Learning and signals in a New Keynesian open economy under discretionary monetary policy

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Abstract

This research points at assessing the importance of communicating central bank's forecasts to private sector under discretionary monetary policy. The practice of issuing signals proves to be effective in reducing a welfare based expected loss measure in a New Keynesian open economy where the private sector learns adaptively by means of recursive least squares.
This issue is addressed both from an analytical and a numerical point of view. Outcomes for a benchmark economy are simulated, then these are compared to outcomes from the same economy under the assumption of prior expectations updated accordingly to a specific signal. Finally, the importance of the degree of openness is assessed.

1 Introduction

This paper studies the implications of communicating central bank's forecasts to private sector in a New Keynesian open economy when the former adopts a discretionary policy that is optimal under rational expectations, and the latter has an imperfect knowledge about the true structural equations underlying the economy. The imperfect knowledge that characterizes the private sector is modeled accordingly to the standard assumptions for an adaptive learning framework (Evans and Honkapohja, 2001). The analytical conditions for stability under learning are figured out, and the short run dynamics have been simulated in order to evaluate the impact of signaling on learning transitory equilibria (along which the parameters of the rational expectation equilibrium are converging). Signaling is incorporated into the model by assuming that the private sector observes a vector of forecasts computed accordingly with the true equations of the economy. The information used in determining the signal is owned by the central bank only, which also utilizes it in determining its policy on a period basis. Monetary policy is intended to optimize in each period the value of the instrument accordingly to a welfare based loss function, under the assumption of the world operating with rational expectation. That is to say, in this particular economy, that the central bank does not know how the private sector computes its own expectations.
In a relevant number of situations it is reasonable to assume that the private sector and the monetary authority have asymmetric information about the environment in which they operate. In this work it is assumed that the central bank knows the exact structural equations of the economy, linearized around the zero inflation steady state (namely, in the New Keynesian environment,

the dynamic IS and the usual Phillips curve), as well as the data until the current period. On the contrary, by assumption the private sector's information set does not include any knowledge about the structure of the economy, although it observes available data with one period lag. Accordingly with the literature on learning, the private sector estimates the DGP of the model by making assumptions about the arguments in the rational expectations equilibrium. The forecasts that "enter" the model are the ones coming from the private sector, whose optimizing decisions in an ideal world with rational expectations are resumed in the linearized equations of the economy. The central bank computes its own forecasts as if the world operates under RE, thus using a rational expectations-consistent DGP. It is worth emphasizing that both the economic players have a misspecified model, hence, none of their forecasts will be actually correct. In fact, as it is evident, the private sector lacks a relevant piece of information and is not able to formulate expectations rationally. On the other hand, the central bank's representation of the economy is misspecified because, in evaluating the optimal response, it assumes that the world operates under rational expectations, which is in contrast with the private sector's behavior.

The motivation of this work lies in the possibility, that it offers, to reduce the misspecification of the representations that economic players have of the economy, moving from basic assumptions about information endowments that are standard in the literature about learning for the same class of models. The key feature of this work is the presence of a mechanism which allows the private sector to extract some information from the central bank forecasts. The mechanism described below in the text implies that the final expectations used by agents are a combination of a prior, determined by a usual recursive least squares estimation, and a signal constituted by central bank forecasts. The relative weights are determined endogenously as a result of the historical effectiveness of the signal in being relatively more informative than the prior. This last feature is particularly important as it allows the private sector to decide wether to give a large importance to the signal or not, by using the same tools that are used in the estimation of the prior, i.e. statistical comparison with actual data. Absent this feature, the private sector's choice of how much to evaluate the signal's importance would remain exdogenous, or at least its proper definition would require some additional assumption. In fact, the usual assumptions about the information endowment of the private sector hamper it from judging wether the signal comes from a different (possibly larger) information set. The private sector does not have the proper tools to assess the relative accuracy of the signal with respect to the prior, as it cannot recognize wether the central bank's behavior is driven by a scheme derived from a larger information set. On the other hand, as remarked by Eusepi and Preston (2010), "the presence of unanchored private beliefs that are not consistent with monetary policy strategy represents a challenge for stabilization policy, so that the way is opened for the design of the role of communication in monetary policy. "Hence, if it is reasonable to assume the existence of heterogeneous information across players, and if it is reasonable one's interest in spreading different knowledges across players, then, it may also be plausible the assumption that the signal is filtered by a bayesian mechanism of signal extraction, that is precisely what this paper proposes.

An extensive literature analyzes monetary policy in economies characterized by imperfect knowledge and learning. Moving from the seminal works by Evans and Honkapohja (2003a), Bullard and Mitra (2002) and Bullard and Mitra (2007), some recent examples are Bullard and Schaling (2009), which provides a suitable analytical framework for learning in open economies á la Galí and Monacelli (2005), Milani (2011), which examines the role of expectation shocks as a determinant of the business cycle in a learning environment, Berardi (2009), figuring out the implications of heterogeneous learning agents with misspecified expectations. Honkapohja and Mitra (2005) determine the E-stability conditions in a basic New Keynesian economy under the assumption that both central bank and private sector compute their own expectations by a recursive learning algorithm, and no communication is allowed. Gaspar et al. (2010) provide a numerical analysis with useful insights about optimizing policies computed assuming that agents learn adaptively the correct DGP of the economy. Ferrero and Secchi (2010) consider the role of central bank 's macroeconomic projections in a fashion that is similar to what is done in this paper but with substantial differences, namely the presence of exogenous weights to the prior and the signal while in this work these are completely endogenous, the monetary policy regime based on a Taylor rule whereas this paper treats discretional optimal (under RE) monetary policy, the absence of the open economy extension and the focus on the speed of learning, while this paper focuses on short run dynamics.
The contribution of this work to the existing literature consists in trying to lessen a restriction that arises when different information sets coexist, and each player uses its own knowledge (although incomplete) to produce forecasts. In particular, the restriction here addressed is the one that does not allow players in the economy to exchange information in a reasonable way. By reasonable it is meant a procedure that is able to define the information extraction procedure in terms of the knowledge owned by the involved players. In many cases it would make sense the existence of some form of communication, for example across private agents or between central bank and private sector, like the case that is here analyzed. This paper proposes a simple mechanism to model unidirectional communication of a particular kind, i.e. forecasts about the endogenous state variables. In particular the communication takes the form of a signal issued by the central bank and utilized by the private sector to extrapolate some kind of relevant information, and this is done in a fully specified New Keynesian model. The kind of restriction to communication here addressed is quite relevant in the literature and is common to environments where central bank behaves optimally with respect to some benchmark, and such a benchmark is not inferable or replicable by the private sector - as is the case of learning agents and policies that are optimal under RE. Some example in the literature can be found, among others, in Evans and Honkapohja (2003b), Berardi (2009), Preston (2006).
With respect to the information structure of learners in this paper, it is here assumed that the private sector can observe the realization of the state variables with one period lag, which is a typical assumption in an adaptive learning environment. On this basis, the private sector utilizes a statistical algorithm to predict the variables for which they do not know the true DGP. In order to achieve this goal, the private sector estimates the parameters of the guessed rational expectation

