Optimal tax on Colombia's capital inflows

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Abstract

We estimate the optimal tax value on capital inflows so that private agents do internalize the social costs of their borrowing decisions in an economy with financial constraints. Using Colombian data for the 1996-2011 period (which includes the crisis of 1998-1999), we find that the tax would be set around 1,2%.

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1. INTRODUCTION

In the traditional view of financial crises, free capital flows are desirable and any government intervention is considered as inefficient for capital markets. In contrast, recent economic literature presents new arguments in favor of the idea that financial crises may be a result of the fact that private agents do not internalize the contribution of their decisions to aggregate financial instability. Consequently, some policy interventions may improve social welfare.

The present paper analyzes the impact of capital flows regulation on social welfare. The analysis is based on the theoretic formalization made by previous literature[2] of a common argument nowadays, which states that overborrowing was one of the main causes of the recent global financial crisis. Private agents rationally underestimated the social cost of debt repayments.

Since private participants have a negligible impact on the market, it is rational for them to take prices as given. However, in aggregate, economic agents' decisions have an effect on prices and, in turn, these changes in prices affect economic agents. This is why debt/consumption decisions give rise to pecuniary externalities. Because this type of externalities operates through the price system, its effects are innocuous under complete and competitive markets since such effects efficiently reflect the relative scarcity of goods. In contrast, when markets are imperfect, pecuniary externalities may have real and distortionary effects. This is precisely the case of economies that are subject to financial constraints as explained in the following lines.

[2] Among others Jeanne and Korinek (2010a), Jeanne and Korinek (2010b), Korinek (2010), Mendoza (2010), Bianchi (2011), Bianchi and Mendoza (2011), Korinek (2011a) and Korinek (2011b).

When the financial constraint is binding, debt repayments and the limited access to credit force agents to reduce consumption. This reduction on aggregate demand has a negative effect on asset prices. Since assets serve as collateral for debt, the borrowing capacity is further reduced[3]. This negative feedback among aggregate demand, asset prices and limited access to credit gives rise to a downward spiral in the economy through a debt-deflation amplification mechanism[4].

The problem arises from the combination of market imperfections (the financial constraint) and private agents not internalizing the social cost of their debt decisions. On aggregate, these decisions have an effect on prices and ultimately on the economy's borrowing capacity (when the economy is financially constrained), and hence imply a social cost. From this perspective, financial crises are defined as situations in which economies experience a debt-deflation spiral as described above.

Since private agents underestimate the social cost of their decisions, the related literature suggests adopting prudential capital controls so as to reduce financial vulnerability in facing crisis events. One specific proposal is to regulate capital flows by imposing a pigouvian tax on inflows in order to reduce the impact of capital outflows when a financial crisis materializes. The present paper is based on this view and, with the purpose of calculating the optimal tax level, sets up a model that exhibits financial amplification dynamics. Overborrowing is measured as the difference between the level of debt that

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[3] There is an alternative interpretation with similar results. In order to honor their debts private agents need to sell part of their assets. This fire-sale implies further reductions in asset prices and, as a result, additional sales of assets … (see Aghion et. al, 2004; Mendoza, 2010; Bianchi and Mendoza, 2011; Stein, 2012).

[4] Two seminal works on the idea that financial restrictions may give rise to an amplifying mechanism of business cycles are Bernanke and Gertler (1989), and Kiyotaki and Moore (1997).

would be acquired by a central planner (who internalizes the social cost of its decisions) and the one that is acquired by private agents in a decentralized economy.

The model is simple and highly tractable. It is based on Korinek (2010, 2011a). He differentiates the optimal tax according to the debt risk profile. Specifically, he estimates the externality imposed by different financial instruments during the Indonesia crisis (1997-98). However, this estimation is made in the empirical part, assuming that the externality grows one to one with the volatility of the financial instrument but without any support on the theoretical model. As an additional contribution, we incorporate into the model financial instrument volatility so as to give theoretical support to our empirical estimation.

Calculating the optimal tax is highly relevant for some emerging economies which after the recent financial global crisis have become an attractive alternative for investment and, as a result, are experiencing strong capital inflows. Adopting prudential controls may allow these countries to reduce the future social cost derived from any sudden change in the trend of international investment[5].

We make use of historical data for the Colombian economy so as to calculate the optimal tax. Our results suggest that the optimal tax on capital inflows is about 1,2%. This result is similar to those reported by related literature which estimates the optimal tax for other countries.

We calculate the optimal tax for two types of debt, CPI-indexed Colombian peso debt such that its real value for the borrower does not vary and dollar debt whose real value for the borrower changes with movements in the nominal exchange rate and the price level. We

[5] Aghion et al. (2004) draw attention to the fact that economies at an intermediate level of financial development are the most vulnerable to financial instability events.

find that the optimal tax for the first case is about 1,1% and about 1,2-1,3% for the second one. The difference between the two is small since in the last decades Colombia did not experience large values of devaluation minus inflation[6].

In the following section we describe the model and in section 3 we obtain the equilibrium for both the decentralized and the central-planner cases. In section 4 we derive an expression for the optimal tax that implements the central-planner solution in a decentralized economy. In section 5 we estimate the value of the optimal tax for the Colombian case. Section 6 concludes.

