

Does trend inflation make a difference?

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Abstract

The average inflation in the postwar period of developed countries was greater than zero and varied across countries. However, much of the extensive literature on monetary policy rules employed models approximated around a zero-inflation steady state. Comparing three estimated medium scale NK DSGE models with real and nominal friction, we want to shed a light on the quantitative implications from omitting trend inflation, that is a positive level of inflation at the steady state. We compare some population characteristics and the IRFs for the three models applying two loss functions, based on a point distance criterium and on a distribution distance criterium, respectively. Finally we compare the RMSE forecasts. We repeat the analysis for three subperiod: the Great inflation, the Great Moderation and the union of the two periods. We do not find a clear evidence that a model with trend inflation should be always preferred. Nevertheless, considering trend inflation could be relevant for the analysis of the Great Inflation. In fact, for this period, considering a positive level of steady state inflation induces a better fit with the data in terms of marginal data density and the lowest forecast RMSE, especially for the inflation time series.

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1 Introduction

Looking at the macroeconomic data of developed countries, a stylized fact is that the average inflation in the postwar period was greater than zero and varied across countries. However, much of the extensive literature on monetary policy rules employed models approximated around a zero-inflation steady state. Ascari and Ropele (2007) suggest that the monetary policy literature has focused on this particular assumption, even if it is both empirically unrealistic and theoretical special, for two reasons: it is analytical convenient and price stability is the optimal prescription in a cashless economy¹. Relaxing the zero steady state inflation assumption, new insights emerge. First, Ascari and Ropele (2007, 2009) show that even low trend inflation can affect optimal monetary policy and the dynamics of inflation, output and interest rates in a standard New Keynesian model. Moreover, trend inflation shrinks the determinacy region of a basic New Keynesian model when monetary policy is conducted by a contemporaneous interest rate rule². Second, as shown by Cogley and Sbordone (2008) in a small scale models the inclusion of time varying trend inflation seems to eliminate the need to include partial indexation schemes to produce a backward looking dynamics.

Given the empirical practice and these theoretical caveats, the goal of this analysis is to shed a light on the quantitative implications of omitting trend inflation in a medium-scale DSGE. We compare a NK DSGE model log-linearized around a zero steady state inflation and with partial indexation to past inflation with two NK models with trend inflation, indeed both of these two models imply a generalized Phillips. The first one includes partial indexation to past inflation³, while the second does not.

The model is based on two workhorse medium-scale DSGEs: Smets and Wouters (2007) and Schmitt-Grohe and Uribe (2004). These NK DSGE models add both real and nominal frictions to the standard textbook model. The real frictions are monopolistic competition in goods and labour markets, habit formation in preferences for consumption, capital utilization and investment adjustment cost. The nominal frictions are due to the Calvo mechanism for nominal price and wage. This model is able to replicate the main features of the US macro data and it is characterized by a good fit of the observed data.

We analyze three different periods: from 1966 to 2004, before the Great Moderation (from 1966 to 1982) and the Great Moderation's years (from 1983 to 2004). These periods are characterized by different average levels of inflation and therefore we have the possibility to test the quantitative implications of trend inflation for different levels of inflation at the steady state. We compare the cross-correlations and the IRFs for the three models applying the evaluation method proposed by Schorfheide (2000). In particular, we compare the models using two types loss function. The first one is based on a point distance criterium, as in Schorfheide (2000). The second, proposed as a novelty in this study, is distribution distance criterium based on the idea of entropy

¹See Woodford (2003).

²Other papers study the effects of changes in trend inflation like Hornstein and Wolman (2005) and Kiley (2007), concluding that the Taylor principle breaks down when trend inflation rate rises and that a more aggressive policy in reaction to inflation to insure the determinacy is required.

³Ascari and Ropele (2007) show that under full indexation in the Calvo pricing scheme the log-linearization around zero trend inflation or positive trend are identical. In this case the distortions due to a positive trend inflation disappear when all the non-reoptimizing firms re-adjust their price to the past inflation and/or to the trend inflation.

proposed by Ullah (1996). Moreover, since one of the advantages of DSGE model is its use for forecasting purposes, we compare the ‘in-sample’ forecast RMSE between the three DSGE models.

