Lee and Salyer (2006) fix this problem by assuming that the business cycle is driven by shocks to the uncertainty on the idiosyncratic shock while Covas and Den Haan (2011) fix it by adding an equity contract in which the cost of issuing equity is countercyclical. CHECK IF EFFECT STILL IN PLACE FOR MONETARY SHOCK AND IF CAN RECONCILE BOTH

[6]The analysis is limited to a positive explanation of one possible determinants of the empirical results by the authors, but does not speak to the policy implications of it because in this model default increase as an optimal decision of agents. A normative analysis would require a much more complicated modeling device and is left out of the analysis COMMENT STEIN, KASHAP AND STEIN, EXPLAIN PREFER REMAIN POSITIVE BUT MICROFOUNDED THAT THE OPPOSITE

that the indirect leverage effect commented above dominates, potentially contributing to explain the empirical result on new loans by Jimenez *et al.* (2007). The effect is driven by asymmetric information, because if information was symmetric, the Modigliani-Miller theorem would imply the lenders' indifference to the leverage ratio of the borrower and agents would sign state-contingent contracts that would rule out defaults as a possible state. The second part of the paper, developed in section 3, argues that the effect on new loans remains in place in a dynamic general equilibrium framework. Additionally, a general equilibrium effect on asset prices reduces the default probability of outstanding loans because monetary policy expands aggregate demand and unexpectedly increases firms' revenues, hence generating also the result by Jimenez *et al.* (2007) on outstanding loans.[7]

This paper is related to the literature on the risk taking channel of monetary policy started Rajan (2006) and Borio and Zhu (2008) that argues that the marginal costs of monetary expansions might go well beyond inflationary pressures. Existing theoretical contributions usually take a reduced-form approach and stress the role of banks' incentives along borrowers that exogenously differ with regard to their riskiness. To do so, credit demand effects are typically shut down by either assuming some exogenous investment possibilities (Acharya and Naqvi (2009), Agur and Demertzis (2012)) or a monopolistic bank facing an exogenous demand curve for loans (Dell'Ariccia, Laeven and Marquez (2010) and Valencia (2011)).[8] A more microfounded approach is taken by Angeloni and Faia (2011), who nevertheless are interested in the default probability of banks instead of firms. This paper studies

[7]The model can only replicate one small fraction of the heterogeneity of firms in the dataset used by the authors. In the model, loans are financial, non-collateralized loans issued by an intermediary, they pay a fixed interest rate and have maturity of 1 quarter. In the dataset, 60 % of loans are financial loans, almost 85 % have no collateral, around 90 % are issued by a commercial or saving bank and most have non adjustable rates (see Jimenez and Saurina (2004)). While these features are overall shared between my model and the dataset by the authors, loans in their dataset have an average maturity of 5 quarters, with some with maturity above 5 years. In the calibration I adjust for the difference in maturities by matching the empirical default probability of loans with maturity of 1 quarter, as explained in section 3.4 and 3.7.

[8](*add Cucioba, Shukayev and Ueberfeldt (2011), Drees, Eckwert and Vardy (2010)*)

to which extent the dynamics in the borrower's leverage ratio can contribute to explain the link between monetary policy and firms' defaults. The results are in line with an emerging empirical literature (Maddaloni and Peydro (2011), Altunbas, Gambacorta and Marquez-Ibanez (2010) and Paligorova and Santos).[9]

2 The costly state verification model in partial equilibrium

This section uses the costly state verification model in partial equilibrium to formalize the key intuition of the paper, which is that monetary expansions might increase firms' default probabilities on new loans by leading firms to take more leverage. I first highlight the key features of the model and then use graphs and numerical examples develop the intuition behind the result.

2.1 Environment

The partial equilibrium model consists of one period. At the beginning of the period risk neutral agents receive a non-consumable endowment which can be transformed into end-of-period consumption through either a risk-free bond or through a linear production technology. No aggregate uncertainty enters the partial equilibrium model. Production is affected by an idiosyncratic shock and yields an expected return that exceeds the safe return on the risk-free bond. The combination of risk neutrality and higher expected return on production implies that it would be optimal for the economy to invest the entire endowment in the risky technology.

[9] The empirical literature studies in which way the equilibrium effects observed in the data comes from demand relative to supply effects. The model in this paper is not reach enough to guide the empirical identification on this matter since it uses an optimal contract which does not allow for demand and supply curves to be defined independently on the other. The model isolates a potentially interesting leverage effect but is silent about whether this effect is generated by an incentive of the borrower relative to the incentive of the lender.

Whether credit markets allocate resources such to achieve this equilibrium or not depends on whether the information between agents is symmetric or not (more on this later).

Agents are heterogeneous with respect to both investment possibilities and initial endowment. Assume that a continuum of agents called *entrepreneurs* have access to the risky production function and receive limited net worth, while a continuum of agents called *lenders* have only access to the risk-free bond and receive an abundant endowment. One can think of entrepreneurs as agents with limited funds but creative business ideas and lenders as savers with limited entrepreneurial skills but abundant funds.[10]

Entrepreneurs borrow from lenders on competitive markets.[11] In this setting competition should be thought of as lenders competing among each other to provide loans to entrepreneurs, but it does not imply price-taking behavior because the borrowing rate is optimally set in the debt contract together with the amount borrowed.[12] Perfect competition among lenders implies that the contract maximizes the expected profits of the entrepreneur under the condition that the lender is indifferent between issuing the loan and investing in the risk-free bond. I follow the literature and assume that monetary policy affects the opportunity cost of lending by either controlling the real risk-free interest rate directly (in partial equilibrium,

[10]Excluding entrepreneurs from the risk-free bond market does not impose a loss of generality to the model, since production dominates the risk-free bond in equilibrium and the constraint on investing in the production technology rather than in the risk free bond would not bind. On the contrary, excluding lenders from accessing the production technology is required to generate borrowing and lending in equilibrium without modeling explicitly how agents end up on one side or the other in the credit market. See Quadrini (2011) for a review of the role of heterogeneous agents in economic models with credit markets.

[11]Assume for convenience that each entrepreneur borrows only from one lender. Whether lenders provide loans to only one entrepreneur or are allowed to diversify across loans is irrelevant with regard to the positive result on equilibrium default rates, although it would matter for a normative analysis since it affects whether lenders cover the opportunity cost of lending in expected value or in every state of the world. The aim of this paper is limited to isolate an economic force that potentially explains the empirical result by Jimenez *et al.* (2007), but is not suitable for a normative analysis as to the policy implications of this effect.

[12]I follow Carlstrom and Fuerst (1997) who consider this setting plausible by thinking of entry into lending as easier that entry into entrepreneurial activity.

this section) or by affecting it indirectly through some nominal price rigidities (in general equilibrium, section 4).[13]

Information is symmetric ex ante but asymmetric ex post. At the beginning of the period the contract is signed when idiosyncratic shocks on revenues have not been drawn yet. If at the end of the period the idiosyncratic shock was costlessly observed by both parties, state-contingent contracts would be available at the beginning of the period, allowing for the borrowing rate to be some optimal function of *ex post* entrepreneur's revenues. The costly state verification approach assumes instead that at the end of the period the entrepreneur observes this shock costlessly, while the lender observes it only if he pays an auditing dead-weight loss. This assumption captures the richer set of information that borrowers typically have relative to lenders. Under this assumption the debt repayment can be, at best, a function of the *expected* entrepreneur's revenues because a contingency of the borrowing rate on ex post revenues would lead the entrepreneur to opportunistically under-report the realization of ω in order to pay back a lower interest rate.

2.2 Maximization problem

Define R the opportunity cost of lending and assume the following linear production function:

$$y = \omega R^K K$$

[13] The result of the paper of a negative relation between the policy rate and firms' default probabilities does not rely on the assumption of perfect competition. In Dell'Ariccia, Laeven and Marquez (2010) the same result is driven by the fact that monetary expansions reduce the net return on lending by reducing the borrowing rate and increasing the monopolist bank's leverage ratio, which reduces the incentive to monitor and increases the default probability of the firm. In Valencia (2011) the result is driven by the fact that a lower opportunity cost of lending leads the monopolistic bank to extract more rent from firms by increasing lending, and this increases firms' leverage ratio and defaults. The advantage of the approach followed in this paper is not in the assumption of perfect competition, which is not necessarily more realistic that a monopolist, but that in the fact that the result is derived from first principles.

$R^K > R$ stands for the deterministic aggregate return on the risky technology and in this section is assumed known at the beginning of the period, while ω is the idiosyncratic shock with $\omega \in [0, \infty)$, $E(\omega) = 1$ and known cumulative distribution function $\Phi(\omega)$.[14] The realization of ω is costlessly observed by the entrepreneur, but is not observed by the lender unless he pays a constant fraction $\mu < 1$ of ex post revenues. Production is subject to full depreciation.[15]

Townsend (1979) shows that in this setting the optimal contract takes the form of a risky debt contract.[16] The entrepreneur borrows $K-N$ at the non-contingent gross interest rate R^B, where N represents the exogenous entrepreneurial net worth.[17] Given ex post revenues $\omega R^K K$ and $\omega \in [0, \infty)$, there exists an endogenous threshold value $\bar{\omega}$ of ω pinned down by $\bar{\omega} R^K K = R^B(K - N)$ below which revenues are not high enough to cover the debt repayment obligation. At the end of the period ω is realized. If $\omega > \bar{\omega}$ the entrepreneur pays back $R^B(K - N)$ and keeps profits $\omega R^K K - R^B(K - N)$. If instead $\omega < \bar{\omega}$, the entrepreneur defaults and the lender recovers $(1 - \mu)\omega R^K K$.

The contract maximizes the expected profit of the entrepreneur under the condition that the lender is indifferent between issuing the loan and investing at the risk-free rate. The maximization problem can be written as

[14] In this section I assume that the price of capital equals one. In the general equilibrium version of the model R^K is unknown when the contract is signed and will reflect the realization of aggregate productivity and monetary policy shocks.

[15] The result does not depend on the linearity assumption nor on the assumption of full depreciation, which only make the intuition more legible. Covas and Den Hann (2011) show that if the production function is concave, the entrepreneur's default probability becomes a function of net worth. Depreciation will be added in the general equilibrium version of the model, although for convenience it will not enter the production function but the aggregate return on capital R^K, as in Bernanke, Gertler and Gilchrist (1999). COMMENT IF ADD IT HERE. SOMEWHERE COMMENT THAT IT IS BETTER TO LEAVE LINEAR PRODUCTION FUNCTION

[16] The key intuition of the result by Townsend (1979) reliex on the fact that existence of a deadweight monitoring loss implies that it is efficient to minimize the probability of incurring it. A very brief explanation of this intuition is offered in Piffer (2013).