2. THE MODEL

The model is based on Korinek (2010, 2011a). As an additional element, we incorporate the volatility of debt in order to give theoretical support to our empirical estimation differentiated by debt risk profile.

It is a stylized three-period model of financial amplification for a small open endowment economy inhabited by a representative consumer. In periods one and three, for simplicity, there is just one type of good, a tradable good (T) which is the numeraire. In the second period there is an additional good, a non-tradable (N) with a relative price p_2, and therefore $1/p_2$ can be interpreted as the real exchange rate. In the first period there are no endowments and consumption must be financed by debt. In the second period the consumer obtains endowments of both types of goods (y_T and y_N) and in the third period he obtains an endowment y_T of tradable goods.

[6] For instance, during the last crisis (1999), the value of (devaluation-inflation)/(1+inflation) was 24,1%. The value for Indonesia during its crisis period was 118%.

Periods one and three are only needed to start the economy with debt and to guarantee all debt is paid back, respectively. This is why these periods are modeled in the simplest possible way and all the interesting elements are incorporated into the second period.

The representative agent maximizes the following utility function (marginal utility is assumed to be positive and decreasing, i.e. $u'' < 0 < u'$):

$$U = u(c_{T,1}) + \beta u(c_2) + \beta^2 c_{T,3} \quad \textbf{(1)}$$

where β is the discount rate and $c_2 = c_{T,2}^{\sigma} c_{N,2}^{1-\sigma}$ a consumption index which aggregates tradable (c_T) and non-tradable consumption (c_N) with shares σ and $1-\sigma$, respectively.

The consumer may acquire (tradable) debt in periods one (d_1) and two (d_2). The value of d_1 is subject to uncertainty because the final payment depends on the state in period two. We assume there are two states both with the same probability (1/2). In the 'adverse' state the consumer repays $d_1(1+\theta), \theta > 0$, and in the 'favorable' state he repays $d_1(1-\theta)$. Note that the expected value of first-period debt is d_1 and its standard deviation is $d_1\theta$. This is why we interpret θ as debt volatility[7]. Debt acquired in period two is not subject to uncertainty.

The assumption about different debt states tries to capture the fact that different asset categories which represent debt may be subject to uncertainty and its real value may change from the moment debt is acquired until it is finally paid back. Incorporating the debt risk

[7] We assume there is only one type of debt d_1 in the model; however it can be verified that conclusions are still the same if we introduce different types of debt (with different levels of risk), such that the total amount is equal to the sum of all types. Given the concavity of the utility function (risk aversion), the consumer acquires different types of debt only if higher volatility of debt implies a lower interest rate (r). For instance, if there are two types of debt, one with high volatility θ_h and the other with low volatility θ_l, $\theta_h > \theta_l$, it is required that $r_h < r_l$. In practice, this kind of situation can be observed when an agent may borrow in dollars at a low interest rate (but with high foreign exchange risk) or may borrow in local currency (avoiding exchange risk) at a higher interest rate.

profile is relevant for the purpose of calculating the optimal tax because, as remarked by Korinek (2010) and Jeanne and Korinek (2011), this volatility has an impact on the size of the externality generated by private decisions on debt. In practice there are different asset categories associated to debt (e.g. GDP-indexed, CPI-indexed, Dollar debt…), and each of them represents a different risk profile for the borrower (and the lender). Changes in local or international economic conditions may produce significant variations in the real value of debt repayments. Although these variations may be related to endogenous variables (e.g. the real exchange rate) for simplicity we assume debt states are exogenously determined.

The interest rate is r_1 in period one and r_2 in period two. We also define gross interest rates $R_1 \equiv 1 + r_1$ and $R_2 \equiv 1 + r_2$. Taking these rates as given, the representative consumer chooses, in period one, the level of d_1 (which has to be paid back in period two but can be at least partially refinanced by d_2) and, in period two, the level of d_2 (which has to be paid back in period three).

Considering all the elements mentioned above, budget constraints for periods one, two and three can be expressed as follows:

$$c_{T,1} = d_1/R_1 \tag{2}$$

$$c_{T,2}^i + p_2^i c_{N,2}^i + d_1\left(1 + \theta I^i\right) = y_T + p_2^i y_N + d_2^i/R_2 \tag{3}$$

$$c_{T,3}^i + d_2^i = y_T \tag{4}$$

where the superscript $i \in \{A, F\}$ indicates the debt state in period two, adverse or favorable, and I is a binary variable, $I^A = 1$ and $I^F = -1$. In the first period, the consumer must finance consumption by borrowing. In the second period, consumption and the debt repayment (whose value depends on the state, A or F) are financed by income (endowments) and new debt. Note that consumption and debt decisions in the second and third periods depend on the state observed in period two. In the third and last period, the endowment is used for financing consumption and paying back all remaining debt.

We assume that the credit market is subject to a moral hazard problem. In particular, borrowers could threaten default and due to imperfect legal enforcement lenders can recover only a fraction k of the borrower's income (net of previous-period debt) in the second period. As a result, in order to prevent fraud, lenders are not willing to lend more than the level they can recover and the financial constraint is

$$d_2^i/R_2 \leq k\left(y_T + p_2^A y_N - d_1(1+\theta)\right) \quad \textbf{(5)}$$

where $k < \sigma/(1-\sigma)$ [8].