equilibrium, which can be represented as a VAR. The standard assumption in the literature is that the perception of the REE is not misspecified, such that the true variables are in the estimated equation. Thus, what remain unknown are the parameters only. Hence, the private sector's prior forecasts are based on their own VAR model. A different approach in this sense was undertaken in Berardi (2009), where it is analyzed a framework where the estimated DGP is misspecified. The monetary authority has a different model, which, on the contrary, is a correct representation of the structural linear version of the economy's equations. Therefore, the central bank is able to observe shocks' as they realize and endogenous state variables with a one period lag. Hence it relies on the true rational expectations-consistent DGP of the economy, and is assumed to make efforts to induce the private sector to utilize its predictions. Nevertheless the private sector is not able to recognize the superiority of the central bank's forecasts, or the correct model underlying them. The best it can do is to compute *ex post* the prediction error of the signaled forecasts and then determine a weight to the signal based on its past effectiveness. Then, the prior is improved by the weighted signal. This composite expectation is the result of the combination of information from different sources, and is then utilized from agents to make their economic decisions. These expectations enter the model, then actual values of the endogenous state variables realize. It is worth to remark that the perceived law of motion of the economy will not be equal to the actual law of motion until the estimated parameters have converged to the REE value, such that the true DGP and the one used by private sector coincide. The parameter's convergence in real time learning and the E-stability conditions are checked and verified in the text.

The numerical analysis conducted in this work compares simulated results from a benchmark model and from a model with expectation improvement by signal extraction. The benchmark model is the model with adaptive learning without signal, i.e. private sector utilizes only its own model to predict relevant variables. Issuing the signal as it is determined in the model, i.e. such that the composed exclusively by central bank's expectations, is welfare improving in the limit. Output gap's average is higher while inflation's level is lower and both are are close to the respective target values. Both the endogenous variables show lower variances in the model with expectation improvement. The simulation of short run dynamics show a smoother economy's response under the signal hypothesis.

The remainder of the paper is organized as follows. Section 2 introduces the baseline equations and the way expectations are formed. In Section 3 the methodology used to simulate the model presented in Section 2 is explained, then the results of the numerical analysis are showed. Section4 concludes.

2 The model

The true structure of the economy is a Galí and Monacelli (2005) open economy. The baseline model is augmented for adaptive learning as in the model analyzed by Bullard and Schaling (2009). The original model from Bullard and Schaling (2009) is built as a two-country model with flexible exchange rates, where the two economies are perfectly symmetric. One of the findings of Bullard and Schaling's paper is that the world economy's matrix, i.e. a matrix containing the two economies' structural equations is block diagonal, unless the monetary policy in the two countries is determined on a cooperative basis. In what follows, cooperation among different central banks is not treated, such that the results obtained for a single open economy can be extended to the other economy in the model, by appropriate parameters' notation adjustments. The economy is described by the following equations, representing the usual dynamic IS and the New Keynesian Phillips curve

$$x_t = Ex_{t+1} - \sigma_0^{-1} \left(i_t - E\{\pi_{t+1}\} - \bar{rr}_t \right), \tag{1}$$

$$\pi_t = \beta E \pi_{t+1} + \lambda_0 y_t + u_t, \tag{2}$$

where $\kappa_0 = \gamma (\sigma - 1)$, $\sigma_0 = \sigma - \kappa_0$, $\kappa = \sigma - \kappa_0 + \phi$, $\lambda_0 = \delta\kappa$, $\delta = \theta^{-1} (1 - \theta) (1 - \beta\theta)$. The variable x_t represents the domestic output, while the variable π_t represents the domestic producer price inflation. The parameter γ represents the degree of openness, such that for $\gamma \to 0$ the foreign economy gets increasingly relatively smaller, and all goods tend to be produced and consumed abroad. The parameter θ represents the probability that a firm will not be able to change its price in the current period. The term u_t is an exogenous disturbance following an AR(1) process, such that

$$u_t = \rho u_{t-1} + \epsilon_t,$$

and ϵ_t is white noise. The domestic real rate of interest rr_t , conditional on the foreign output is determined by the following equation:

$$\bar{rr}_t = \sigma_0 E \Delta \bar{y}_{t+1} + \kappa_0 E \Delta y^*_{t+1}, \tag{3}$$

where $\Delta \bar{y}$ represents the rate of growth of the natural level of output and Δy^* represents the rate of growth of the level of foreign output. The nominal exchange rate evolves accordingly with a CPI-based purchasing power parity

$$e_t = p_t - p^*_t + \bar{\eta}_t, \tag{4}$$

where p_t and p^*_t represent respectively the domestic and the foreign producer price level.[1] The relationship between terms of trade (η_t) and output differentials is linear, as in the following equation:

$$\eta_t = y_t - y^*_t + \bar{\eta}_t, \tag{5}$$

where $\bar{s}$ is the natural level of the term of trade.

In each period the monetary authority determines the instrument interest rate in order to minimize the following welfare based loss function:

$$L_t = (1 - \gamma) \Lambda E \sum_{\tau=t}^{\infty} \beta^{\tau - t} 0.5 \left[\pi_\tau^2 + \alpha_0 x_t^2 \right], \tag{6}$$

[1] The linkage to the CPI is worked out in Bullard and Schaling (2009), Section 2.2.

where $\Lambda = \xi/\delta$ and $\alpha_0 = \lambda_0/\xi$ and the target values for inflation and output gap are set equal to zero. It is worth remarking that the minimization of the loss function (6) would be optimal only if the world was composed by fully rational agents with a complete knowledge of the economy. In the adaptive learning framework that is used in this paper, the minimization of the aforementioned loss function does not necessarily results in optimal policies. However, as argued by Gaspar et al. (2010), the fundamental policy prescriptions under model consistent expectations (in this case rational expectations) still hold under adaptive learning. In particular they are even strengthened due to the increased central bank's power to anchor inflation expectations.