[17] In the general equilibrium version of the model N evolves through retained earnings. For simplicity the model does not allow for outside equity, as for instance in Gertler and Kyiotaki (2010). This remains an important limitation of the model, although it is shared with many other contributions in the literature.

$$\max_{\{\bar{\omega}, R^B, K\}} \int_{\bar{\omega}}^{\infty} \omega R^K K - R^B(K-N) d\Phi(\omega)$$

subject to

$$\bar{\omega} R^K K = R^B (K - N) \tag{1}$$

$$[1 - \Phi(\bar{\omega})] R^B + \Phi(\bar{\omega})(1-\mu) \frac{E(\omega \mid \omega < \bar{\omega}) R^K K}{K - N} \geq R \tag{2}$$

Equation (1) defines the threshold value $\bar{\omega}$ as a function of R^B and K. Equation (2) guarantees the indifference condition of the lender by ensuring that the expected return on lending (left-hand side) is at least as high as the opportunity cost of lending (right-hand side). Note that the expected return on lending equals the weighted average of what the lender obtains ex post depending on whether the default occurs or not.

To help develop the intuition and solve the maximization problem, define $F(\bar{\omega})$ and $G(\bar{\omega})$ the shares of expected revenues $R^K K$ to respectively the entrepreneur and the lender. These shares can be computed by substituting equation (1) into (2) and into the objective function.[18] $F(\bar{\omega})$ and $G(\bar{\omega})$ determine the allocation of net expected output (i.e. expected output net of expected monitoring costs) between the borrower and the lender, as shown in equation (5). An increase in the share of expected revenues promised to the lender ($G(\bar{\omega})$) implies a decrease in the share that goes to the entrepreneur ($F(\bar{\omega})$) and is associated with a decrease in the default threshold because it is harder for the entrepreneur to meet the higher repayment obligation to the lender.[19] The key result of the paper relies on this positive first

[18] Simple algebra gives

$$F(\bar{\omega}) = \int_{\bar{\omega}}^{\infty} \omega d\Phi(\omega) - [1 - \Phi(\bar{\omega})]\bar{\omega} \quad ; \quad G(\bar{\omega}) = 1 - F(\bar{\omega}) - \mu \int_0^{\bar{\omega}} \omega d\Phi(\omega) \tag{3}$$

with $F'(\bar{\omega}) < 0$, $F''(\bar{\omega}) > 0$ and, under Assumption 1, $G'(\bar{\omega}) > 0$, $G''(\bar{\omega}) < 0$.

[19] More precisely, $G(\bar{\omega})$ is increasing in $\bar{\omega}$ only in the lower support of $\bar{\omega}$, where the higher promised repayment share more than offsets the higher probability that the repayment will not

derivative of $G(\bar{\omega})$, because, other things equal, an increase in $G(\bar{\omega})$ on the one hand implies that a default is more likely, but on the other hand it is required to increase investment.

$$\underbrace{\left[1-\int_0^{\bar{\omega}} \omega d\Phi(\omega)\right] R^K K}_{\text{net expected output}} = R^K K \left[\underbrace{F(\bar{\omega})}_{\substack{\text{share to the} \\ \text{entrepreneur}}} + \underbrace{G(\bar{\omega})}_{\substack{\text{share to the} \\ \text{lender}}} \right] \tag{5}$$

$$\text{with} \quad F'(\bar{\omega}) < 0 \quad \text{and} \quad G'(\bar{\omega}) > 0$$

We can use equations $F(\bar{\omega})$ and $G(\bar{\omega})$ to eliminate R^B from the maximization problem and rewrite it as

$$\max_{\bar{\omega}, K} \quad F(\bar{\omega}) R^K K$$

$$\text{subject to} \quad \frac{G(\bar{\omega}) R^K K}{K-N} \geq R \tag{6}$$

Constraint (6) ensures that the expected return on lending (left-hand side) is at least as high as the opportunity cost of lending (right-hand side). Note that the expected return on lending: 1) increases in the share $G(\bar{\omega})$ to the lender because, given K, he receives a higher share of expected revenues and 2) it decreases in K because, given $G(\bar{\omega})$, an increase in investment pushes up the entrepreneur's leverage, implying a smaller buffer offered by the exogenous net worth to the risky loan.

be met. Since it would be suboptimal to agree on a $\bar{\omega}^*$ where $G(\bar{\omega})$ decreases (both parties would strictly benefit from a reduction in $\bar{\omega}$) we can disregard the decreasing part of $G(\bar{\omega})$. As shown in Bernanke, Gertler and Gilchrist (1999), condition $G'(\bar{\omega}^*) > 0$ is guaranteed by assuming that

$$\frac{d}{d\omega} \omega \frac{d\Phi(\omega)}{1-\Phi(\omega)} > 0 \tag{4}$$

which is satisfied for standard distributions, including the log normal distribution used later.

To solve the maximization problem substitute the constraint in the objective function and derive the optimality condition for $\bar{\omega}^*$: [20]

$$-F'(\bar{\omega}^*) = F(\bar{\omega}^*)\frac{G'(\bar{\omega}^*)}{\left(\frac{R^K}{R}\right)^{-1} - G(\bar{\omega}^*)} \quad ; \quad \text{implying} \quad \frac{d\bar{\omega}^*}{dR} < 0 \tag{7}$$

Equation (7) pins down the default threshold $\bar{\omega}^*$ (and hence the default probability $\Phi(\bar{\omega}^*)$) as a function of the discounted return to capital R^K/R. To close the model, substitute $\bar{\omega}^*$ in constraint (6) and compute the equilibrium level of investment K^*. Last, given $\bar{\omega}^*$ and K^*, compute the equilibrium borrowing rate R^B from equation (1).

2.3 Interpreting the optimality condition

The threshold value $\bar{\omega}^*$ pinned down by equation (7) is a decreasing function of R (see for instance Covas and Den Haan (2001), appendix C). This means that a decrease in the opportunity cost of lending *increases* the default probability of entrepreneurs, which is consistent with the empirical evidence by Jimenez *et al.* (2007) on new loans.

To understand the economic intuition behind this result it is convenient to isolate two different effects that follow a reduction R, the *direct effect on the borrowing cost* and the *indirect leverage effect.* To anticipate the intuition, the direct effect reduces the default probability because the lower opportunity cost of lending reduces the borrowing rate through perfect competition, and this mitigates the entrepreneur's debt burden. At the same time, though, the same reduction in the opportunity cost

[20] Assume that the maximization has an interior solution for K, which requires that, in the optimum, $\bar{\omega}$ satisfies

$$R^K < \frac{R}{G(\bar{\omega})}$$

If this was not the case, the aggregate return R^K would be high enough to make the asymmetric information irrelevant because the lender would be willing to supply an infinite amount of credit.

of lending increases the discounted return to capital and increases the optimal level of investment, which pushes up the leverage ratio of the entrepreneur and *increases* defaults. The negative equilibrium relationship between $\bar{\omega}$ in R reflects the fact that in the costly state verification model the indirect effect always dominates, independently on the parameter values.

Isolating the direct effect

To isolate the direct effect on the borrowing cost, start from the initial equilibrium and shut down the indirect leverage effect by assuming that the level of investment K remains fixed at the initial equilibrium level. Under this assumption, study the effect of a reduction in the opportunity cost of lending.

Figure 1: Direct effect: $R \downarrow$, K constant

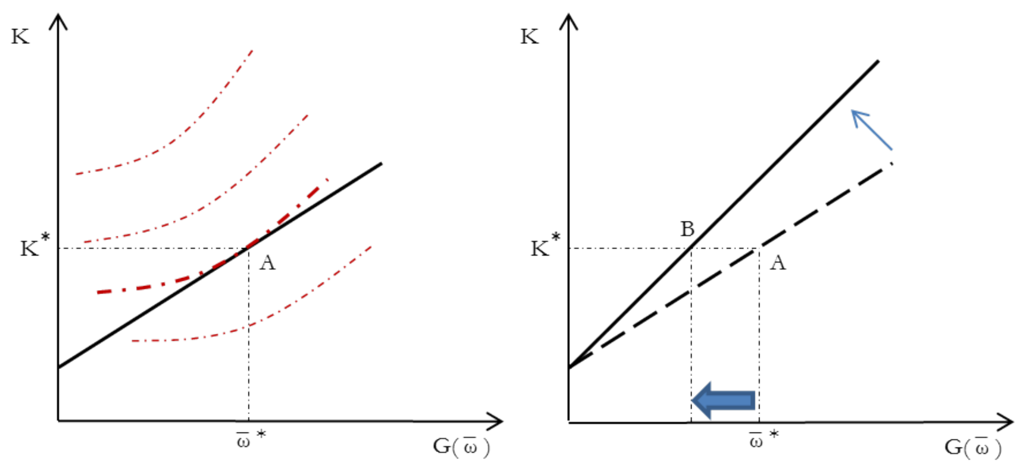
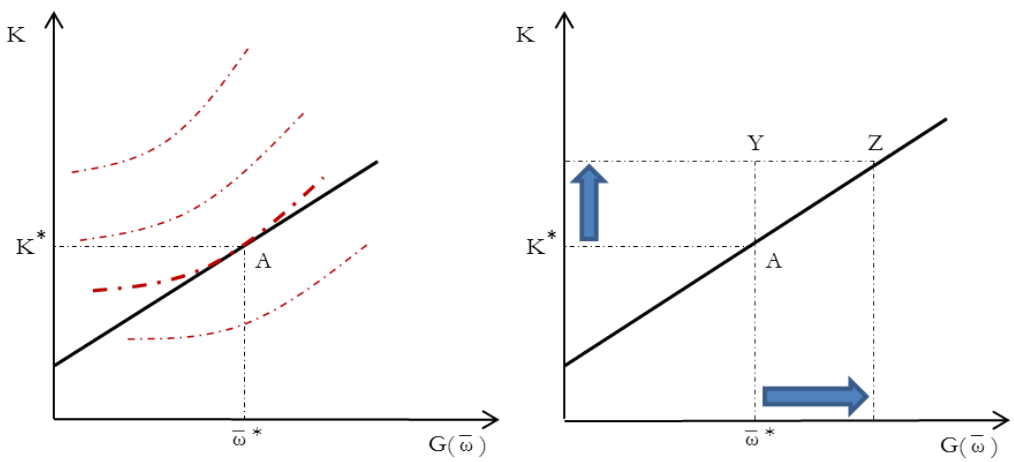


Figure 1, left panel, shows the initial equilibrium (point A) as the point on constraint (6) (upward-sloping line) that corresponds to the optimal level of $\bar{\omega}$ pinned down by equation (7). The combinations of $\{K, G(\bar{\omega})\}$ below the line represent combinations that satisfy the indifference condition of the lender. When R decreases (right panel) the constraint shifts upwards and increases the sets of possible combinations because the lender requires now a lower expected return on lending. If K is arbitrarily constant the upward shift in the constraint moves the equilibrium to point B where $G(\bar{\omega})$ and the corresponding default probability have decreased due to the lower cost of borrowing. If this was the only effect in place, the reduction in R would decrease the default probability.