The above equation also introduces an important feature of the model which is that, in assessing the debt capacity of potential borrowers, lenders have not yet observed the state of period two and their assessment has two important features: i) it takes into account not only the borrowers' assets (endowments) but also the borrowers' previously acquired debt and ii) it incorporates some degree of risk aversion by only considering the adverse scenario (this is the reason for only using subscript A in equation (5) and, implicitly, only

[8] Since there is an amplification effect, this condition guarantees the total effect on consumption of changes in initial debt converges to a finite value. See footnote 12.

$\mathrm{I}^{\mathrm{F}} = -1$ is considered)[9]. In this way, we incorporate the fact that higher volatility (θ) may have a negative impact on the financial constraint.

Equation (5) introduces the amplification effects: the debt capacity of borrowers depends on the value of their assets (net of debt). It also incorporates the effect of the real exchange rate on the borrowing capacity and, as mentioned by Korinek (2010), captures the common notion that depreciations ($\downarrow p_2$) may contribute to the contraction of emerging economies.

3. EQUILIBRIUM

The model is solved by backward induction, and therefore firstly solved for periods two and three, taking the initial debt level d_1 as given. In making consumption decisions for periods two and three, the representative consumer has already observed the state of period two, and therefore this part of the solution does not imply uncertainty. We obtain solutions for both the decentralized case and the one in which a central planner decides the consumption level. By comparing these two solutions we can show, when the economy is financially constrained, that private agents undervalue liquidity relative to the central planner. Given this result and solving for period one (in which there is uncertainty about the state of period two) it is shown that the decentralized solution implies overborrowing or, in other words, that a central planner would acquire a lower debt level in period one. This latter result rationalizes the introduction of a policy action which may reduce the size of the externality produced by private agents' decisions.

[9] This assumption is made for the sake of simplicity. Lenders could consider both states, adverse and favorable. All we require in order to introduce volatility in the model is that lenders, as a form of risk aversion, give to the adverse state a higher weight than it has in the probability distribution of states.

3.1. Decentralized Equilibrium

Substituting (4) into (1) and then maximizing subject to constraints (3), (5) and the market-clearing conditions ($c^i_{N,2} = y_N$, for non-tradables and $c^i_{T,2} + d_1(1 + \theta I^i) = y_T + d^i_2/R_2$ for tradables) we can obtain the following first-order conditions with respect to $c_{T,2}$, $c_{N,2}$ and d_2 for state $\mathrm{i} \in \{\mathrm{A}, \mathrm{F}\}$:

$$u'(c^i_2)\sigma\left(\frac{y_N}{c^i_{T,2}}\right)^{1-\sigma} = \mu^i \qquad \textbf{(6)}$$

$$u'(c^i_2)(1-\sigma)\left(\frac{c^i_{T,2}}{y_N}\right)^{\sigma} = p^i_2\mu^i \qquad \textbf{(7)}$$

$$R_2\beta + \lambda^i = \mu^i \qquad \textbf{(8)}$$

where μ and λ are the Lagrange multipliers associated to the budget constraint (3) and the financial constraint (5), respectively. Condition (6) equalizes the marginal utility of consumption to the shadow value of current wealth. Condition (7) equalizes the marginal rate of substitution of goods (tradable and non-tradable) to their relative price. Equation (8) is the Euler equation for assets. If the financial constraint is binding, there is a gap between the shadow value of current wealth and the value of transferring income between periods due to the shadow price of relaxing the financial constraint (λ^i). Using (6) and (7) we find the following expression for the price of non-tradable goods:

$$p^i_2 = \frac{1-\sigma}{\sigma}\frac{c^i_{T,2}}{y_N} \qquad \textbf{(9)}$$

When the initial debt level d_1 is sufficiently low (see equation (10) below), private agents are not financially constrained, and therefore $\lambda^i = 0$, $\mu^i = R_2\beta$. In this case, from (6), we can deduce the equilibrium value for $c_{T,2}$ as a function of $R_2\beta, \sigma$ and y_N, i.e. $\bar{c}_{T,2}(R_2\beta, \sigma, y_N)$ and the corresponding equilibrium value of debt is $\bar{d}_2^i/R_2 = \bar{c}_{T,2}(R_2\beta, \sigma, y_N) + d_1(1 + \theta I^i) - y_T$. Using these values back in (5), we can express the financial constraint as:

$$d_1^* \leq \frac{(1+k)y_T - \left(1 - k\frac{1-\sigma}{\sigma}\right)\bar{c}_{T,2}(R_2\beta, \sigma, y_N)}{1 + \theta I^i + k(1+\theta)} \quad \textbf{(10)}$$

where d_1^* is the debt level that private agents would optimally choose when they are not constrained (see section 3.3).

In the present document we define crisis as a situation in which the economy is financially constrained. It can be seen, from equation (10) and similarly to Korinek (2011a), that crisis events are associated to periods in which income is sufficiently low. In particular, taking as given the values of the remaining parameters, sufficiently low values of y_T imply that (10) is never satisfied, and therefore the economy is always financially constrained. In contrast, sufficiently high values of y_T imply that (10) holds and the economy is never constrained.