As remarked in Clarida et al. (1999), the discretionary monetay policy problem above stated can be approached as a sequence of static problems. In this case the central bank utilizes the indirect control variable $y_\tau = \{t, t+1, \ldots, \infty\}$ to minimize L while treating $E\pi_{t+1}$ as given. In order to compute the RE-optimal response function the central bank minimizes (6) s.t. equations (1), (2) and (3), given the definition for u_t. By combining the first order conditions of the lagrangian, it is obtained the typical "leaning against the wind" optimality condition,

$$y_t = -\frac{\lambda_0}{\alpha_0}\pi_t, \tag{7}$$

that is used to determine the appropriate instrument rule. By substituting equations (7) into (1) and substituting the result into (2) by using (3), then solving for r_t, it is possible to extrapolate the instrument rule that always satisfies the RE-optimal condition. As remarked by Evans and Honkapohja (2003b) and Evans and Honkapohja (2006) about a similar closed economy model, in order to avoid indeterminacy and e-instability, the appropriate RE-optimal instrument rule should take the expectations' formation mechanism outside the determination of the rule. Hence, expectations should be considered as given. Therefore, the following instrument rule is going to be used in the model

$$i_t = \bar{rr}_t + \phi_x x^{ex}_{t+1} + \phi_\pi \pi^{ex}_{t+1} + \phi_u u_t, \tag{8}$$

where

$$\phi_x = \sigma_0 \quad ; \quad \phi_u = \frac{\sigma_0\lambda_0}{\alpha_0 + \lambda_0^2}; \quad \phi_\pi = \frac{\lambda_0\sigma_0\beta}{\alpha_0 + \lambda_0^2} + 1.$$

After substituting the instrument rule into the IS (1) curve and again the IS into equation (2), the state space representation of the economy is then

$$y_t = MEy_{t+1} + Pu_t,$$

where $y_t = (x_t, \pi_t)'$ and

$$M = \begin{pmatrix} 0 & \frac{-\beta\lambda_0}{\alpha_0+\lambda_0^2} \\ 0 & \frac{\alpha_0\beta}{\alpha_0+\lambda_0^2} \end{pmatrix} \qquad P = \begin{pmatrix} \frac{-\lambda_0}{\alpha_0+\lambda_0^2} \\ \frac{\alpha_0}{\alpha_0+\lambda_0^2} \end{pmatrix}.$$

The non-zero eigenvalue of M is

$$0 < \frac{\alpha_0 \beta}{\alpha_0 + \lambda_0^2} < 1,$$

such that the RE equilibrium is unique. In particular, the RE solution of the model, i.e. the equilibrium arising when both the private sector and the central bank have a perfect knowledge of the economy's structure, can be identified by the method of undetermined coefficients assuming that the solution has the form

$$y_t = \mathbf{A} + \mathbf{C} u_t. \tag{9}$$

The REE coefficients, i.e. the limiting value of the parameters estimated by the private sector, are $\mathbf{A} = (0, 0)'$ and

$$\mathbf{C} = \left(\frac{-\lambda_0}{(1 - \beta\rho)\,\alpha_0 + \lambda_0^2}, \frac{\alpha_0}{(1 - \beta\rho)\,\alpha_0 + \lambda_0^2} \right)'.$$

2.1 Timing of the model

As a consequence of the information structure of the economy, when the current value of the state variables realize, it is not immediately observable in the current period by both private sector and central bank (even though the latter would have enough information to correctly compute it if it could compute the effect of the signal on $E^{ps} y_{t+1}$). The value of the state variables becomes observable only with a one-period lag. The only difference between private sector and central bank in terms of information set is that the latter knows exactly the structural form of the economy, while the former has to generate expectations by estimating in each period a guessed REE. Once the shocks are observed, the private sector is able to compute its prior about expectations by the standard real-time learning mechanism. In the meanwhile the monetary authority computes expectations too, which are based on information from the knowledge of the model's structure. Central bank's expectations are communicated to private agents, which in turn update their prior by conditioning old gaussian information to new gaussian information. Then, the relevant private sector's expectations are determined. These are the only relevant expectations because only these are utilized to make allocation choices, and therefore are the only entering the structural equations. Once the private sector's expectations are computed, the central bank is able to determine the interest rate that optimizes its loss function, according to the RE-optimal monetary policy regime. Hence y_t realizes and the period ends.

2.2 Private sector's prior

The private sector does not know the full structure of the economy, but is endowed with a guess solution that is presumed to mimic the actual law of motion of the economy. Such a guess has, by assumption, the exact linear form of the REE in equation (9), and includes also the same arguments (i.e. u_t). Private agents utilize this statistical models to predict the future value of the state variables by estimating the parameters c_t in the following guess solution:

$$y_t = c_t u_t. \tag{10}$$

Moving from this perceived representation of the economy, the private sector is able to consistently compute forecasts of y_{t+1} such that

$$E^{ps} y_{t+1} = \rho c_t u_t$$

where the superscript denotes the different type of expectation relatively to the RE-consistent central bank forecasts. These assumptions comply to the existing literature about adaptive learning quoted in Evans and Honkapohja (2001) and in a large part of the subsequent research. However, in this paper the role assigned to E^{ps} is quite different. In particular in each period it constitutes the private sectors's prior, which, within the same period, has to be updated according to new incoming information from an external source, that is the central bank.

2.3 Signals

In this model the central bank has a complete knowledge of the economy except for the expectations formation method adopted by the private sector. The central bank's information set includes the endogenous state variables y as well as the exogenous cost shock u up to one period in advance with respect to the period in which expectations are formed. Together with the knowledge of the economy's equations, this information set allows the central bank to determine monetary policy accordingly to a loss function which is minimized period by period taking private sector's expectations as given. The central bank is able to compute expectations according to its information set and to communicate them to the public. Nevertheless, the private sector does not have enough analytical tools to perceive that the central bank's forecasts are originated from the true economy's structure. Hence, in this economy the central bank acts as an information disseminator, as it makes efforts in spreading throughout the economy its somehow "privileged" information. The private sector observes the state variables with a one period lag with respect to the current period, just like the central bank, yet it is aware of the limitedness of its global knowledge about the economy's structure. For this reason central bank's forecasts are not rejected *a priori*, and, on the contrary, are included in the information set and utilized accordingly to the value that private sector is able to recognize. The importance the private sector assigns to central bank's forecasts may vary in each period as it is based on their past accuracy. In particular, the information extraction procedure is structured like a conditional mean of a gaussian variable (i.e. the private sector's prior) on another gaussian (i.e. the central bank forecast) as follows:

$$Ey_t = Ey_{t+1}^{ps} + \Phi_t \left(s_t - s_t^{ex} \right), \tag{11}$$

where E^{ps} stands for the private sector's prior expectation, s_t is the signal, i.e. a bivariate vector containing the central bank's forecasts of y_{t+1}. In particular the central bank's forecasts are based on the RE representation of the economy's law of motion given in equation (9), such that $s_t = \rho C u_t$. Φ is a matrix containing the time dependent weights attached to the signal with respect to its expected value s_t^{ex}. The time t value of Φ is determined as follows:

$$\Phi_t = Cov_t\left(y, s\right)' Var_t\left(s\right)^{-1}, \tag{12}$$

where all the required data are available to private sector at the moment of the computation. The expected value of the signal is conditioned to the private sector' s information set, therefore it coincides with the prior on the underlying variable, such that $s_t^{ex} = \rho c_t u_t$. The updating procedure is implemented in each period.