Isolating the indirect effect

To isolate the indirect leverage effect, start from the initial equilibrium and shut down the effect on the borrowing cost by keeping the opportunity cost of lending unchanged. Consider then what happens when the entrepreneur is unsatisfied with the level of K and decides to borrow more and increase investment.

Figure 2: Indirect effect: R constant, $K\uparrow$



Constraint (6) shows that if the constraint is initially binding, an increase in K is incompatible with the indifference condition of the lender unless the share $G(\bar{\omega})$ of expected revenues promised to the lender increases. The reason of this effect can be seen by rewriting constraint (6) in terms of an upward limit to the entrepreneur's leverage ratio:

$$\frac{K}{N} \leq \frac{1}{1 - \frac{R^K}{R} G(\bar{\omega})} \tag{8}$$

From equation (8) it is immediate to see that the maximum entrepreneurial leverage that the lender is willing to accept is an increasing function of $G(\bar{\omega})$. This is intuitive: given net worth, the entrepreneur can invest more only by borrowing more, but this increases his leverage which indirectly reduces the relative buffer that net worth provides to the risky loan. To compensate the lender for this leverage effect the borrower must pay a leverage premium that takes the form of a higher $G(\bar{\omega})$. The increase in $G(\bar{\omega})$ pushes up the default rate because it is harder for the lender to meet the higher repayment obligation, but it is necessary to convince the lender to issue more credit and allow the entrepreneur to expand investment.[21]

This intuition is shown in figure 2. Given an unchanged level of R, if $G(\bar{\omega})$ remains constant the increase in K moves the equilibrium above the constraint to point C where the indifference condition of the lender is violated. To convince the lender to issue more credit to fund the higher level of investment an increase in $G(\bar{\omega})$ is required, as shown in point D.

Combining direct and indirect effect

Consider now the direct and the indirect effects together. We have already seen in figure 1 that following a decrease in R, perfect competition pushes down the expected cost of borrowing and shifts the constraint upwards. At the same time, though, it increases the optimal level of investment because it increases the discounted value R^K/R of end-of-period aggregate return to capital R^K. Whether one effect is stronger than the other depends on the relative importance attached

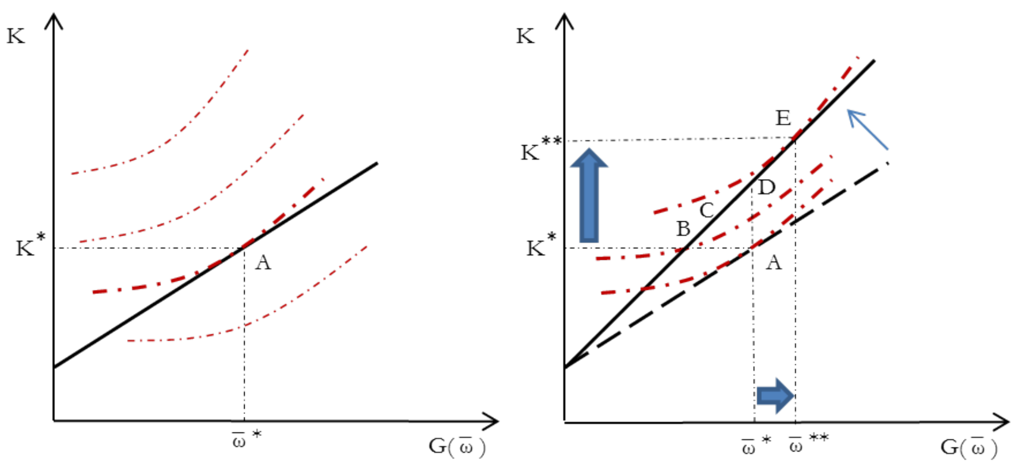
[21] The leverage premium is defined here as the semi-elasticity of $G(\bar{\omega})$ with respect to the leverage ratio of the entrepreneur. The implicit function theorem implies

$$\frac{dG(\bar{\omega})}{\frac{dK/N}{K/N}} = \underbrace{\left(\frac{R^K}{R}\right)^{-1} - G(\bar{\omega})}_{\text{Leverage Premium}} > 0$$

i.e., a 1 % increase in the leverage ratio of the entrepreneur requires that the share of expected revenues to the lender increases by $\left[\left(\frac{R^K}{R}\right)^{-1} - G(\bar{\omega})\right] * 100$ basis points. This premium decreases in the discounted return to capital and in the share$G(\bar{\omega})$.

by the entrepreneur in running a bigger investment relative to accepting a higher repayment rate and a higher default probability. This trade-off was captured in the optimality condition.[22] As clearly shown in figure 3, in this model when R decreases, the entrepreneur is willing to substitute out from small investment into higher default probability in order to take advantage of the higher discounted return to capital over and above what would be implied by the simple decrease in R (point F). In fact, the optimality condition (**??**) prescribes a new equilibrium at a higher level of $\bar{\omega} = \bar{\omega}^{**} > \bar{\omega}^{*}$, as shown in point D. This is the key result of the paper, synthesized in Proposition 1 below:

Figure 3: Both direct and indirect effect: $R \downarrow$



PROPOSITION 1: *In partial equilibrium a decrease in the opportunity cost of lending pushes down the borrowing rate and, other things equal, decreases the default probability of entrepreneurs by attenuating their debt burden. At the same time, though, it increases the discounted return to capital and induces the entrepreneur to borrow more and increase leverage. To compensate the lender for a higher leverage ratio*

[22] From equation (7), a marginal increase in ω decreases the expected share to the lender by $F'(\bar{\omega})$but increases investment by $\frac{G'(\bar{\omega}^*)}{\left(\frac{R^K}{R}\right)^{-1} - G(\bar{\omega}^*)}$, which marginally increases the expected profits of the lender by $F(\bar{\omega})$.

a leverage premium must be paid, which dampens the reduction in the borrowing cost and increases the default probability. In the costly state verification model the indirect leverage effect always dominates on the direct effect on the borrowing cost, implying that defaults increase following a reduction in the opportunity cost of lending.

2.4 Numerical exercise

In this section I do a numerical exercise on the model from the previous section to help develop of the intuition behind Proposition 1. The full calibration of the model will be presented in section 3, after introducing its elements of dynamic general equilibrium. Here, the calibration is chosen to simplify the intuition as far as possible. To this purpose, net worth is normalized to 1. The initial opportunity cost of lending is arbitrarily set at 2 % and the return to capital at 4 %, implying a discounted return to capital Rk/R of 1.0196, i.e. 1.96%. The variance of the idiosyncratic shock and the observation cost for the lender are calibrated to match an equilibrium leverage ratio equal to 2 and an initial default probability of 3 %. Note that the normalization of net worth to 1 implies that investment and leverage ratio coincide.

The calibration and the initial equilibrium is shown in table 1, while table 2 shows the case of a 1 % decrease in R. All combinations from figures 1 and 3 are reported, where combinations differ for how much the entrepreneur increases investment in response to the higher discounted return to capital. Basis point variations from the initial equilibrium are reported in parenthesis to simplify the comparison.

In the initial equilibrium, the entrepreneur borrows 1 from the lender at an interest rate of 2.78 % and invests almost 2. Expected revenues from investment

Table 1: Calibration

	Parameter	Value
Opportunity cost of lending	R	1.02
Aggregate return to capital	R^K	1.04
Entrepreneurial net worth	N	1
Variance of e^ω	σ	0.3434
Observation cost for the lender	μ	0.1655
	Variable	**Value**
Investment	K	2
Leverage	K/N	2
Borrowing rate	R^B	1.0278
Repayment share	$G(\bar{\omega})$	0.4901
Expected revenues	$R^K K$	2.08
Repayment threshold	$\bar{\omega} R^K K$	1.0278
Default threshold	$\bar{\omega}$	0.4938
Default probability	$\Phi(\bar{\omega})$	0.03

equal 2.08, but ex post revenues could range from 0 to infinity depending on the realization of the idiosyncratic shock. At the end of the period, if ex post revenues are above the repayment obligation (1.0278) the entrepreneur pays back his debt and retains profits, otherwise he defaults and the lender obtains revenues net of monitoring costs. The probability that the entrepreneur will default equals 3 %.[23]

Consider what happens when the opportunity cost of lending decreases from 2 % to 1 %. If the entrepreneur decides to invest the same amount (point B), perfect competition reduces the borrowing rate by 107 basis points, which pushes down defaults by 20 basis points. Despite being theoretically convenient, a constant level of investment is unlikely to be an optimal choice, because the reduction in the risk free rate has pushed the discounted return to capital up from 1.96 % to 2.97 %, making each unit of investment more productive in real expected terms. The entrepreneur takes advantage of this by investing more, but this increases his leverage and attenuates the reduction in the borrowing rate because the lender anticipates the fact that net worth provides now the same buffer to a bigger loan.

[23]The condition for an internal solution is satisfied, since $G(\bar{\omega})R^K = 0.8695 < 1.0200 = R$.

Table 2: A 1 % decrease in the opportunity cost of lending

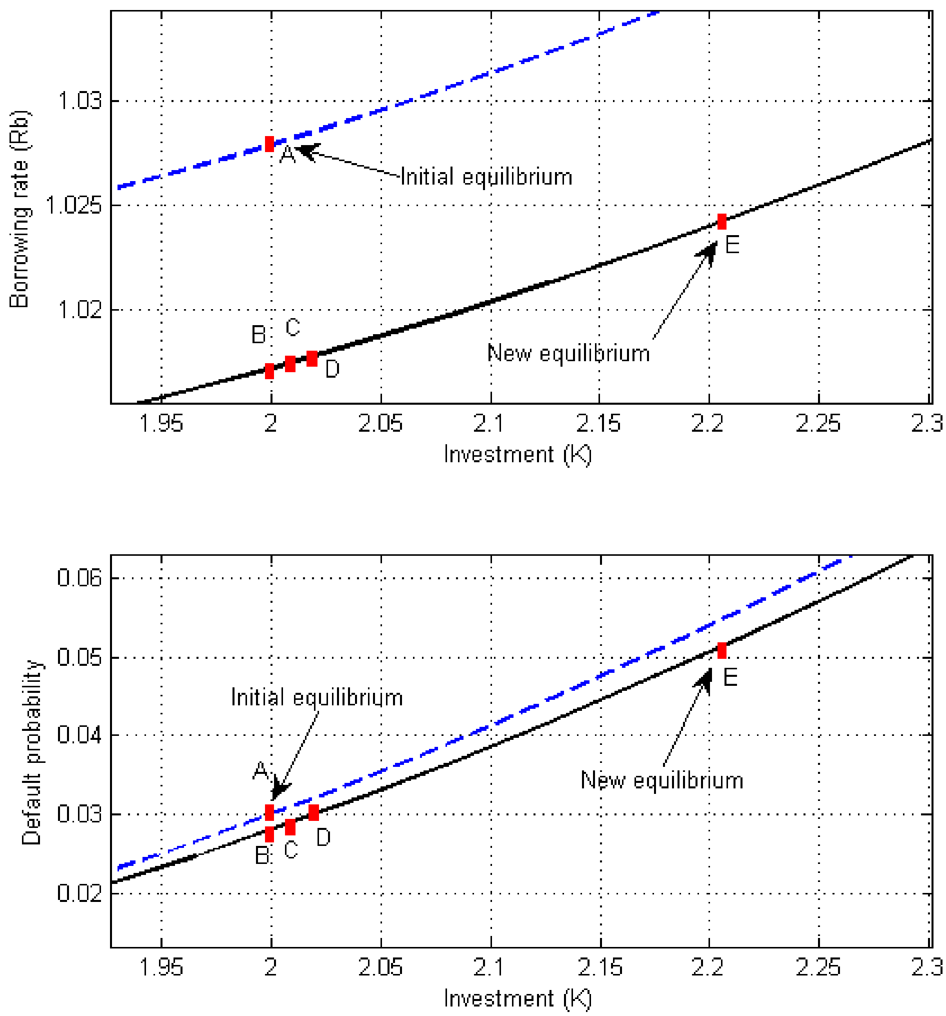
	R	$K \equiv K/N$	R^B	$G(\bar{\omega})$	$\Phi(\bar{\omega})$	$F(\bar{\omega})R^K K$
A	1.02	2	1.0278	0.4901	0.03	1.0556
B	1.01	2	1.0172	0.4853	0.0280	1.0659
		(0)	(-107)	(-48)	(-20)	
C	1.01	2.01	1.0175	0.4878	0.0290	1.0660
		(50)	(-104)	(-24)	(-10)	
D	1.01	2.02	1.0178	0.4901	0.03	1.0661
		(100)	(-101)	(0)	(0)	
E	1.01	2.2057	1.0242	0.5309	0.0514	1.0671
		(1034)	(-36)	(407)	(214)	