As an addition to Korinek (2011a), crises in the present model also depend on the state (adverse or favorable) faced by agents at the moment of paying back debt in period two. We focus on the case in which the economy is in crisis (constrained) only in the adverse state and hence crisis is not perfectly predictable, since in period one there is uncertainty about the state of period two. The situation in which the economy is constrained independently of the state (when income is too low), is less relevant for our purposes; however, for the sake of completeness, some comments are made about this case[10].

When the financial constraint is binding, equation (5) determines the debt level, $d_2^A/R_2 = k\left(y_T + p_2^A y_N - d_1(1+\theta)\right)$ and tradable consumption is determined in equation (3), $c_{T,2}^A = y_T + d_2^A/R_2 - d_1(1+\theta)$. Using these equations and the equation for the price of non-tradable goods (9), we obtain that in the constrained equilibrium[11]:

$$\frac{d_2^A}{R_2} = \frac{\mathrm{y_T} - d_1(1+\theta)}{\sigma - (1-\sigma)k} k \qquad \textbf{(11)}$$

$$c_{T,2}^A = \frac{\mathrm{y_T} - d_1(1+\theta)}{\sigma - (1-\sigma)k} (1+k)\sigma \qquad \textbf{(12)}$$

Additionally, we can see from (8) that the value of μ^A is greater for the constrained equilibrium ($\lambda^A \geq 0$). Given this result and taking into account condition (6) and the assumption that marginal utility is decreasing, we can conclude that both the levels of tradable consumption $c_{T,2}^A$ and of debt d_2 are lower when private agents are constrained.

[10] Since we are interested in cases in which there is a positive probability of crisis we do not present the analysis of the case where income is sufficiently high such that the economy is never constrained. Additionally, it is not possible to have a case in which the economy is in crisis only in the favorable state. This can be seen from (10). Suppose that the economy is constrained in the favorable state. If this is true for $\mathrm{I^F} = -1$, it must also be true, for the same set of parameter values, that the economy is restricted in the adverse state as well, since the denominator in the RHS of (10) is greater for such state ($\mathrm{I^A} = 1$).

[11] Notice that in case we had included different debt types, the equations would be similar but $\mathrm{d_1}(1+\theta)$ should be substituted by $\sum_\mathrm{j} \mathrm{d_1^j}\left(1+\theta_\mathrm{j}\right)$ where j indicates a specific asset category with a particular level of volatility (θ_j).

Furthermore, since $0 < k < \sigma/(1-\sigma)$ and $\partial c_{T,2}^{A}/\partial d_1 = -(1+k)(1+\theta)\sigma/(\sigma-(1-\sigma)k) < -1$, increments in the level of initial debt have a negative and amplified effect on consumption[12].

3.2. Central Planner´s Equilibrium

The previous section describes the equilibrium reached when agents take aggregate variables as given, in particular the price of non-tradable goods. In this section, the case of a benevolent central planner (CP) with restricted planning abilities is considered. Specifically, it is assumed that the CP is subject to the same financial constraint and uncertainty conditions as private agents, but she is capable of internalizing the effect of borrowing decisions on prices.

Unlike a private consumer, the CP takes into account the effect of borrowing and consumption decisions on the exchange rate $1/p_2$. In particular, the CP realizes that a lower debt level mitigates the decline in the prices of non-tradable goods and can prevent too great a fall in the borrowing capacity when the financial restriction is binding.

The level of non-tradable consumption is not important for the CP decision since in the aggregate the condition $c_{N,2} = y_N$ is always satisfied, independently of the debt level. Taking this into account, and using the subscript "CP" to indicate the Lagrange multipliers associated with the CP´s problem, the associated Lagrangian is:

$$\mathcal{L} = \beta u(c_2^i) + \beta^2(y_T - d_2^i) - \beta\mu_{CP}^i\left(c_{T,2}^i + d_1(1+\theta I^i) - y_T - d_2^i/R_2\right)$$

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[12] Using $d_2^A/R_2 = k\left(y_T + p_2^A y_N - d_1(1+\theta)\right)$ and $c_{T,2}^A = y_T + d_2^A/R_2 - d_1(1+\theta)$, note that the initial effect of increasing d_1 (in one unit) on $c_{T,2}^A$ is $(1+\theta)(1+k)$. This effect on consumption reduces p_2^A (equation (9)) by $(1-\sigma)/\sigma$ and consumption is again reduced by $k(1-\sigma)/\sigma$. The final effect on $c_{T,2}^A$ is $(1+\theta)(1+k)(1+k(1-\sigma)/\sigma+[k(1-\sigma)/\sigma]^2+\dots)$ which, under the condition that $k(1-\sigma)/\sigma < 1$, converges to $(1+\theta)(1+k)\,\sigma/(\sigma-(1-\sigma)k)$ (see equation (12)).

$$-\beta\lambda_{CP}^{i}\left[d_2^i/R_2 - k\left(y_T + \frac{1-\sigma}{\sigma}c_{T,2}^A - d_1(1+\theta)\right)\right] \quad \textbf{(13)}$$

The first order conditions with respect to $c_{T,2}$ and d_2 for the favorable state are as in (6) and (8) and, therefore, $\mu_{CP}^{F} = \mu^{F}$ and $\lambda_{CP}^{F} = \lambda^{F}$. Consumption, debt and valuation of liquidity are equal to those of the decentralized case. Since it is assumed that the financial constraint is not binding in the favorable state of the economy, then $\lambda_{CP}^{F} = 0$ and $\mu_{CP}^{F} = R_2\beta$.