2.4 Adaptive Learning

The real time learning is implemented according to the guidelines provided in chapters 6 and 10 in Evans and Honkapohja (2001) and in Evans and Honkapohja (1998). The gain from learning decreases with time, such that the estimates converge to OLS values. In addition to the algorithm in the cited works, there are the equations related to the dynamics of the parameters in Φ_t. The moment matrix updating rule is

$$R_t = R_{t-1} + t^{-1}\left(z_t z_t' - R_{t-1}\right), \tag{13}$$

where $z_t = \left(1, v_t'\right)$. In order to convert the system in a convenient standard form, I make a timing change on R_t, i.e. $S_{t-1} = R_t$, so that

$$S_t = S_{t-1} + t^{-1}\left(z_t z_t' - S_{t-1}\right) + t^{-2}\left(\frac{-t}{t+1}\right)\left(z_t z_t' - S_{t-1}\right). \tag{14}$$

The parameters updating equation is

$$\xi_t = \xi_{t-1} + t^{-1} R_t^{-1} z_{t-1}\left(y_{t-1} - \xi_{t-1}' z_{t-1}\right)', \tag{15}$$

The parameter ξ_t includes the matrix of parameters to be estimated, such that $\xi_t' = (0, c)$.
A recursive formulation of the functions determining the current value of $\Phi_t = C_t' V_t^{-1}$ adds to the system of ODE, where $C_t = cov(y, s)$ and $V_t = var(s)$:

$$C_t = C_{t-1} + t^{-1}\left(y_t s_t' - C_{t-1}\right), \tag{16}$$

$$V_t = V_{t-1} + t^{-1}\left(s_t s_t' - V_{t-1}\right). \tag{17}$$

The real-time learning algorithm above is in the form

$$\theta_t = \theta_{t-1} + t^{-1} H\left(\theta_{t-1}, X_t\right) + t^{-2}\rho_t\left(\theta_{t-1}, X_t\right),$$

where $\theta_t = (\xi_t, S_t, V_t, R_t)$ and $X_t = (1, v_t, v_{t-1})$, whose convergence has been demonstrated to be driven by E-stability under conditions A and B in Section 6.2.1 in Evans and Honkapohja (2001). In order to prove that also the algorithm (14-17) converges, and that the convergence is driven by the E-stability conditions provided in Section 2.5, I need to analyze the equations governing vector X_t, that I put in the following form

$$X_t = A(\xi_{t-1}) X_{t-1} + B(\xi_{t-1}) W_t,$$

where

$$W_t^{'} = \begin{pmatrix} 1 & \epsilon_t^{'} \end{pmatrix},$$

$$A = \begin{pmatrix} 0 & 0 & 0 \\ T_a(\xi_{t-1}) & T_c(\xi_{t-1}) & 0 \\ 0 & F & 0 \\ 0 & 0 & 0 \\ 0 & I & 0 \end{pmatrix},$$

$$B = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & I \\ 0 & 0 \\ 0 & 0 \end{pmatrix}.$$

and zeros and I represent respectively matrices of zeros and identity matrices of appropriate dimensions.

By substituting $y_t = T(\xi_t) z_t$ into Equation 15, I obtain

$$\xi_t = \xi_{t-1} + t^{-1} S_{t-1}^{-1} z_{t-1} z_{t-1}^{'} (T(\xi_{t-1}) - \xi_{t-1}).$$

The stability of the above system of equations has to be intended as local around a fixed point of $T(\xi)$. Following Evans and Honkapohja (1998) I have to define a domain D of the algorithm. Let $\bar{\xi}$ be a a fixed point of $T(\xi)$. Assume also that the eigenvalues of b are strictly inside the unit circle. Given that $z_t'(\xi) = \left(1, y_{t-1}'(\xi), v_t'\right)$, and recalling that $y_{t-1} = T(\xi_{t-1}) z_{t-1}$, then $z_t(\xi)$ is a stationary process for all ξ sufficiently near $\bar{\xi}$. Let $M_z(\xi) = E(z_t(\xi) z_t'(\xi))$, and let $\hat{D}$ be an open set around $\left(\bar{\xi}, \bar{S}\right)$, where $\bar{S} = E(\bar{z}_t(\xi) \bar{z}_t'(\xi))$ so that for all $(\xi, S) \in \hat{D}$, $\bar{\xi}$ is the only fixed point of T, S and V are invertible, the roots of b are strictly inside the unit circle. Given that the decreasing gain function is t^{-1}, with this construction the conditions A in Section 6.2.1 are met. Provided that $\hat{D}$ is sufficiently small, conditions B are met since the moments of W_t are bounded.

The associate ODE of the algorithm (14-17) are computed taking expectations and limits, as follows:

$$\frac{d\xi}{d\tau} = S^{-1} M_z(\xi) (T(\xi) - \xi),$$

$$\frac{dS}{d\tau} = M_z(\xi) - S,$$

$$\frac{dC}{d\tau} = T(\xi) M_z(\xi) K' - C,$$

$$\frac{dV}{d\tau} = K M_z(\xi) K' - V,$$

where $K = (0, M\mathbf{B} + N, M\mathbf{C} + P)$ is such that $s_t = Kz_t$. The first two equations are standard, and as long as $S \rightarrow M_z$, convergence is driven by conditions $T(\xi) - \xi$, which turns out to be the E-stability condition of the model under examination. The third equation shows that the recursively estimated covariance converges to its theoretical value, as $Cov(T(\xi) z, Kz) = T(\xi) Cov(z, z) K'$. As long as $Var(Kz) = KVar(z) K'$, the fourth equation too converges to its asymptotic value. The algorithm (13, 15) is used to simulate the economy under learning in the following section. For further details about the link between e-stability and real-time learning I refer to the original book from Evans and Honkapohja.

2.5 E-stability

In the previous section it was showed that the recursive least squares learning is locally convergent to the REE. Also, the stability of the main associated ODE is driven by equation

$$\frac{d\xi}{d\tau} = T(\xi) - \xi,$$

where $\xi = (0, c)$. In this section it will be shown that such a condition corresponds to the E-stability condition for model (1) - (8), (11) and (12). Then I derive such conditions in closed form. The analysis of the stability under learning is conducted according to the methodology proposed in Evans and Honkapohja (2001). According to Bullard and Schaling (2009) the model without signal is determinate under RE and is e-stable for all parameter values. The actual law of motion of the economy (ALM) is

$$y_t = MEy_{t+1} + Pu_t$$

where

$$Ey_{t+1} = E^{ps} y_{t+1} + \Phi_t \left(s_t - s_t^{ex}\right) = \rho c_t u_t + \Phi_t \left(\rho \bar{c} u_t - \rho c_t u_t\right).$$

It follows that the mapping describing the evolution of the estimated parameters is

$$T(c) = \left(M[\rho (I - \Phi_t) c_t + \rho \Phi_t \bar{c}] + P\right).$$

By computing the jacobian ot vecT_c around the fixed point $c_t = \bar{c}$, I can compute the stability conditions of the following system:

$$\frac{d}{d\tau}(c) = T(c) - (c).$$

Thus, the model under examination is asymptotically stable under learning if the eigenvalues of the following matrices have real parts less than unity:

$$DT_c(\bar{c}) = \rho M.$$

Such a condition is met for every values of the structural parameters. Thus, the model that incorporates the prior updating procedure is always e-stable, and the evolution of the relative weights between the prior and the signal do not affect the asymptotic convergence to the REE.