The overall effect on defaults will depend on whether the increase in leverage is strong enough to offset the direct effect on the borrowing cost.

If the entrepreneur increases investment by 0.01 (point C) the increase in leverage is not strong enough to dominate and defaults still decrease, although by less (10 basis points instead of 20). Of course the entrepreneur is not constrained to borrow only 0.01 more. If he finds it optimal to borrow 0.02 more his leverage ratio would increase even more rapidly, leading to a even smaller reduction in the borrowing rate. The key intuition of the paper becomes then clear: if the increase in leverage is strong enough to offset the direct effect of a lower opportunity cost of lending, the equilibrium default probability would *increase* despite the ultimate decrease in the borrowing rate. In this model it is optimal for the entrepreneur to increase investment by 10.34 % (point E) despite the fact that this increases leverage by enough to dampen the reduction in the borrowing cost and ultimately lead to an increase in the default probability of 214 basis points. Note that it is optimal to increase investment by more that what merely implied by the decrease in R (point D). The lender is indifferent as which level of investment the entrepreneur chooses, since the loan will be priced accordingly to meet the indifference condition.

The entrepreneur instead is better off by accepting a higher default probability in exchange of a higher investment up to point E, as shown by the increase in his expected profits (last column of table 2).

Figure 4: A 1 % decrease in the opportunity cost of lending



This result becomes clearer in figure 4, which shows the combinations of investment and borrowing rates that satisfy the indifference condition of the lender (top graph) and the corresponding entrepreneur's default probability (lower graph). The initial equilibrium is shown in point A, as in table 1 and figure 1. When R decreases from 2% to 1% the lender is willing to accept a lower borrowing rate for each level of investment and this shifts down the indifference condition and the default curve. Potentially, the entrepreneur could stick to the same level of investment and benefit from a lower default probability (point B). In the model it is instead optimal to move along the new indifference condition in order to benefit from the higher discounted return to capital. The new equilibrium is found at point E, where the borrowing rate has decreased by less if compared to the case of a constant leverage, and the default probability has ultimately increased.

2.5 Comparing the model to the case of symmetric information

The two previous sections argued that in partial equilibrium the indirect leverage effect that follows a reduction in R exerts an upward pressure on equilibrium defaults which dominates in the costly state verification model and pushes up equilibrium default. In this section I show that the result is driven by asymmetric information because if information was symmetric, the indirect leverage effect would disappear. This result is well known in the literature and follows from the Modigliani-Miller theorem.

So far a default was defined as a state where the lender incurs an observation cost to uncover the realization of the idiosyncratic shock. With symmetric information this observation cost becomes irrelevant and agents sign a contract in which the borrowing rate is set contingent on the realization of ω.

With symmetric information the contract solves

$$\max_{K,R^B,\bar{\omega}} \int_0^\infty \omega R^K K - R^B(\omega)(K-N)d\Phi(\omega)$$

$$\text{subject to} \int_0^\infty R^B(\omega)d\Phi(\omega) \geq R \tag{9}$$

$$\omega R^K K \geq R^B(\omega)(K-N), \forall\omega \tag{10}$$

Constraint (9) guarantees the indifference condition of the lender, constraint (10) ensures that the repayment scheme is feasible.

Substitute constraint (9) in the objective function and rewrite the maximization problem as

$$\max_K \ (R^K - R)K + RN \tag{11}$$

$$\text{subject to } \omega R^K K \geq R^B(\omega)(K-N), \ \forall\omega$$

Following from agents' risk neutrality, the entrepreneur is indifferent to the specific repayment scheme agreed in the contract as long as the borrowing rate does not exceed R in expectations. As for the optimal repayment scheme, there exist infinitely many sets of $\{R^B\}_{\omega=0}^{\infty}$ that satisfy the feasibility constraint. One possible scheme is to repay the proportion $\varphi < 1$ of ex post revenues, i.e. $R(\omega) = \ \omega R^K K$. This scheme is obviously feasible. Substituting it in the optimality condition and computing the optimal φ gives $R^B(\omega) = \omega R$.

It is important to note that the optimal level of borrowing is infinite under the assumption of $R^K > R$, independently on the specific repayment scheme $\{R^B(\omega)\}_{\omega=0}^{\infty}$. We saw that under asymmetric information the lender requires a leverage premium as compensation for a higher entrepreneur's leverage ratio. With symmetric information, instead, this effect disappears because the borrower's leverage becomes irrelevant from the point of view of the lender, as long as the repayment scheme equals R in expectation. This is just an application of the Modigliani Miller theorem.[24] Under symmetric information the entire endowment of the economy (the entrepreneurs' net worth and the entire lenders' wealth) is invested in the production function. This was not the case in section 2.2, where credit markets reached an equilibrium in which part of the lender's wealth is still invested in the risk-free bond. The role of asymmetric information in delivering the result from Proposition 1 is synthesized in Proposition 2:

> PROPOSITION 2: *If information was symmetric, defaults would not occur since agents would agree on a state-contingent repayment scheme, the Modigliani-Miller theorem would hold and the lender would be indifferent to the level of entrepreneur's leverage, as long as the opportunity cost of lending is covered in expected value.*

[24] With decreasing returns to investment the maximization problem rewrites as

$$\max_{K,R^B(\omega)} \int_0^\infty \omega R^K K^\alpha - R^B(\omega)(K-N)d\Phi(\omega)$$
$$\text{subject to } \int_0^\infty R^B(\omega)d\Phi(\omega) \geq R$$
$$\omega R^K K \geq R^B(\omega)(K-N),\ \forall\omega$$

which in equilibrium implies

$$K = \left(\alpha \frac{R^K}{R}\right)^{\frac{1}{1-\alpha}} \quad ; \quad R(\omega) = \omega R$$

The entrepreneur optimally demands a level that increases in the discounted return to capital. Note that the repayment scheme is independent on the entrepreneur's leverage.

3 A dynamic general equilibrium extension of the model

So far I have departed from the intuition that lower policy rates decrease default probabilities on new loans by making debt cheaper and I have argued that this effect does not dominate if the borrower increases his leverage ratio too much. The analysis kept constant some important variables of the debt contract in order to isolate the forces in play. In doing so it derived a result that is static and could potentially be lost if one accounts for the dynamic response of entrepreneurial net worth from the second period onwards. Additionally, the result might be not robust to general equilibrium effects, for reasons that will be explained shortly.

In this section I address such concerns by extending the model to a dynamic general equilibrium model. The exercise delivers results that are in line with the empirical finding by Jimenez *et al.* (2007). In the model, the default probability of loans taken *after* the monetary expansion occurs still increases, suggesting that dynamic general equilibrium considerations do not invalidate Proposition 1; at the same time the default probability of loans taken *before* the shock and still outstanding when the monetary expansion occurs *decreases* due to a general equilibrium effect on the price of capital.

3.1 Why a dynamic general equilibrium model is needed

To see the usefulness of extending the partial equilibrium model to a dynamic general equilibrium framework, synthesize the partial equilibrium result by rewriting the default probability $\Phi(\bar{\omega}^*)$ from section 2 as a function of the discounted return to capital and the entrepreneur's leverage:[25]

$$\Phi(\bar{\omega}^*) = f\Big(\frac{R^K}{R}, \frac{QK}{N}\Big), \quad \text{with} \quad \underbrace{f'_{R^K/R} < 0}_{\text{Direct effect}}; \ \underbrace{f'_{QK/N} > 0}_{\text{Indirect effect}} \tag{12}$$

Q stands for the price of capital, which in the previous section was set equal to 1. If leverage QK/N remains constant, a decrease in R increases the discounted return to capital R^K/R and reduces the borrowing cost and the default probability (direct effect). Since investment is not constrained to be constant, the entrepreneur reacts to a decrease in R by borrowing more and increasing his leverage, leading the lender to demand a leverage premium which pushes up the default probability (indirect effect). In partial equilibrium, the indirect leverage effect prevails, as synthesized in Proposition 1.

From equation (12) we see that whether the default probability still increases in the dynamic general equilibrium depends on the behaviour of N, Q and R^K. There are at least three forces that potentially revert the result.

First, it is unlikely that R^K and Q remain constant after a monetary expansion. If capital was the only input and one assumes decreasing returns to capital, a monetary expansion would likely reduce R^K, dampening the direct effect on the borrowing cost. Instead, if R^K is positively affected by the monetary expansion, say because the price of capital increases or because more labour is used in production, a monetary expansion might *increase* R^K, implying that the direct effect might be stronger than in partial equilibrium and could potentially prevail on the indirect leverage effect.

[25] WRITE DOWN EXPLICIT FUNCTION USING DIFFERENCE FROM STEADY STATE AND EXPRESSION

$$\Phi(\bar{\omega}^*) = \int_0^{\Gamma^{-1}} (\frac{R^K}{R}) f(\omega) d\omega$$

where $f(\omega)$ is the probability density function of the idiosyncratic shock and $\Gamma^{-1}(\bar{\omega})$ is the inverse function of $\Gamma(\bar{\omega}) = \Big(G(\bar{\omega}) - \frac{G'(\bar{\omega})}{F'(\bar{\omega})} F(\bar{\omega})\Big)^{-1}$, with $\Gamma'(.) > 0$.