In the adverse state, the first order conditions for the CP are:

$$u'(c_2^A)\sigma\left(\frac{y_N}{c_{T,2}^A}\right)^{1-\sigma} + \lambda_{CP}^A k\frac{1-\sigma}{\sigma} = \mu_{CP}^A \quad \textbf{(14)}$$

$$R_2\beta + \lambda_{CP}^A = \mu_{CP}^A \quad \textbf{(15)}$$

where $k(1-\sigma)/\sigma$ indicates by how much the value of the collateral changes in equilibrium when there is a change in tradable good consumption. Note that this factor is directly proportional to the fraction of current income net of liabilities that agents can use as collateral (k), and to the relative size of second-period non-tradable consumption.

Similarly to the decentralized case, when the financial restriction is binding, equation (5) determines the debt level and tradable consumption is set by equation (3). Therefore, given an initial debt level d_1, second-period tradable consumption and debt will be equal to those in the decentralized equilibrium. However, the valuation of liquidity differs. By comparing the first order conditions (6) and (14), it is possible to see that in a constrained economy ($\lambda_{CP}^{A} \geq 0$) the valuation of liquidity is greater or equal when there is a CP, that is, $\boldsymbol{\mu_{CP}^{A}} \geq \boldsymbol{\mu^{A}}$. This happens because the CP takes into account the indirect effect of an increase in tradable consumption ($\lambda_{CP}^{A}k(1-\sigma)/\sigma$), which increases the price of non-tradable goods and looses the financial restriction for all agents by $k(1-\sigma)/\sigma$, which has a shadow value of λ_{PC}^{A}.

This implies that the planner chooses an initial debt level lower than the one chosen by a decentralized consumer, as shown in the next section.

3.3. Initial debt level

In the initial period the function to maximize with respect to d_1, both for the decentralized consumer and the CP, is:

$$u(d_1/R_1) + \frac{1}{2}V^A(d_1) + \frac{1}{2}V^F(d_1)$$

where $V^i(d_1) = \beta u\left(c_2^i(d_1)\right) + \beta^2 c_{T,3}^i(d_1)$, $i \in \{A, F\}$, is the value function from the utility maximization in periods 1 and 2. The first order condition is:

$$u'(c_1) = -\frac{R_1\beta}{2}\sum_{i=A,F}\left(u'(c_2^i)\frac{dc_2^i}{dc_{T,2}^i}\frac{dc_{T,2}^i}{dd_1} + \beta\frac{dc_{T,3}^i}{dd_1}\right) \quad \textbf{(16)}$$

In the favorable state the economy is not constrained thus the solution uses the fact that, in equilibrium, second period consumption can be expressed as $\bar{c}_{T,2}(R_2\beta, \sigma, y_N)$; this is associated with an equilibrium level of debt $\bar{d}_2^F/R_2 = \bar{c}_{T,2}(R_2\beta, \sigma, y_N) + d_1(1+\theta) - y_T$, as explained above. The solution is the same (d_1^*) for both the decentralized consumer and the CP. As mentioned in section 3.1, if the value d_1^* does not satisfy condition (10), private agents cannot obtain this debt level and therefore the economy will be financially constrained.

When the economy is constrained, we should take into account that $d_2^A/R_2 = k\left(y_T + p_2^A y_N - d_1(1+\theta)\right)$, $c_{T,2}^A = y_T + d_2^A/R_2 - d_1(1+\theta)$ and $c_{T,3}^A = y_T - d_2^A$. Using these equations, the first order condition for the decentralized economy (where p_2^i is taken as given) is:

$$u'(c_1) = \frac{R_1\beta}{2}\left[\left((1+k)\mu^A - kR_2\beta\right)(1+\theta) + R_2\beta(1-\theta)\right] \quad \textbf{(17)}$$

However for the CP the effects on p_2^i are taken into account. Equations (11), (12), (14), and (15) show that:

$$u'(c_1) = \frac{R_1\beta}{2}\left[\left((1+k)\mu_{PC}^A - kR_2\beta\right)(1+\theta) + R_2\beta(1-\theta)\right] \quad \textbf{(18)}$$

for the CP in the constrained case. Given that $\mu_{CP}^A \geq \mu^A$, the level of initial debt chosen by the CP would be lower than the one chosen by decentralized agents ($\boldsymbol{d_{1,CP} \leq d_1}$)[13].

4. Externality and Pigouvian Tax

As shown in the previous section, decentralized agents undervalue the social cost of debt. For them it is completely rational to take the real exchange rate as given since the impact of their individual actions on it is essentially zero. This behavior does not represent a problem in an economy with complete and distortion-free markets. However, when there are financial constraints, the impact of past debt on asset prices and the exchange rate ends up affecting the agents' borrowing capacity as well.