3 Numerical analysis

In this section the model presented above is simulated by Montecarlo methods. The purpose of this exercise is a comparison between the model where priors are updated and the original models with no update, namely a RE version and a standard adaptive learning version of the model.

3.1 Methodology

The starting point of the simulation exercise consists in generating pseudo-random structural shocks, i.e. a time series u_t. Then, the general layout of the simulation in each period follows essentially three steps. The first step determines the signal from central bank and the private sector's prior expectation. In the second step the prior is updated accordingly with the signal and relevant expectations are computed. The third step determines the response of the central bank and the actual response of the economy, such that the current realizations of the state variables are generated.

The sequence above reflects the timing of the model, such that the output is determined only once expectations are formulated. As monetary policy reacts to private sector's expectations, that are formulated only after having received the signal, a simultaneous realization of the temporary equilibrium would not make much sense.

In this particular specification of the model the central bank is not interested in evaluating *ex ante* the impact of the signal on private sector's expectations, even though it has the tools to do so.

Before starting utilizing the signal to improve the prior, it is assumed that the private sector collects and estimates twenty periods (five years) of data. In this time interval, the central bank keeps sending the same kind of signal that is supposed to send in the following periods. Thus, the private sector utilizes the signal to produce a consistent estimate of the matrix Φ, without changing the prior. Once the five years have passed, and consequently a consistent estimate of Φ has been computed, the private sector starts using the signal in order to improve the prior, and also to improve Φ. From this period on, this is how the model normally runs. This assumption about different use of collected data in the early iterations of the model is useful to get rid of small sample issues related to estimation of the variances and covariances of states and signals.

Agents begin learning adaptively immediately, also during the data collection period. The starting point of their estimation is an arbitrary initial condition in which the economy is at its steady state, the priors about the MSV coefficients are actually equal to the RE solution and the initial moment matrix is a symmetric matrix of random realization of zero-mean variables with arbitrary variance smaller that the variance of the economy's shocks. The gain from learning is a decreasing function of time, such that $(t+5)^{-1}$. Thus, the learning improvements affect the prior less as a consequence of the simple passage of time. The presence of a constant that adds the value of t helps in reducing the relevance of the very early iteration of the learning algorithm that may be affected by small sample issues.

In each period the private sector estimates Φ_t by using sample covariance estimator, such that $\Phi_t = \mathrm{Cov}'_t(y,s)\,\mathrm{Var}_t(s)^{-1}$. In order to avoid unreasonable results due to singularity of matrix Φ, a projection facility is included in the algorithm routines that computes Φ. It is imposed that if the covariance matrix of the signal vector, $\mathrm{Var}_t(s)$, is nearly singular, it is substituted by $\mathrm{Var}_{t-1}(s)$. The reasonability of assumption is due to the nature of the estimation. Throughout the model PS acts as an econometrician, and as a such, it may be reasonable to subordinate the estimation tools to the economic meaning and usefulness of the estimated object. In this case, a singular Σ_t^s would not be of any help in adding economic meaning to the signal, that is the purpose of its estimation. Nevertheless, the consequences of a singular Σ_t^s would be devastating for the dynamics of the model, as it would let expectations diverge, reflecting this tendency to the whole economy. If this facility were used consistently, such that the validity of the model would be heavily questioned, it would make sense to get rid of it and to re-think the model. But, according to the simulations' results, its influence is very small in every configuration of the model that I tested.

In each experiment I control for the stream of pseudo-randomly generated numbers, such that they vary across simulations in each experiment, but in each experiment what actually changes is only the signalling regime. Also the random shock are all the same than in every experiment.

In conducting the numerical simulations, it has been made use of a calibration of the structural parameters of the model that has already appeared in the literature. In particular, following Bullard and Schaling (2009), the values of the structural parameters have been chosen so that, for $\gamma \to 0$, the values for the domestic economy collapse to the values applied to the closed economy studied in Woodford (2003). The simulation is conducted on a quarterly basis, such that the discount factor $\beta = 0.99$. The other structural parameters are summarized in Table 1.

In the performed exercise the length of each simulation is set equal to 2000, and the number of simulations is 500. The benchmark for the role of signaling is the "pure" adaptive learning economy, with no prior updating. The RE economy, with no learning, is utilized as a benchmark in the short run period analysis in order to assess the smoothing effect due to learning and how it affects learning itself.

3.2 Results

This subsection shows some results from simulations conducted in the long run and in the short run. The former type is interesting because it allows a comparison between pseudo-asymptotic values,

θ	ϕ	γ	σ	ρ	ψ
0.745	0.11	0.4	0.157	0.165	7.88

Table 1: Calibrated parameters

	No signal	Priors updating
$\bar{x}$	-0.0028	-0.0028
$\bar{\pi}$	$3.587E-04$	$3.566E-04$

Table 2: Means

in which the effect of the initial conditions fades away. Differences emerge also between the paths followed by output gap and inflation are along the transition to the REE. The short run analysis is helpful in understanding how the economy reacts to shocks and how quick is the reversion to the initial condition.
In Tables 2, 3 and 4 are shown the Montecarlo averages for mean values and variances of output gap and inflation, as well as the expected loss, computed accordingly with equation (6).The average value of the output gap is the same under both the learning regimes.
On the other hand, inflation's average value is lower under the assumption of bayesian prior's update. The target values of monetary policy for output gap and inflation are both equal to zero. Also the variability of the two endogenous state variables is different across the two experiments. Both the variances of output gap and inflation are lower if the private sector updates its prior with central bank's forecasts. These results appear to be consistent with the expected loss in Table 4, which is lower where the private sector updates its prior.
The long term simulation provides some support in favor of signaling the RE-consistent expectations to private sector, where available. The lower variance of the state variables results in a lower value of the welfare based loss function, although in both cases the largest part of the transition occurs within the first 150 periods, as shown in Figures 1 and 2. In particular, Figures 1 and 2 show the difference, in each period, between the value of output gap and inflation realized under a learning regime and the value that they would assume, after the same path of disturbances, if expectations were formed rationally. These figures depict the convergence of the economy towards the REE, as the estimated parameters converge to their RE value. Both the simulation exercises, with and without signal, converge to the REE as expected from the analysis in Section 2.5. The parameters' evolution is illustrated in Figures 3 and 4.