Second, adjustment costs on investments are considered important and realistic features of investment decisions by firms. This view has been well-received in the macroeconomic theory at least since Tobin developed his investment theory. If the increase in capital is subject to investment adjustment costs, the build up of leverage might be less rapid than in the partial equilibrium model, dampening the intensity of the indirect leverage effect.

Third, firms' net worth does not remain constant following a monetary expansion because the increase in aggregate demand realistically increases firms' revenues. Other things equal, if this leads to a significant accumulation of net worth, the entrepreneur's leverage might ultimately decrease, making the indirect leverage effect work towards a *decrease* the default probability.

As I will argue, the first remark is correct and is at the origin of the decrease in the default probability of outstanding loans. The intuition behind this result is that in the model a monetary expansion unexpectedly increases asset prices and delivers an ex post aggregate return to capital that exceeds its expected value for the first period after the shock, generating an unexpected increase in revenues that reduces defaults of outstanding loans. The second and third remarks, instead, are irrelevant for the validity of Proposition 1 because in this model what matters for defaults is the leverage ratio, not the level of capital in itself, and because the accumulation of net worth leads capital to increase even more, implying that leverage still increases.

3.2 The full model

The main departure of the model in this paper from existing general equilibrium monetary models is that, on top of stressing defaults instead of inflationary pressures as the marginal costs of monetary expansions, it takes the policy rate as exogenous and chooses the calibration is order to match moments of the Spanish economy. Both of these choices, commented in greater detail below, are necessary to establish a quantitative comparability between the model and the empirical results by Jimenez *et al.* (2007). For the rest, the model is fairly standard and shares features of many existing models.[26] Standard features will only be briefly commented here, unless they have an important role in driving the result. The equations of the model are provided in appendix A.

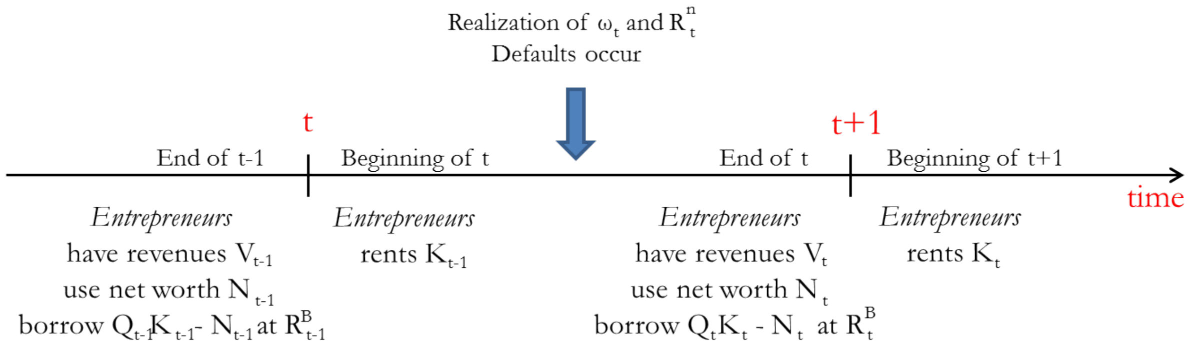
Timing assumption

The timing of the model is crucial in generating the result and is taken from Bernanke, Gertler and Gilchrist (1999) and is shown in figure 5. Entrepreneurs borrow from lenders to buy capital one period before it is used. Debt contracts are signed based on the mathematical expectation of the return to capital. Agents know that the ex post realization of the return to capital can potentially differ from its expected value because aggregate uncertainty enters the general equilibrium model due to the policy shock. Entrepreneurs are willing to bear this uncertainty due to their risk neutrality, implying that the borrowing rate is not a function of the realization of the aggregate shock. The ex post return to capital is also subject to idiosyncratic uncertainty. Since information on this shock is asymmetric as in section 2, the borrowing rate cannot be a function of the idiosyncratic shock either. A non-contingent value of R^B on neither the idiosyncratic shock nor the aggregate shock implies that the default threshold $\bar{\omega}$ is a function of the aggregate shock, as can be seen from equation (1).

The dependency of the default threshold on the ex post realization of R^K has an important implication in generating the result on outstanding loans. When the contract is signed agents form an expectation of the default probability. Whether the actual rate of default will be below or above the expected one depends on whether the ex post return to capital is respectively above or below its expected

[26]The paper is mostly related to Bernanke, Gertler and Gilchrist (1999), but also CEE and SW? Woodford, standard New Keynesian?

Figure 5: Timing assumption



values. This effect is quite realistic and detects the fact that unexpected increases in firms revenues reduce their default probability.[27]

Agents

The model consists of 6 agents: households, lenders, entrepreneurs, capital producers, intermediate good producers and retailers.

Households are risk averse, they derive utility from a basket of substitutable consumption goods and from leisure. They have a log utility function which displays habit formation. They are infinitely lived and postpone consumption through the financial services of lenders who act as intermediaries.[28] These services take the form of deposits, which pay the nominal risk-free rate R_t^n directly controlled by the central bank. R_t^n is defined as the risk free rate between period t and period $t+1$.

Lenders are risk neutral, they raise deposits from households and provide loans for the amount of $Q_{t-1}K_{t-1} - N_{t-1}$ to entrepreneurs at the borrowing rate R_{t-1}^B.[29]

[27]In the model, the monetary expansion increases aggregate demand, but only its investment components matters in unexpectedly increasing revenues and reducing defaults because only entrepreneurs borrow in this model, not retailers. This assumption simplifies the analysis, although it leaves some interesting part of the effect on defaults unexplored.

[28]By separating households from lenders we can solve the costly state verification contract by assuming that both agents in the contract are risk neutral, as in section 2 while still deriving an Euler equation for the model. Since the return on lending ultimately goes to households, aggregate uncertainty is borne by the entrepreneurs despite the lenders being risk neutral.

[29]The convenience of separating households from lenders is that the debt contract is solve under risk neutrality, while still having a Euler equation

Whether at time t this borrowing rate will be actually paid or not by the entrepreneur depends on whether he defaults or not on the loan. Individual entrepreneurs can potentially default, but lenders never default because they are assumed to perfectly diversify idiosyncratic risk. Since the lender and the entrepreneur shield households from both the idiosyncratic and the aggregate risk and since lending is assumed to be a competitive activity, the relevant opportunity cost of lending is the real value of the nominal interest rate R^n_{t-1}. Lenders do not consume in equilibrium since they make zero profit.

Entrepreneurs are risk neutral. At time $t-1$ they own net worth N_{t-1} accumulated from retained earnings from previous periods and borrow from lenders to buy K_{t-1} units of capital at unit price Q_{t-1} from capital producers. Capital is then rent in time t to intermediate good producers on competitive markets. The ex post return on capital R^K_t equals the rental rate on capital plus the capital gain from non-depreciated capital, as from equation (13):

$$R^K_t = \frac{r^k_t + (1-\delta)Q_t}{Q_{t-1}} \tag{13}$$

The ex post return R^K_t if affected by aggregate uncertainty because an unexpected variation in the nominal risk-free rate R^n_t affects the investment decisions at time t and unexpectedly moves price Q_t (more on this later). The ex post return to capital that the entrepreneur actually receives is $\omega_t R^K_t$, i.e. the aggregate return to capital R^K_t after it has been hit by the idiosyncratic shock ω_t.[30]

Non-defaulting entrepreneurs obtain aggregate revenues

$$V_t = F(\bar{\omega}_t) R^K_t Q_{t-1} K_{t-1}$$

[30] Contrary to the partial equilibrium model, ω_t is not a structural parameter from the production function but enters the model as an *ad hoc* shock to the market return to capital R^K_t. The reason for this is to make the model tractable enough. Covas and den Haan (2011) avoid taking this strong assumption ....

A fraction $1-\gamma$ of such entrepreneurs is assumed to retire and the same mass $1-\gamma$ of entrepreneurs is born to keep the ratio of entrepreneurs to households constant. The literature takes this assumption to avoid that the accumulation of net worth is strong enough to let the entrepreneur entirely self finances investment.[31] Being risk-neutral, the entrepreneurs who remain in business until the next period allocate their revenues into next-period net worth. They also provide labour services and invest their wage W_t in the purchase of units of capital. Entrepreneurial net worth at the end of period t is given by

$$N_t = \gamma V_t + W_t$$

Entrepreneurs rent capital to intermediate good producers, who use it together with labor input in a standard Cobb-Douglas production function. Intermediate goods are then sold to retailers on competitive markets.

Capital producers buy non-depreciated capital from the market, invest in new units of capital and sell the new stock to capital holders. Their investment technology is subject to adjustment costs, a feature that allows a time-varying price of capital. The adjustment cost function is borrowed from Christiano, Eichenbaum and Evans (2005) and is shown in appendix A.

Retailers buy intermediate goods and assembly them into final goods. Since final goods are viewed as imperfect substitutes by households, retailers have some price-making power. Under the assumption of Calvo price setting, this nominal rigidity gives monetary policy real effects in the short term.

[31]The literature refers to these entrepreneurs as dying, although the retiring option is probably more neutral.

3.3 Modeling an exogenous policy rate

The monetary shock enters at time t by changing the risk-free nominal interest rate R_t^n between t and $t+1$. As such, it affects the opportunity cost of lending and the investment decisions starting from period t onwards.

The literature typically assumes that the central bank controls the risk-free rate through a feedback rule in the spirit of Taylor (1983). In this model such an approach would not be advisable because it would make the interest rate partially endogenous, when the result is to be compared quantitatively with the empirical result from Jimenez *et al.* (2007) whose identification strategy relies on the exogeneity of the policy rate. The authors argue this to be a realistic scenario for Spain because the policy rate (proxied by the European market interbank overnight rate) is set in Frankfurt and not in Madrid.

To be consistent with their approach, in this paper I model the evolution of the policy rate by extracting the same time series used in Jimenez *et al.* (2007) in the same time period (Deutsche interbank rate between 1986 and 1998, European Overnight Index Average (Eonia) between 1999 and 2006) and by modeling it as an autoregressive process. The details of this estimation are provided in appendix B. The dynamic equation for the policy rate used in the impulse-response exercise below is

$$\frac{R_t^n}{R_{ss}^n} = \left(\frac{R_{t-1}^n}{R_{ss}^n}\right)^{\rho_i} e^{\epsilon_t^R} \tag{14}$$

with $\rho_i = 0.9914$ the estimated parameter from the autoregressive process of order one and $R_s^n s = 1.04113$ the sample average of the gross policy rate.

3.4 Calibration

Some parameter values are taken from the literature. The rest are calibrated quarterly to match the empirical moments available on Spanish data. The calibration, shown in table 3, should be thought as the midpoint of the wider range of parameter values that will be considered in section 3.6 to assess the sensitivity of the result with respect to the calibration used.