A central planner who is aware of the negative externalities generated by individual's debt and consumption decisions can improve general welfare by choosing a lower initial debt

[13] The same result ($d_{1,CP} \leq d_1$) holds for the case of an economy that is always constrained (independently of state i). In that case, the conditions are: $u'(c_1) = (R_1\beta/2)\left[\left((1+k)\mu^A + k(\mu^F - 2R_2\beta)\right)(1+\theta) + \mu^F(1-\theta)\right]$ and $u'(c_1) = (R_1\beta/2)\left[\left((1+k)\mu_{CP}^A - kR_2\beta + (k\varphi/\sigma)(\mu^F - R_2\beta)\right)(1+\theta) + \mu^F(1-\theta)\right]$, where $\varphi = \sigma/(\sigma - (1-\sigma)k)$. Note that these conditions are equal to (17) and (18) when $\mu^F = R_2\beta$ y $\mu_{CP}^F = R_2\beta$, respectively.

level and thereby enhancing future levels of liquidity, borrowing capacity and aggregate demand. This will also mitigate the negative amplification effects of debt on the economy.

One way of implementing the central planner solution is by imposing a tax τ on the level of initial debt (which could be returned as a lump sum transfer T to the consumers). In this case, equation (2) becomes $c_{T,1} = d_1(1-\tau)/R_1 + T$. τ must be set so that condition (17) with the tax included results in the same level of debt as in condition (18)[14].

Using the first order conditions given by equations (6), (8), (14) and (15), the multipliers μ^i_{CP} and μ^i can be written in terms of those associated with the financial constraint for a decentralized economy (λ^i). Then the optimal tax can be expressed as:

$$\tau_\theta = 1 - \frac{(\lambda^A/\mu^A)[(1+k)(1+\theta)-2]+2}{(\lambda^A/\mu^A)[\varphi(1+k)(1+\theta)-2]+2} \tag{19}$$

where $\varphi = \sigma/(\sigma-(1-\sigma)k)>1$. Note that expression (19) gives a theoretic foundation to the discrimination of the optimal tax level by debt type through incorporating the volatility of debt (θ)[15].

Korinek (2010, 2011a) does not include the volatility θ in his theoretical analysis. Once he obtains an expression for the optimal tax level he performs an empirical estimation for each type of debt assuming that for two assets with volatilities $\theta_a > \theta_b = 0$, then $\tau_{\theta_a} =$

[14] The value of τ is obtained by solving:

$$\frac{\big((1+k)\mu^A - kR_2\beta\big)(1+\theta) + R_2\beta(1-\theta)}{1-\tau} = \Big((1+k)\mu^A_{CP} - kR_2\beta\Big)(1+\theta) + R\beta(1-\theta) + \mu^F_{PC}(1-\theta)$$

[15] The parameter θ enters equation (19) not only explicitly but also implicitly through λ^A/μ^A. However, in order to differentiate the tax by asset (or debt) type, λ^A/μ^A is irrelevant because it does not depend on the volatility of a specific asset but on the summation $\sum_j d^j_1(1+\theta_j)$ (see footnote 11) and, therefore, does not vary by asset type.

$(1 + \theta_a)\tau_{\theta_b}$. By contrast, the analysis in this paper shows that the ratio $\tau_{\theta_a}/\tau_{\theta_b}$ does not necessarily increase one to one with the value of $1 + \theta_a$, as explained in the next section.

5. Empirical approximation

In the spirit of the approximation of sufficient statistics described by Chetty (2009), equation (19) has been derived so that on the one hand θ is shown separately in order to being able to calculate the tax by asset type and, on the other hand, all the other structural parameters of the model are kept grouped to use the agents' first order conditions.

The main idea of this approximation is that instead of trying to identify all the relations that are part of the model structure, the calibration is concentrated on those variables relevant for the study (in this case, optimal tax value). In this way it is possible to reduce the number of components to be identified while keeping the primitive parameters of the model grouped in elements directly related to the first order conditions (e.g., elasticities or, as is the case in the present paper, Lagrange multipliers).

By not requiring that the variables to estimate be directly related to specific parameters and, therefore, to the exact structure of the model, the theoretical results can be presented in a simple and tractable way[16]. Calibration results can also be valid for more general forms of the same model or for models with similar structures. Furthermore, by reducing the number of elements to identify, the sufficient statistics approach allows for minimum data availability requirements[17].

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[16] In equation (19): the expression for λ^A/μ^A in terms of primitive parameters has a closed form only for very particular cases, e.g. logarithmic utility functions.

[17] For example, to calculate the optimal tax in this document by estimating primitive parameters would require, among other things, to estimate the value of interest rates and volatility of every single asset relevant

The aforementioned advantages certainly come with some costs. The main drawback of not expressing the tax in terms of the structural parameters of the model is that it becomes impossible to perform an appropriate sensibility or counterfactual analysis of results[18]. As a way of compensating for this disadvantage, and to have a benchmark of how reasonable or robust the estimation is, where possible the intermediate results or final values are compared to those obtained in other recent studies related to this document.