As far as the closed version of the economy is concerned, similar results are obtained. The

	No signal	Priors updating
σ_x^2	1.9846	1.9669
σ_π^2	0.0320	0.0317

Table 3: Variances

	No signal	Priors updating
Expected Loss	230.36	223.95

Table 4: Expected loss

presence of the signal seems to be effective in reducing the variability of the endogenous state variables. As a consequence, the economy's welfare is improved. Therefore, the isomorphism between the closed and the open economy in this New Keynesian framework, outlined in Galí and Monacelli (2005), is supported numerically also in the respect of a learning economy with priors updating. Tables 5 and 6 report the mean value and average variance throughout the simulations, generated by calibrating parameters as in Table 1 whilst assuming instead that $\gamma = 0$. The evidence from simulation suggests that reducing the degree of openness is generally detrimental to the economy, as, on average, it will experience lower and more volatile income, as well as higher and more volatile inflation. As a result, expected loss is higher in the $\gamma = 0$ case. Nevertheless, the effect of updating priors by including information from a correct structural model is beneficial to the economy in all the respects. Generated aggregate income is higher and inflation is lower than in the no-signal case. Both are less volatile, such that expected loss is lower. Interestingly, in the closed economy the effect of updating priors seems to be more pronounced.

A second branch of the analysis here conducted is concerned with short run simulations. Its purpose is to assess wether and how the utilization of the signal affects the way economy reacts to productivity shocks. The simulation experiment figures out the the reaction of inflation and output gap to a one percent cost push shock. The experiment consists of 1000 simulations along 500 periods. The shock occurs at time 20, i.e. after learning already showed a reversion to the REE values of the MSV solution and at the same time the private sector starts utilizing the signal to update its prior. Actually, in the priors updating version the private sector estimates the matrix Φ for 20 periods before taking into account the signal. Thus, the beginning of the usage period of the signal is contemporaneous to the shock. At the 20^{th} iteration the endogenous states are artificially taken to the steady state value, and the same is done until two periods earlier. This procedure is necessary in order to get rid of the transitional effect of learning on the state variable (temporary equilibrium) and then to compare the REE to the learning economy for a given value of the estimated parameters. The matrix c_t, i.e. the parameters of the guessed equilibrium, estimated by the private sector is unchanged with respect to the value resulting from simulations., so that the effect of learning is isolated, while left unaltered its effect on parameters estimations.
The main result emerging from the simulation is that the economy's responses are attenuated by the priors updating mechanism, as it is shown in Figures 5 and 6. As expected, when expectations are formed by adaptive learning, the response of the economy appears smaller than if they were formed according to the RE paradigms. The reversion to the initial (steady state) values is faster than in the RE model. There is no form of smoothing across periods. In each period the response under learning is lower in absolute value than the RE case. Among the learning versions of the

	No signal	Priors updating
$\bar{x}$	-0.0035	-0.0034
$\bar{\pi}$	$4.415E-04$	$4.35E-04$

Table 5: Means

	No signal	Priors updating
σ_x^2	3.0112	2.9378
σ_π^2	0.0485	0.0473

Table 6: Variances

economy, the "learning plus update" mechanism contributes to a further reduction in the response to the shock.

The same experiment ran under the assumption of closed economy, i.e. where $\gamma = 0$, shows isomorphic results. As it can be observed in Figures 7 and 8, the lowest responses in absolute value are obtained, respectively, in the "updating" case and in the standard AL mechanism.

By shifting ahead the time of the cost push shock, the response of the economy tends to be similar in the three environments. As time passes, the learning transition effect vanishes as the estimated MSV solution gets closer and closer to the REE. Figures 9 and 10 show the economy's responses to a shock occurring at time 200. The three curves are hardly distinguishable. The most plausible interpretation is that once the learning transition is completed, the impact of the signal on economy's dynamics vanishes, as the state variables move accordingly to the REE.

4 Concluding remarks

Given an economy in which central bank and a private sector have different information sets, this paper investigates the consequences of communicating central banks forecasts to the private sector. In the environment analyzed in this research the central bank has full knowledge of the structural equations that underlie the economy. Thus, it forms expectations consistently with its own representation of the economy, thus, in a different way than the private sector. Therefore, the central bank's expectations are used by the private sector as a signal about the future state of the economy. Hence, the private sector encompasses the signal about forecasts into the expectations formation mechanism, in order to improve its own prior expectations, which in turn are formed according to an adaptive learning procedure. From an analytical point of view, the presence of the signal in this particular model does not have implications on the stability under learning of

	No signal	Priors updating
Expected Loss	481.53	452.91

Table 7: Expected loss

the economy. The asymptotic convergence is granted by the usual e-stability conditions. As the dynamics of the economy shows convergence in the long term, some numerical experiments have been ran in order to assess the impact of the signal on the relevant statistics and on the short run dynamics, compared to the standard adaptive learning and the RE cases. The simulations, conducted by Montecarlo method, show that issuing this kind of signal is beneficial to the economy in terms of expected value of a welfare based loss function. When the private sector updates its prior by using the signal, the average values of output gap and inflation are closer to the target values that are used into the loss function than in the case of "pure" adaptive learning. Also the variability of output gap and inflation is lower in the priors updating case. Similar "isomorphic" results are obtained for the closed version of the economy. Nevertheless, the simulations show that in this case the welfare improvement generated by the signal is higher than the case of open economy. According to the simulations, issuing the signal is beneficial to the economy also in terms of short term response to cost push shocks. The presence of a signal implies that the economy responds less strongly whenever hit by a shock, such that the reversion to the initial conditions is faster.

References

Berardi, M. (2009). Monetary policy with heterogeneous and misspecified expectations. *Journal of Money, Credit and Banking*, 41(1):79–100.

Bullard, J. and Mitra, K. (2002). Learning about monetary policy rules. *Journal of Monetary Economics*, 49.

Bullard, J. and Mitra, K. (2007). Determinacy, learnability, and monetary policy inertia. *Journal of Money, Credit and Banking*, 39(5):1177–1212.

Bullard, J. and Schaling, E. (2009). Monetary policy, determinacy, and learnability in a two-block world economy. *Journal of Money, Credit and Banking*, 41(8):1585–1612.

Clarida, R., Gali, J., and Gertler, M. (1999). The science of monetary policy: A new keynesian perspective. *Journal of Economic Literature*, 37(4):1661–1707.

Eusepi, S. and Preston, B. (2010). Central bank communication and expectations stabilization. *American Economic Journal: Macroeconomics*, 2(3):235–71.

Evans, G. and Honkapohja, S. (2001). *Learning and expectations in macroeconomics*. Princeton University Press.

Evans, G. W. and Honkapohja, S. (1998). Economic dynamics with learning: New stability results. *Review of Economic Studies*, 65(1):23–44.

Evans, G. W. and Honkapohja, S. (2003a). Adaptive learning and monetary policy design. *Proceedings*, pages 1045–1084.