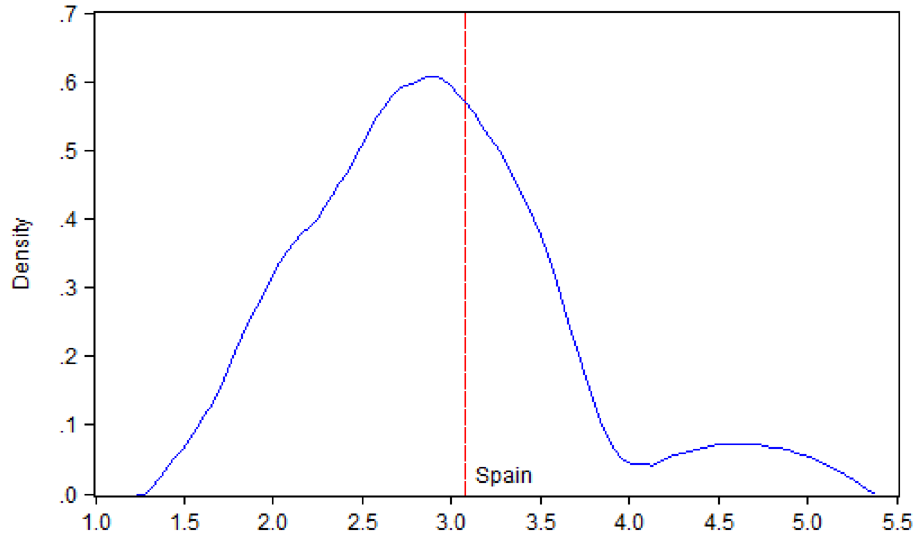
Table 3: Calibration

	Parameter	**Value**	**Calibrated**	**Source**
Habit formation	h	0.5	✗	-
Fisher elasticity of labour	η	1	✗	-
Marginal product of K	α	0.35	✗	-
Depreciation rate	δ	0.25	✗	-
Capital adjustment cost	ν	3	✗	-
Prob of price non-optim.	ψ	0.75	✗	-
Habit formation	h	0.5	✗	-
El of subst. of varieties	ϵ	6	✗	-
Discount factor	β	0.9898	✓	-
Variance of e^{ω}	σ	0.1562	✓	-
Observation cost	μ	0.10	✓	-
Prob. entrepreneur retires	$1-\gamma$	0.0210	✓	-
Weight on disutility of labour	χ	17.4319	✓	-
Policy rate smoothing	ρ_i	0.9914	✓	-
	Moment	**Model**	**Data**	**Source**
Annual default rate	$\Phi(\bar{\omega})$	0.0240	0.0240	JOPS (2007)
Leverage ratio in Spain	K/N	2.9490	3.0236	KOSY (2011)
Annual risk free rate	R^n	0.0411	0.0411	Bundesbank
Discounted return to K	R_K/R	1.0049	1.0049	BGG (1999)

Calibrated parameters

The discount factor of households is set so that in steady state the nominal risk-free rate R^n equals the quarterly sample average of the German and European interbank lending rate. In the model, inflation is zero in steady state, implying that in steady state R^n and R coincide. Despite being unconventional, I choose to calibrate β to

Figure 6: Distribution of median leverage ratios, European Monetary Union



match the moment of the nominal rather than the real interbank rate because the impulse response functions of the model will be compared with the empirical hazard function in Jimenez *et al.* (2007), which is estimated using the nominal rather than the real interest rate. This point is commented further in section 3.7. The annual nominal interbank interest rate in the relevant period equals 4.1131 %, implying a quarterly discount rate of 0.9898.

The spread between the real return on the risk free bond and the average return on capital is set equal to 200 basis points annually, as in Bernanke, Gertler and Gilchrist (1999) (BGG in table 3). An alternative approach would be to calibrate the moment to match the empirical borrowing rate R^B instead of the aggregate return to capital. This strategy was not viable because, to the best of my knowledge, information on the borrowing rate charged by Spanish banks on loans is not publicly available.

The variance of the idiosyncratic shock and the observation cost μ affecting the costly state verification model are calibrated so that the firms' default probability and their leverage ratio matches the empirical moments for Spain. Specifically, the annual loan default probability of firms is taken from the estimation by Jimenez, Ongena, Peydro and Saurina (2007) (JOPS in table 3), who find a quarterly default probability of 0.6 % for loans of the median firm in their dataset (COMMENT 1 QUARTER VS 5 QUARTERS MATURITY) (BGG use 0.0075, i.e. 3 % annually). The leverage ratio of Spanish firms is taken instead from Kalemli-Ozcan, Sorensen and Yesiltas (2011) (KOSY in table 3). The authors construct a dataset that is particularly appropriate to match the leverage ratio in my model because, by comprising a wide heterogeneity of firms, their dataset is comparable to the dataset used by Jimenez *et al.* (2007).[32] The value that I used to match the steady state leverage ratio in the model is the weighted average of the median value of listed and non-listed non-financial Spanish firms, which equals 3.0785.[33] The value lies in the average median leverage ratio of non-financial firms in the European Monetary Union, as shown in the distribution shown in figure 6 computed using a Kernel density estimation.[34] The Spanish median leverage ratio is above the one for the US, which equals 2.0524.[35] BGG calibrate the model around a leverage ratio equal to 2.

Non-calibrated parameters

The rest of the parameters values are borrowed from the literature that calibrates models on the US economy, for I do not have reasons to believe that they would significantly differ with respect to the Spanish economy. These parameters are

[32]The inconvenience of the dataset by Kalemli-Ozcan, Sorensen and Yesiltas (2011) is that it covers the period between 2000 and 2009, which falls short of the period for which I calibrate the default probability of firms. Other available empirical estimates would arguably be less suitable for the calibration of this paper because they range on an even shorter period of time and focus on only one type of firms :Reverte (2009) studies firms listed in the IBEX35 index in year 2005 and 2006, Inchausti (1997) concentrates on firms listed on the Valencia Stock Exchange, Garcia-Teruel and Martinez-Solano (2007) focus on small and medium enterprises in the manufacturing sector, Ferreira and Vilela (2004) study publicly traded firms between 1987 and 2000.

[33]The mean value was 9.0443, which was found to be heavily biased upwards due to relatively few outliers.

[34]The dataset does not include Cyprus, Greece and Malta.

[35]I am grateful to the authors for having kindly provided me with these statistics.

standard in the literature and are reported in table 3. (COMMENT EACH AND COMPARE TO THE LITERATURE)

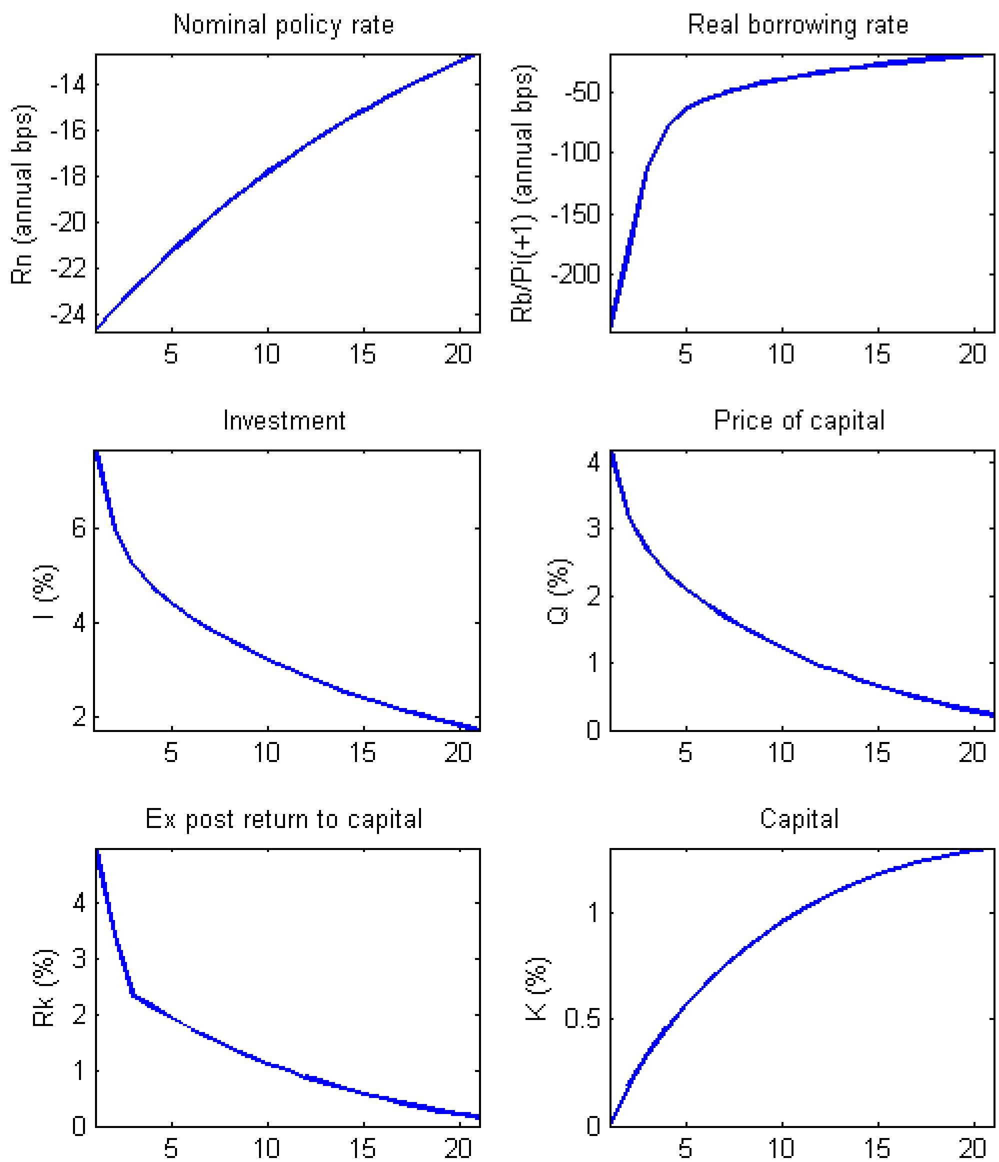
3.5 Results

Figures 7 and 8 show the effects of a 25 annual basis point decrease in the nominal interest rate R_t^n for four years after the shock. The horizontal axis represents quarters. All variables are displayed in basis points or percentage point deviations from the steady state, as specified for each variable. Interest rates, inflation and the default rate are reported in annualized terms.

When the nominal interest rate decreases, capital has already been chosen from the previous period. At time zero the real borrowing rate decreases from 4.46 % to 2 % annually both due to the lower nominal opportunity cost of borrowing and to the expectation of inflationary pressures. The decrease in the real borrowing rate increases end-of-period investment by 7.7 % and pushes up the price of capital by 4.19 %. This unanticipated increase in the price of capital unexpectedly increases the return to capital above steady state by 4.99 % starting from the same quarter in which the monetary shock occurs. This increase is mainly driven by the capital gain on non-depreciated capital, but also reflects a 10.69 % increase in the rental rate of capital generated by a 2.56 % increase in labour in production (unreported in the figures). Capital starts accumulating from the second quarter onwards.