To calculate the value of the optimal tax, the reference point is information taken from the 1998-1999 period which is considered the most recent period of financial crisis in Colombia (see, e.g. Villar et al., 2005; Gómez and Kiefer, 2006), characterized by economic recession, a collapse in the exchange rate band and a strong credit contraction.

Based on equation (19), the estimation of the tax value requires calculating five components[19]:

- The tightness of constraints, λ^A/μ^A. This factor measures the marginal increase in utility that comes from loosening the financial constraint, normalized by the marginal valuation of liquidity. Similarly to Korinek (2010), it is assumed a utility function with constant relative risk aversion (CRRA) and the change in marginal

for the entry of capital flows to Colombia. This data requirement comes from the fact that λ^A/μ^A incorporates information on $c^A_{T,2}$ and d^A_2, and estimating them calls for information on assets previously mentioned (see footnote 11).

[18] For example, it is not appropriate to analyze the changes in the optimal tax value because of changes in k using equation (19) if it is not taken into account the fact that k affects the value of λ^A/μ^A.

[19] The model in Korinek (2011b) requires the estimation of 9 parameters, 10 in that of Mendoza (2010), 12 in that of Bianchi and Mendoza (2011), and 15 in that of Bianchi (2011). None of these models differentiates the value of the tax by the type of capital flows. However, some of them make a stochastic modeling of income flows (e.g., 8 out of the 15 parameters estimated by Bianchi (2011) are required to this end).

utility is approximated by the change in real consumption during the crisis. Using γ as the risk aversion coefficient, the estimation for Colombia is[20]:

$$\frac{\lambda^{A}}{\mu^{A}} \approx -\gamma \cdot \Delta C^{\text{crisis}} + \frac{\gamma(1+\gamma)}{2}\left(\Delta C^{\text{crisis}}\right)^{2}$$

$$\approx -\left(\frac{1}{0{,}42} \cdot -4{,}1\%\right) + 4{,}02 \cdot (-4{,}1\%)^{2} \approx 9{,}1\%,$$

using the value estimated by Prada and Rojas (2010) for the intertemporal elasticity of substitution ($1/\gamma$) for Colombia.

- The proportion of non-tradable consumption, $1-\sigma$. It is estimated as an average of the ratio of real production of non-tradable goods to real total consumption for the period 1996Q1-2011Q3, $1-\sigma \approx 71\%$ [21].
- The proportion k which measures the borrowing capacity of the economy. This value, as suggested by the model, will only be observable during a crisis[22] (when the financial constraint is satisfied with equality). To estimate it, the ratio between foreign debt in constant pesos and real GDP for 1999 is used[23]. In this case[24]:

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[20] The calibration of this ratio is made based on a second order Taylor approximation of the Euler equation in Korinek (2010), which is similar to that in this document (equation (8)), but more general because it corresponds to a model with infinite periods. In Korinek (2010), the equation is $\lambda/\mu = 1 - u'(c_{t+1})/u'(c_t)$, under the postulate that βR =1. The second order Taylor approximation around $c_{t+1} = c_t$ results in $\lambda/\mu \approx -\gamma(\Delta c_{t+1}/c_t) + \gamma(1+\gamma)(\Delta c_{t+1}/c_t)^2/2$. Korinek (2010) recurs to a first order approximation. However, in the present document, the second order effects are significant for the estimation at the second-decimal level.

[21] The value is highly stable throughout the period reaching a minimum of 69% and a maximum of 73%. The value is also similar to the one calibrated by Bianchi (2011) for Argentina (69%).

[22] For its numerical examples, Korinek (2010) takes the maximum value of debt over GDP estimated by Reinhart et al. (2003) in 50%. However, maximum values could be reached at times when the economy is not going through a crisis (e.g. times of high liquidity and international financial boom) and, therefore, because the economy is not financially constrained, those values would not be a good approximation of k.

[23] When solving equation (5) for k (when the restriction is binding) the result suggests that this parameter should be estimated as the ratio between the current debt level and current income net of liabilities. However, the simplifications in the model do not make that approach necessarily the most precise one. For example, in the model, debt is paid back in one period, but in reality it could be amortized at different time frames. For this reason, a small modification is made, and the borrowing capacity of a constrained economy is measured

$$k \approx \frac{Debt_{99}}{GDP_{99}} \approx 33{,}6\%$$

- The volatility by debt or asset type, θ. In the model, θ represents the volatility in the real value of debt (net of interest) with respect to its original value. To illustrate the effect of this volatility on the optimal tax, this document assumes two asset types. On the one hand, the estimation is made for a claim (noted type *b*) contracted in pesos indexed to CPI so that its real value does not change. In this case, $\theta_b = 0$. On the other hand, the estimation is made for a claim (type *a*) contracted in dollars, thus its real value is affected by changes in both the exchange rate and the CPI. With data for 1996-2011, the average absolute value of the ratio (devaluation-inflation)/(1+inflation) is $\theta_a \approx 9{,}5\%$. Note that estimating θ as an average for the whole period is probably the approach most consistent with the model, where it is assumed that debt value variations do not depend on a crisis occurring (but the latter does depend on the former). However, as an alternative interpretation and additional exercise, a higher value of θ from the crisis is used. For the Sept. 1998 - Aug. 1999 period, the value of (devaluation-inflation)/(1+inflation) is $\theta_{a'} \approx 24{,}1\%$.