Evans, G. W. and Honkapohja, S. (2003b). Expectations and the stability problem for optimal monetary policies. *Review of Economic Studies*, 70(4):807–824.

Evans, G. W. and Honkapohja, S. (2006). Monetary policy, expectations and commitment. *Scandinavian Journal of Economics*, 108(1):15–38.

Ferrero, G. and Secchi, A. (2010). Central banks macroeconomic projections and learning. National Bank of Poland Working Papers 72, National Bank of Poland, Economic Institute.

Galí, J. and Monacelli, T. (2005). Monetary policy and exchange rate volatility in a small open economy. *Review of Economic Studies*, 72(3):707–734.

Gaspar, V., Smets, F., and Vestin, D. (2010). Inflation expectations, adaptive learning and optimal monetary policy. volume 3 of *Handbook of Monetary Economics*, pages 1055 – 1095. Elsevier.

Honkapohja, S. and Mitra, K. (2005). Performance of monetary policy with internal central bank forecasting. *Journal of Economic Dynamics and Control*, 29(4):627–658.

Milani, F. (2011). Expectation shocks and learning as drivers of the business cycle. *Economic Journal*, 121(552):379–401.

Preston, B. (2006). Adaptive learning, forecast-based instrument rules and monetary policy. *Journal of Monetary Economics*, 53(3):507–535.

Woodford, M. (2003). *Interest and Price*. Princeton University Press.

A Figures

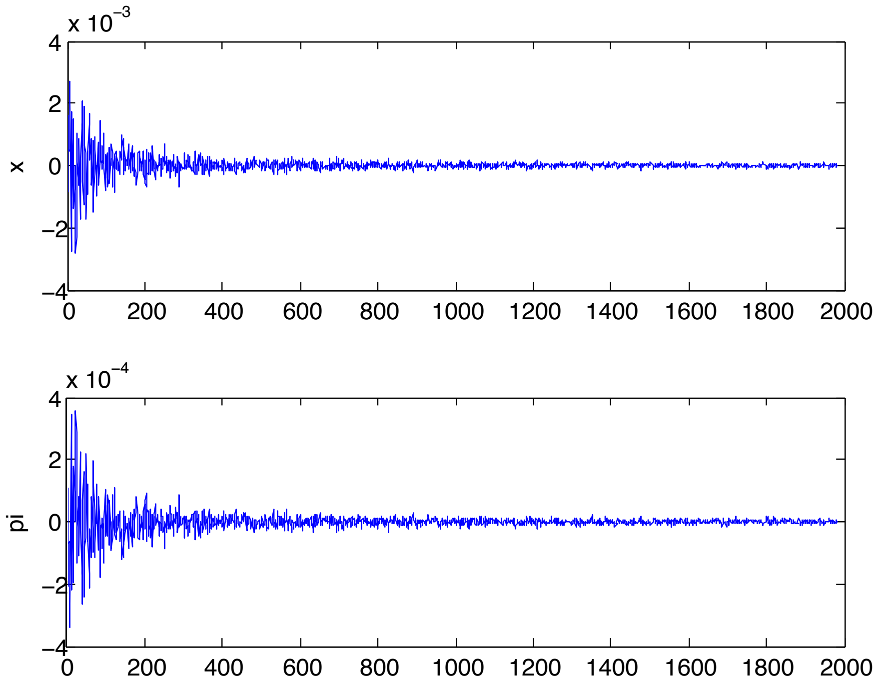

Figure 1: Difference between AL and RE realizations with no signal.