We do not find a clear evidence that a model with trend inflation should be preferred. In all our various comparisons the presence of trend inflation does not give results strongly different from the classical model, apart for the case in which we suppress indexation to past inflation. The major differences appear in the full period, where the standard model seems to describe the data better. This can be due to the fact that we loglinearize the trend inflation model around a time invariant level of steady state inflation, and the model suffers the change of regime, that occurs at the beginning of the 80’s.

These results are consistent with those in Ascari, Branzoli and Castelnuovo (2011): they study the determinacy of the equilibrium in a calibrated medium scale New Keynesian framework, and conclude that trend inflation does not seem to offset the determinacy region when real frictions are included.

Nevertheless, it is important to highlight that considering trend inflation could be relevant for the analysis of the Great Inflation. In fact, for this period, considering a positive level of steady state inflation induces a better fit with the data in terms of marginal data density and the lowest forecast RMSE, especially for the inflation time series.

We contribute to the trend inflation literature studying the effect of different levels of inflation in an estimated NK model. Few articles have investigated trend inflation in a calibrated model⁴ and focusing on the determinacy issue. The first paper that examined the effects of trend inflation on the dynamics of the standard New Keynesian model was Ascari (2004), and subsequently Amano, Ambler and Rebei (2007) studied how the business cycle characteristics of the model vary with trend inflation. Ascari and Ropele (2007) analyzed how optimal short-run monetary policy changes with trend inflation, whereas in Ascari and Ropele (2009) moderate levels of trend inflation offset the determinacy region, altering substantially the monetary policy rule. Kiley (2007) investigated how trend inflation influences the determinacy region and the unconditional variance of inflation in a model in which prices are staggered a la Taylor and monetary policy is described by a Taylor rule. Coibion and Gorodnichenko (2011) showed that determinacy in New Keynesian models under positive trend inflation depends not only on the central bank’s response to inflation and output gap, as is the case under zero trend inflation, but also on many other components of endogenous monetary policy.

The chapter is organized as follows. In Section II we introduce the general DSGE model and we sketch the derivation of the New Keynesian Phillips’ Curve and of the wage equation when the inflation rate at the steady state is a positive value. In Section III we present the data and the Bayesian estimations for the parameters, the relative short run dynamics and forecasts. In Section IV we explain the procedure for comparing the models. In Section V we present the results and in Section VI the conclusions.

⁴Few papers that estimate a New Keynesian DSGE model with a loglinearization around a positive trend inflation: for example Aruoba and Schorfheide (2011), and Mattesini and Perricone (2012).

2 The Model

We base our analysis on a medium-sized DSGE model, similar to the well known model estimated by Smets and Wouters (2007), henceforth SW.

Households maximize a non-separable utility function with two arguments (final good and labour effort) over an infinite life horizon. The presence of time-varying external habit formation makes the current consumption depending also on the past. Labour decisions are made by a union, which supplies labour monopolistically to a continuum of labour markets, sets nominal wages à la Calvo and distributes the markup applied over the marginal cost of labour to the households. Households rent capital services to firms and decide how much capital to accumulate, given capital adjustment costs. Capital utilization is variable and chosen by the households according to a cost schedule.

There is a sector of intermediated goods, where a continuum of firms produce differentiated goods in a monopolistic market à la Dixit and Stiglitz, decide on labour and capital inputs, and set prices, again according to the Calvo model. Consumption goods are a composite made of intermediated goods. The final good producers buy the intermediate goods on the market, package them into units of the composite good, and resell them to consumers in a perfectly competitive market. The model used here is identical to the one estimated by SW, except for three departures. First, we assume that the final producers package their goods according to the Dixit and Stiglitz aggregator, instead of the Kimball aggregator. Second, the monetary authority adjusts the nominal interest rate in response to inflation and output growth, while SW use the output gap. Third, we loglinearize the model around a positive level of steady state inflation as in Schmitt-Grohe and Uribe (2004) and in Ascari, Branzoli and Castelnuovo (2011). This implies that the dynamic equations for our model are equal to SW, except for the Taylor rule, the New-Keynesian Phillips curve and the real wage equation. Therefore, we present in the main text the last two equations, leaving the others in Appendix A.