- With the information specified above, enough information is available to calculate the optimal tax rate that would be imposed on the debt level the year previous to a crisis. However, given that it is difficult to anticipate a crisis and, even if it were

in a more standard way and similar to other documents (e.g. Korinek, 2010), using the debt level as a proportion of GDP.

[24] The attained value lies among those estimated by Bianchi (2011) (32%) and by Bianchi and Mendoza (2011) (36%) who calibrated the value of k so that their models would reproduce the likelihood of a crisis observed in Argentina and the United States, respectively. In a similar exercise, Mendoza (2010) estimates a value of 20% for Mexico.

possible, that the tax would be too high if imposed on a specific period (as a percentage of current total debt), it seems convenient to spread it out among all periods. To that end, the externality value is multiplied by the probability of a crisis, assumed to be 5%, the same as in Korinek (2010) for Indonesia, similar to the value used by Bianchi (2011) for Argentina (5,5%) and higher than the value obtained by Bianchi and Mendoza (2011) for the United States (3,2%).

Using equation (19) and the aforementioned values, the results are:

Asset type	***Estimated optimum tax***
In pesos indexed to CPI	$\tau(\theta_b) \approx 1{,}13\%$
In US dollars	$\tau(\theta_a) \approx 1{,}20\%$ $\tau(\theta_{a'}) \approx 1{,}31\ \%$

Those results suggest that the optimum tax value, by asset type volatility, would lie between 1,1% and 1,3%. This value is higher than the one found by Jeanne and Korinek (2010b) for the United States (0,56%), lower than the one by Bianchi (2011) for Argentina (5%) and closer to the estimations made by Bianchi and Mendoza (2011) for the United States (1,1%) and by Korinek (2011b) in a multi-country model with data from the World Bank´s Global Development Finance (1,89%).

Korinek (2010) provides optimum tax estimations by asset type for Indonesia. He obtains a lower tax rate for the asset in local currency (Rupiahs) indexed to CPI (0,7%), but a tax rate closer to the one in the present document for the asset in US dollars (1,54%). It should be noted that for Indonesia the value of $\theta_{a'}$ (118%) is sizably higher than the one for Colombia. As explained above, Korinek (2010, 2011a) does not obtain the discrimination by asset type from a theoretical model, and his empirical calculations implicitly assume that the value of the optimum tax should increase one to one with the value of $1+\theta$ (see Section 4). However, given that private agents are risk averse, some of the increase in the social cost (due to a higher volatility) is taken into account by them and, therefore, the externality in this model increases less than one to one with the value of $1+\theta$.[25]

6. Conclusions

This document follows the recent economic literature which suggests that it is convenient to take prudential policy measures in order to reduce financial vulnerability in times of crisis. The convenience of these measures lies in the existence of international financial market imperfections (e.g., economies face credit constraints) and in the fact that private agents do not internalize the social costs of their consumption/debt plans, which have an effect on prices and, ultimately (when the credit constraint binds), on the borrowing capacity of the economy.

Since decentralized agents (rationally) underestimate the impact of their decisions, there is a negative externality that amplifies social costs in times of crisis. This externality can be corrected by imposing a Pigouvian tax on capital inflows, as suggested in this document.

The model is similar to others described in the related literature and, as a contribution, incorporates the volatility by debt (or asset) type in order to give a theoretic foundation to the estimation of the optimum tax differentiating by volatility.

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[25] In the present document, using equation (19), and remembering that *a* is the asset with higher risk and *b* the asset with lower risk, it happens that:

$$\tau_{\theta_a} = \frac{x_1}{x_1+x_2}(1+\theta_a)\tau_{\theta_b}$$

where $x_1 = (1+k)\sigma(2-\lambda^A/\mu^A) - 2k(1-\lambda^A/\mu^A)$ and $x_2 = \sigma(1+k)(\lambda^A/\mu^A)\theta_a$. Using the estimated values explained in the document, $x_1/(x_1+x_2)$=0,97. Furthermore, the ratio $x_1/(x_1+x_2)$ is decreasing in the volatility level.

Using Colombian data for the 1996-2011 period (and considering 1998-1999 as the period of financial crisis), an empirical exercise is performed to calculate the value of the optimum tax. The results suggest that the tax on capital inflows would be set around 1,2%, which is of similar magnitude to those obtained in related literature.

Considering differences in volatility, the estimation was made for two debt types, CPI-indexed Colombia pesos and debt US dollar debt. The results suggest that the optimum tax value for the first case would be 1,1%, and it would lie in the 1,2-1,3% range for the second type of debt. The difference between the two cases is small since in the last decades Colombia did not experience large values of devaluation minus inflation.

The estimation is based on simplifying assumptions that make the calculation susceptible to improvement in different aspects and that could serve as motivation for future research. Some extensions of this model are, firstly, to model the productive sector and the accumulation of capital in order to endogenize agent's income flows. Secondly, to attach the probabilities of debt states to the conditions of the economy. Thirdly, it could be possible to introduce multiple debt states so that the financial constraint is not necessarily attached to the worst state.

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