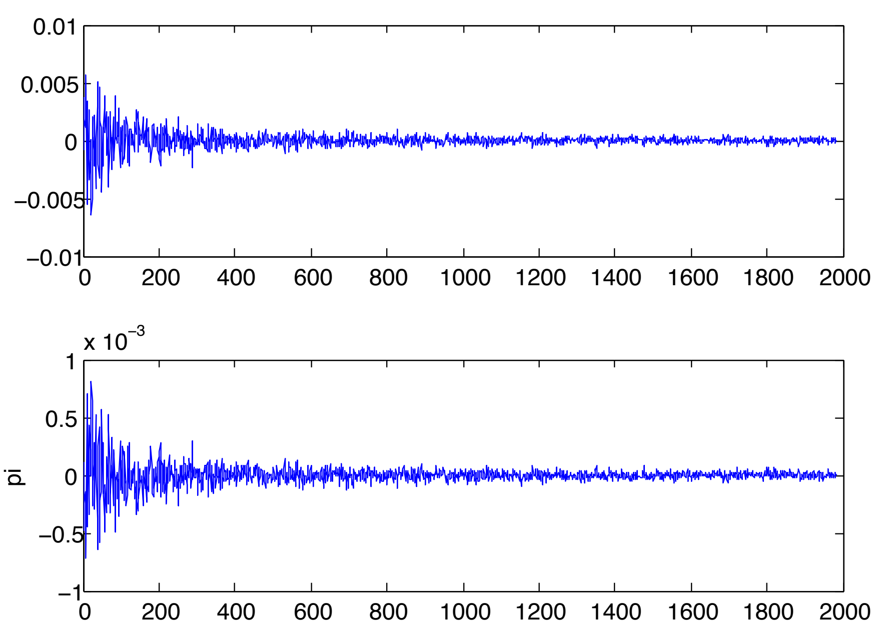

Figure 2: Difference between AL and RE realizations with signal updating

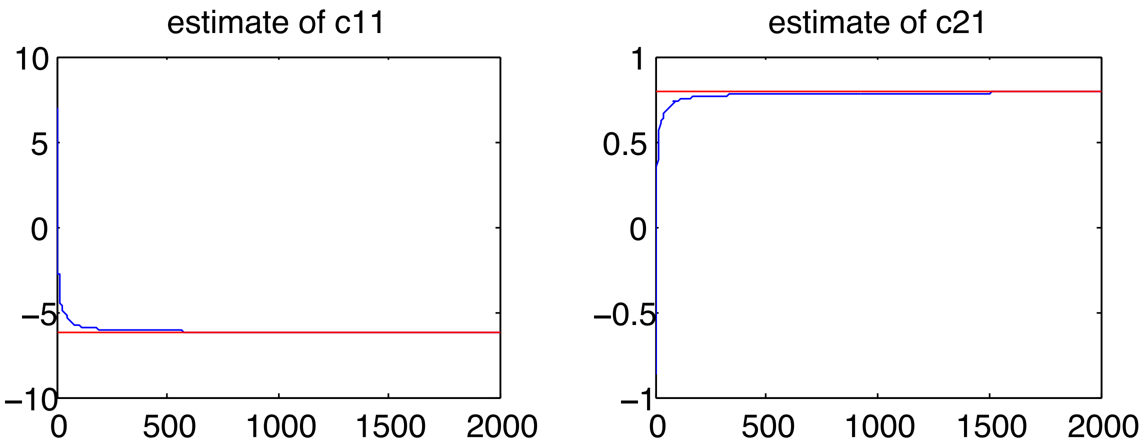

Figure 3: Evolution of parameter c with no signal

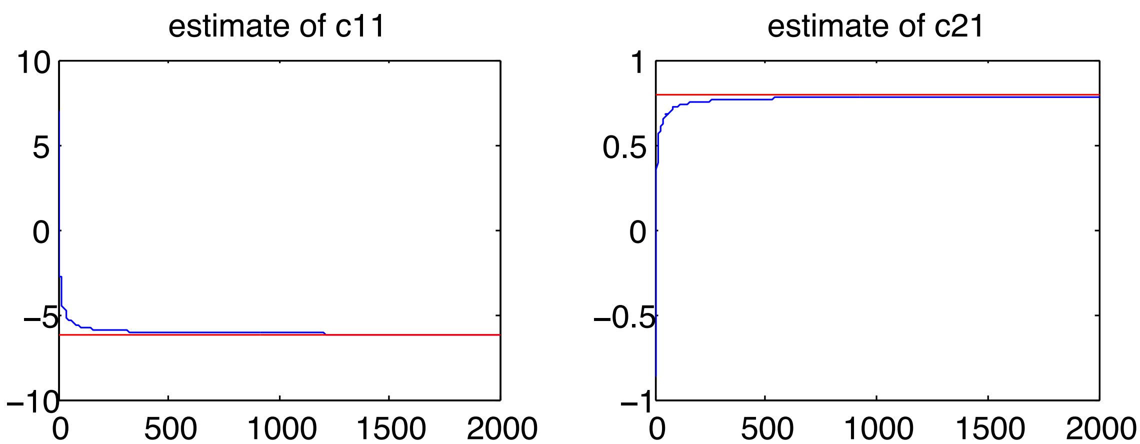

Figure 4: Evolution of parameter c with signal updating

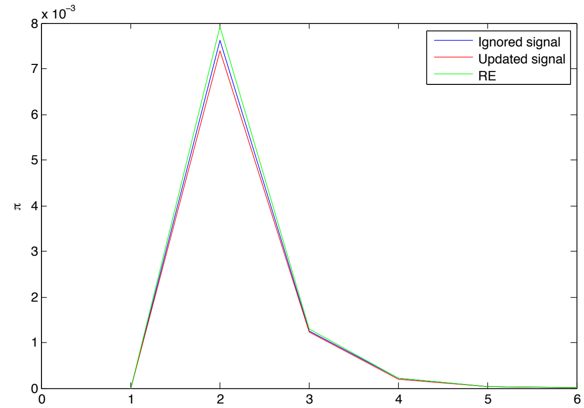

Figure 5: Inflation after a one percent cost push shock

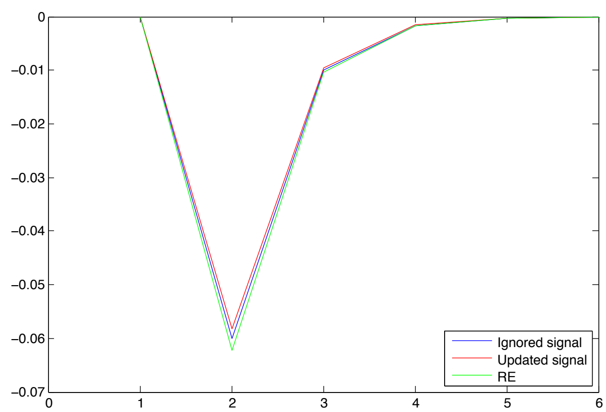

Figure 6: Output gap after a cost push shock.

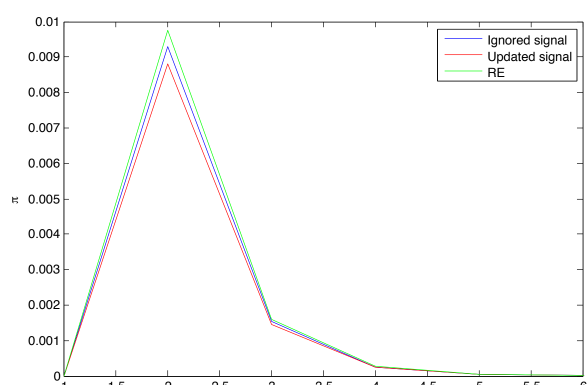

Figure 7: Inflation after a cost push shock in the closed economy version.

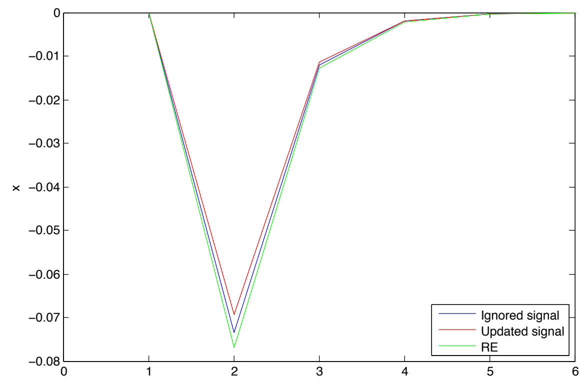

Figure 8: Output gap after a cost push shock in the closed economy version.

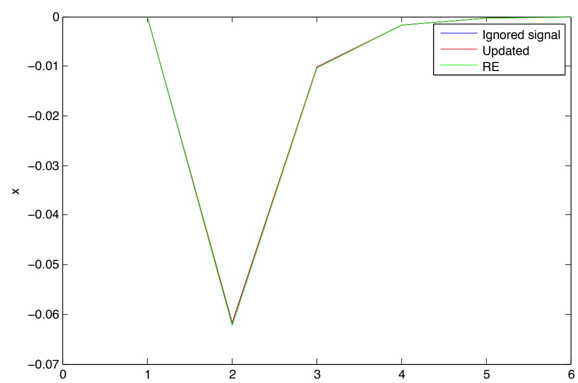

Figure 9: Output gap after a cost push shock. Shock occurring at $t = 200$.

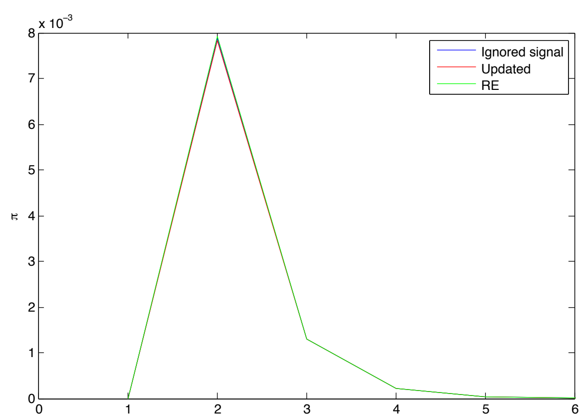

Figure 10: Output gap after a cost push shock. Shock occurring at $t = 200$.