The unexpected increase in the return to capital pushes up entrepreneurs' revenues by 5.43 %. This decreases on impact the default probability by 21.57 basis points below steady state because the debt contracts of outstanding loans had been signed at the steady state non-contingent real borrowing rate of 4.46 %. On impact, the leverage ratio of entrepreneurs at the beginning of the period remains unchanged, because capital was bought at the previous period. At the end of the first period, instead, leverage decreases because the unexpected increase in net worth dominates

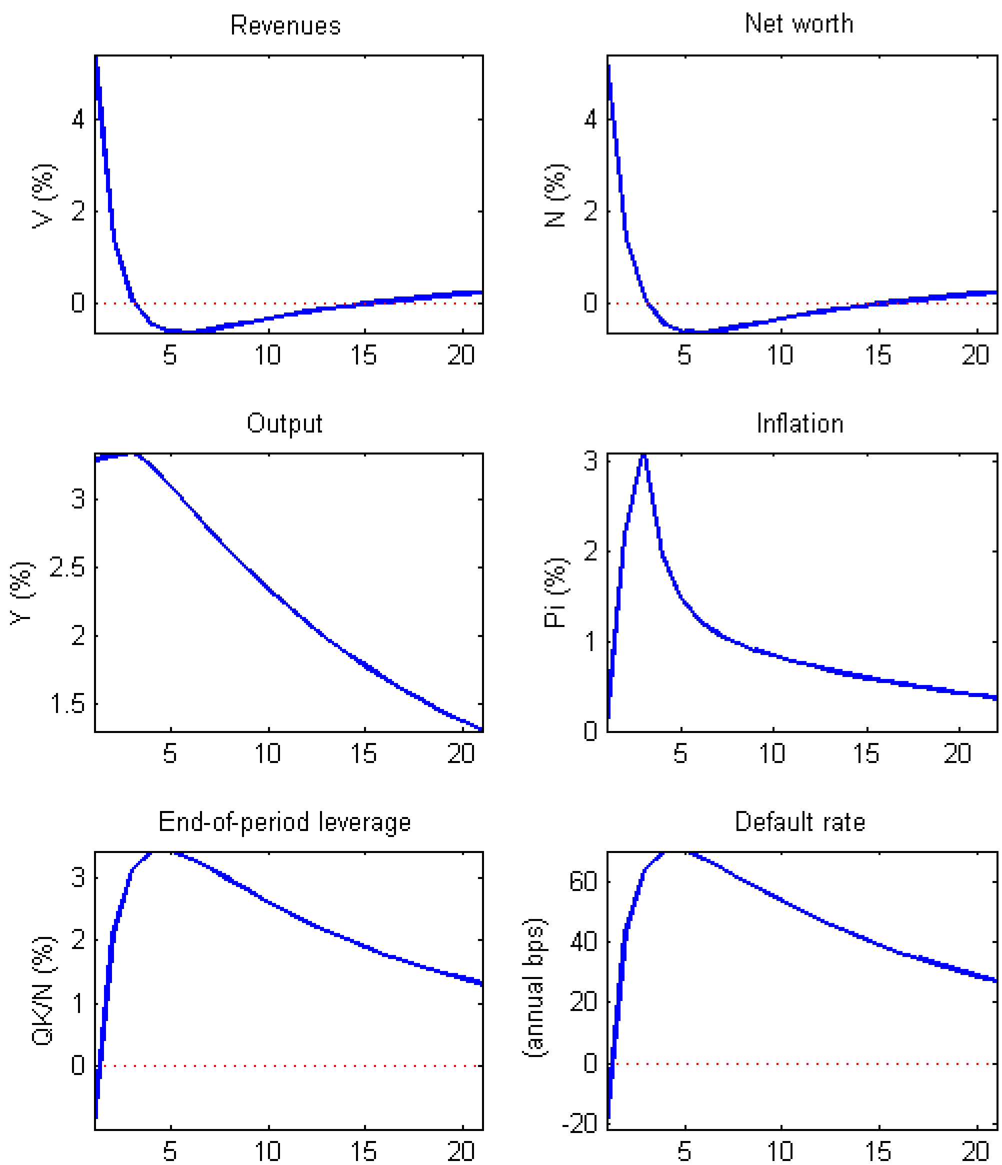
Figure 7: Effects of a monetary expansion



the increase in the price of capital and the slow increase in the purchase of new units of capital for the next period. On impact, output increases by 3.28 % and inflationary pressures arise from the second period onwards, up to 3.09 % annually on the third period.

From the second period onwards no unexpected events occur and the nominal

Figure 8: Effects of a monetary expansion (continues)

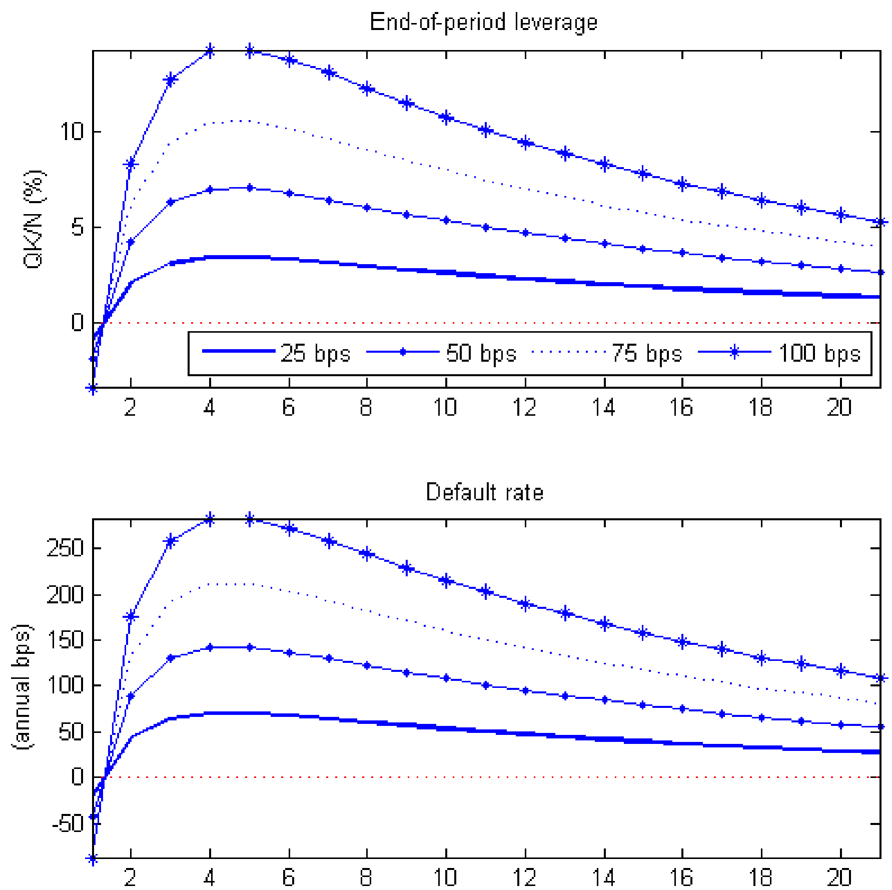


interest rate and price of capital slowly revert to the mean. The accumulation of capital pushes the leverage ratio up above steady state up to 3.34 % on the third period after the shock, carrying with it the default rate. In fact, the behaviour of defaults mimics the one of leverage. Default displays a hump-shaped response and peak to 69.98 basis points on an annual basis above steady state after 4 quarters from the shock. It is the increase in defaults that pushes revenues below steady state after the second period. As the leverage ratio reverts to the mean, the default rate reverts to its steady state. Overall, the 25 basis points decrease in the nominal interest rate has delivered a reduction in defaults of 21 annual basis points for outstanding loans and a subsequent increase in defaults of new loans, with a peak of 75 basis points above steady state in the third quarter after the shock. I borrow from Chari *et al.* the idea of measuring persistence as the time it takes for the effect to decline below its highest value by a certain fraction. I find that it takes respectively 15 and 31 quarters for defaults to revert below 50 % and 20 % the peak value of 75 basis points above steady state reached one quarter after the shock. This strong persistence follows from the assumed persistence in the nominal interest rate.

Figures 7 and 8 studied the effects of a 25 basis point annual decrease in the nominal risk free rate. Figure 9 compares this scenario with the case in which the policy rate decreases by 50, 75 or 100 basis points annually. For simplicity, only the dynamics in leverage and defaults are reported. We see that the stronger the monetary expansion and the higher the initial decrease in defaults due to the stronger unexpected increase in revenues from which outstanding loans benefit. From the second period onwards, the stronger the monetary expansion and the higher the build up of entrepreneurial leverage, implying a more pronounced increase the default rate. For the case of a 1 % annual decrease in the policy rate from 4.113% to 3.113%, the default probability increases by up to 2.82 percentage points above the steady state. The peak is reached after 4 quarters. The degree of persistence remains the same since the threshold value used to compute it moves proportionally.

Before comparing the predictions of the model on default probabilities with the empirical evidence by Jimenez *et al.* (2007), done in detail in section 3.7, it is encouraging to note that the predictions of the model with respect to other

Figure 9: Comparing monetary policy shocks



variables is fairly in line with existing empirical evidence from VAR estimations. (COMMENT, INCLUDING Rb/R)

The result of this section is synthesized in Proposition 3. Some sensitivity analysis is performed in the next section to assess the robustness of the result, before comparing it quantitatively to the empirical finding by Jimenez *et al.* (2007).

PROPOSITION 3: *In a dynamic general equilibrium extension of the model from section 2, the reduction in the policy rate increases asset prices and generates an unexpected increase in revenues which reduces the default probability of outstanding loans. Starting from the second* *quarter onwards, entrepreneurial leverage starts to increase in order to take advantage of the lower real borrowing rate. The increase in leverage pushes up the default rate through a hump-shaped response. A 25 basis point annual reduction in the nominal interest rate delivers an impact decrease in defaults of 21 basis points and a peak in the increase in default rates after 4 quarters of approximately 70 basis points above steady state. A 100 basis point annual reduction in the policy rate decreases defaults of outstanding loans by 87 basis points and increases defaults of new loans up to 282 basis points above steady state 4 quarters after the shock. In both cases, it takes around 30 years for the default probability to revert to the steady state value.*

3.6 Robustness checks

(IN PROGRESS)

ADDRESS ADJUSTMENT COSTS AND OTHER VARIABLES

3.7 A quantitative comparison with Jimenez *et al.* (2007)

(IN PROGRESS)

Jimenez *et al.* (2007) estimate the following hazard function

$$h(t) = \lambda \alpha t^{\alpha-1} e^{\beta_{bo} \cdot i_{bo} + \beta_{ao} \cdot i_{ao} + M \cdot \gamma + C \cdot \delta + A \cdot \sigma} \tag{15}$$

where M includes maturity dummies, C controls bank specific factors, time dummies and other variables for which coefficients are not reported and A controls for aggregate variables: inflation and GDP. In Log this is equivalent to

$$ln(h(t)) = ln(\lambda) + ln(\alpha) + (\alpha - 1)log(t) + \beta_{bo} \cdot i_{bo} + \beta_{ao} \cdot i_{ao} + M \cdot \gamma + C \cdot \delta + A \cdot \sigma \quad (16)$$

We are told that

$$0.6 = \lambda 2.26141^{2.2614-1} e^{-0.127 \cdot 4.1 + 0.293 \cdot 4.1 + \bar{M} \cdot \gamma + \bar{C} \cdot \delta + \bar{A} \cdot \sigma} \quad (17)$$

from which

$$ln(0.6) = ln(\lambda) + ln(2.2614) - 0.127 \cdot 4.1 + 0.293 \cdot 4.1 + \bar{M} \cdot \gamma + \bar{C} \cdot \delta + \bar{A} \cdot \sigma \quad (18)$$

$$\begin{aligned} ln(\lambda) + \bar{M} \cdot \gamma + \bar{C} \cdot \delta + \bar{A} \cdot \sigma &= ln(0.6) - ln(2.2614) + 0.127 \cdot 4.1 - 0.293 \cdot 4.1 \\ &= -2.0074 \equiv step1 \end{aligned}$$

which implies that the loan with median M, C, A the hazard rate function is

$$\begin{aligned} ln(h(t)) &= ln(2.2614) + (2.2614 - 1)log(t) + \beta_{bo} \cdot i_{bo} + \beta_{ao} \cdot i_{ao} + \underbrace{M \cdot \gamma + C \cdot \delta + A \cdot \sigma + ln(\lambda)}_{step1} \\ &= ln\Big(2.2614 t^{2.2614-1} e^{\beta_{bo} \cdot i_{bo} + \beta_{ao} \cdot i_{ao} + step1}\Big) \end{aligned}$$

i.e.

$$h(t) = 2.2614 \cdot t^{2.2614-1} e^{-0.127 \cdot i_{bo} + 0.293 \cdot i_{ao} - 2.0074} \quad (19)$$

This is the hazard function at generic quarter t of a loan with mean characteristics, with mean maturity 5 quarters and if all other regressors stay constant. Note, a ceteris paribus argument also on GDP, inflation and others, although they all move in GE, as in my application.

Estimation in the empirical model uses mean maturity of 5 quarters, but in the model the maturity is one quarter. To fix for this, should down and up the appropriate dummies and adjust $step1$. Additionally, should also adjust for the fact that on average the dummy for collateralized loans in their dataset is 0.077, while in the model loans are not collateralized:

$$step2 = step1 - 1.0987 + 1.182 = -1.8125 \tag{20}$$

With the new maturity period and collateralization, the a hazard function is

$$h(t) = 2.2614 \cdot t^{2.2614-1} e^{-0.127 \cdot i_{bo} + 0.293 \cdot i_{ao} - 1.8125} \tag{21}$$

Note that for this hazard function the rate when both interest rates are at their sample average and $t = 1$ is 0.7291, not 0.60, so need to adjust the calibration.

Fo far I've adjusted $step1$ for M and for C as much as possible. No, take out the elements in A in order to control for them in the general equilibrium exercise

4 Conclusion

(IN PROGRESS) (ADD SPEECH BY DAVID MILES AND FT ARTICLE. EXPLAIN DIFFICULTIES OF MODELLING OUTSIDE EQUITY MARKET)

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5 Appendix A: equilibrium equations of the model

Household

The maximization problem of the households is

$$\max_{C_t,H_t,D_t} \; E_0 \sum_{t=0}^{\infty} \beta^t \left[\log(C_t - hC_{t-1}) - \chi \frac{H_t^{1+\eta}}{1+\eta} \right]$$

$$\text{s.t. } \; P_t C_t + D_t \le R_t^n D_{t-1} + W_t H_t$$

which yields

$$\lambda_t = \frac{1}{C_t - hC_{t-1}} \tag{22}$$

$$\lambda_t = \beta R_t \lambda_{t+1} \tag{23}$$

$$\Pi_{t+1} = \frac{P_{t+1}}{P_t} \tag{24}$$

$$R_t = \frac{R_t^n}{\Pi_{t+1}} \tag{25}$$

$$W_t = \chi \frac{H_t^{\eta}}{\lambda_t} \tag{26}$$

Equation (22) defines the marginal utility of consumption, equation (23) the Euler equation, equation (24) the inflation rate, equation (25) the real gross interest rate and equation (26) the labour supply function.

Entrepreneurs and lenders

Entrepreneurs borrow from lenders to buy capital K_t, which will be used in the following period. The debt contract signed at time t regarding the purchase of capital used in $t+1$ solves

$$\max_{\bar{\omega}_{t+1}(R^K_{t+1}),K_t} \quad E_t\left[F(\bar{\omega}_{t+1})R^K_{t+1}\right]Q_tK_t$$

$$\text{s.t.} \quad E_t[G(\bar{\omega}_{t+1})R^K_{t+1}]Q_tK_t = R^n_t(Q_tK_t - N_t)$$

where expectation in the maximization problem is taken with respect to the aggregate return to capital R^k_{t+1}. The optimality conditions and the corresponding equilibrium equations are

$$\frac{Q_tK_t}{N_t} = \frac{1}{1 - \frac{R^K_{t+1}}{R^n_t}G(\bar{\omega}_{t+1})} \tag{27}$$

$$-F'(\bar{\omega}_{t+1}) = F(\bar{\omega}_{t+1})\frac{G'(\bar{\omega}_{t+1})}{\left(\frac{R^K_{t+1}}{R^n_t}\right)^{-1} - G(\bar{\omega}_{t+1})} \tag{28}$$

$$V_t = \frac{F(\bar{\omega}_t)R^K_tQ_{t-1}K_{t-1}}{\Pi_t} \tag{29}$$

$$CeNR_t = 0 \tag{30}$$

$$CeR_t = (1-\gamma)V_t \tag{31}$$

$$N_t = \gamma V_t + W_t \tag{32}$$

$$Monit_t = \mu\left[\int_0^{\bar{\omega}_t}\omega d\Phi(\omega)\right]Q_{t-1}K_{t-1} \tag{33}$$

$$R^b_t = \frac{E_t[\bar{\omega}_{t+1}R^K_{t+1}]Q_tK_t}{Q_tK_t - N_t} \tag{34}$$

$$R^K_t = \frac{r^K_t + (1-\delta)Q_t}{Q_{t-1}} \tag{35}$$

Equation (27) gives the indifference condition of the lender and corresponds to equation (2), equation (28) gives the optimality condition as in equation (7), equation (29) gives aggregate revenues, equation (30) the aggregate consumption of non-retiring entrepreneurs, equation (31) the aggregate consumption or retiring entrepreneurs, equation (32) aggregate net worth, equation (33) the ex post aggregate monitoring costs, equation (34) the real gross borrowing rate and equation (35) the ex post return to capital.

Intermediate good producers

Intermediate good producers have access to a standard production function, as from equation (36)

$$Y_t = e^{a_t} K_{t-1}^{\alpha} H_t^{1-\alpha} \tag{36}$$

with productivity that evolves according to

$$a_t = \rho^a a_{t-1} + \epsilon_t^a \tag{37}$$

The corresponding optimality conditions are

$$W_t = MC_t(1-\alpha)\frac{Y_t}{H_t} \tag{38}$$

$$r_t^k = MC_t \alpha \frac{Y_t}{K_{t-1}} \tag{39}$$

where equation (38) pins down the labour demand curve and equation (39) the rental rate of capital

Capital producers

Capital producers buy non-depreciated capital from capital holders, invest and sell the new capital stock to capital holders, which will be used in the following period. Investment is subject to capital adjustment costs as in Christiano, Eichenbaum and Evans (2005). The maximization problem of capital producers is

$$
\begin{aligned}
\max_{I_t} \quad & \sum_{t=0}^{\infty} \beta^t \frac{\lambda_{t+1}}{\lambda_t} \left[Q_t K_t - I_t - Q_t(1-\delta) K_{t-1} \right] \\
\text{s.t.} \quad & K_t = (1-\delta) K_{t-1} + I_t \left(1 - \frac{\nu}{2} \left(\frac{I_t}{I_{t-1}} \right)^2 \right)
\end{aligned}
$$

which yields

$$
\frac{\lambda_{t+1}}{\lambda_t} \left[Q_t \left(1 - \frac{\nu}{2} \left(\frac{I_t}{I_{t-1}} \right)^2 \right) - Q_t I_t \nu \left(\frac{I_t}{I_{t-1}} - 1 \right) \frac{1}{I_{t-1}} - 1 \right] + \beta \frac{\lambda_{t+2}}{\lambda_{t+1}} \left[Q_{t+1} I_{t+1} \nu \left(\frac{I_{t+1}}{I_t} - 1 \right) \frac{I_{t+1}}{I_t^2} \right] = 0 \tag{40}
$$

Equation (40) pins down the price of capital.

Retailers

Retailers buy intermediate goods, transform into consumption goods and sell them to consumers. Prices are allowed to be re-optimized according to standard Calvo pricing. The maximization problem of retailers is

$$
\begin{aligned}
\max_{P_t(j)} \quad & E_t \sum_{k=0}^{\infty} (\beta\psi)^k \frac{\lambda_{t+k}}{P_t} \left[P_t(j) Y_{t+k}(j) - MC_{t+k} Y_{t+k}(j) \right] \\
& \text{subject to} \quad Y_{t+k}(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon} Y_{t+k}
\end{aligned}
$$

which gives

$$\frac{P_t(j)^*}{P_t} = \frac{\frac{\epsilon}{\epsilon-1} E_t \sum_{k=0}^{\infty} (\beta\psi)^k \lambda_{t+k} \frac{P_{t+k}(j)}{P_{t+k}} Y_{t+k}(j) \Pi_{t+k}^{\epsilon} mc_{t+k}}{E_t \sum_{k=0}^{\infty} (\beta\psi)^k \lambda_{t+k} \frac{P_{t+k}(j)}{P_{t+k}} Y_{t+k}(j) \Pi_{t+k}^{\epsilon-1}} \tag{41}$$

$$\Pi_t = \frac{P_t}{P_{t-1}} \tag{42}$$

$$P_t = \left[\psi P_{t-1}^{1-\epsilon} + (1-\psi) P_t^{*^{1-\epsilon}} \right]^{\frac{1}{1-\epsilon}} \tag{43}$$

Equation (42) gives the price level of re-optimizing firms, equation (43) defines the aggregate price level.

Market clearing conditions and Taylor rule

The market clearing condition is given by

$$Y_t = C_t + CeR_t + CeNR_t + I_t + G_t + Monit_t$$

6 Appendix B: Modeling an exogenous policy rate

(IN PROGRESS)

(COMMENT LITERATURE ON THE ESTIMATION OF INTEREST RATES)