2.1 Real Wage equation

Labour decisions are made by a union, which supplies labour monopolistically to a continuum of labour markets of measure 1, indexed by $l \in [0, 1]$, and sets wages according to the Calvo model. The problem of the union is

$$\max_{\tilde{W}_t^o(l)} E_t \sum_{s=0}^{\infty} (\omega_w \beta)^s \frac{\tilde{\Xi}_{t+s} \tilde{P}_t}{\tilde{\Xi}_t \tilde{P}_{t+s}} \left[\tilde{W}_{t+s}(l) - \tilde{W}_{t+s}^h \right] L_{t+s}(l)$$

subject to the demand curve

$$L_{t+s}(l) = \left[\frac{\tilde{W}_{t+s}(l)}{\tilde{W}_{t+s}} \right]^{-\theta^w} L_{t+s}^d$$

and the wage setting, which depends on the optimal wage $\tilde{W}_t^o(l)$

$$\tilde{W}_{t+s}(l) = \tilde{W}_t^o(l) \prod_{k=1}^s \gamma \tilde{\pi}_{t+s-k}^{\theta^w}$$

Here $\tilde{W}_t(l)$ denotes the nominal wage charged by the union in labour market l at time t , \tilde{W}_t is an index of nominal wages prevailing in the economy, \tilde{W}_t^h is the nominal wage received by the households, L_t^d is a measure of aggregate labour at time t demand by firms, \tilde{P}_t is the nominal price index, β is the subjective discount factor, ω_w is the probability of not reoptimizing wages, ι_w is the wage indexation on consumer-price past inflation, γ represents the labour-augmenting deterministic growth rate and σ_c is the inverse of the elasticity of intertemporal substitution for constant labour, whereas the stochastic discount factor, $\tilde{\Xi}_{t+s}$, is defined as:

$$\tilde{\Xi}_{t+s|t} = \frac{\tilde{\Xi}_{t+s}}{\tilde{\Xi}_t} \quad \text{and} \quad \tilde{\Xi}_t = \gamma^{-\sigma_c(t)} \tilde{\xi}_t$$

The first order condition is

$$0 = E_t \sum_{s=0}^{\infty} (\omega_w \beta)^s \frac{\tilde{\Xi}_{t+s|t} \tilde{P}_t}{\tilde{P}_{t+s}} \left\{ \tilde{W}_{t+s}(l) - \tilde{W}_{t+s}^h - \theta^w \left[\frac{\tilde{W}_{t+s}(l)}{\tilde{W}_{t+s}} \right]^{-\theta^{w-1}} \frac{\tilde{X}_{t,s}}{\tilde{W}_{t+s}} \tilde{L}_{t+s}^d + \tilde{X}_{t,s} \tilde{L}_{t+s}(l) \right\}$$

where

$$\tilde{X}_{t,s} = \prod_{k=1}^s \gamma \tilde{\pi}_{t+s-k}^{\iota_w}$$

Defining $\frac{\tilde{W}_t}{\tilde{P}_t} = \gamma^t \tilde{w}_t$, after some algebra

$$0 = E_t \sum_{s=0}^{\infty} (\omega_w \beta \gamma^{1-\sigma_c})^s \tilde{\xi}_{t+s|t} \left(\frac{\tilde{w}_t^o}{\tilde{w}_{t+s}} \right)^{-\theta^w} \left(\prod_{k=1}^s \frac{\tilde{\pi}_{t+k-1}^{\iota_w}}{\tilde{\pi}_{t+k}} \right)^{-\theta^w} \tilde{L}_{t+s}^d \left[\frac{\theta^w - 1}{\theta^w} \tilde{w}_t^o \left(\prod_{k=1}^s \frac{\tilde{\pi}_{t+k-1}^{\iota_w}}{\tilde{\pi}_{t+k}} \right) - \tilde{w}_{t+s}^h \right]$$

Starting from the first order condition, we can write the wage equation as

$$\frac{\theta^w - 1}{\theta^w} \tilde{w}_t^o \tilde{f}_t^{1,w} = \tilde{f}_t^{2,w}$$

where $\tilde{f}_t^{1,w}$ and $\tilde{f}_t^{2,w}$ are defined as

$$\tilde{f}_t^{1,w} = \left(\frac{\tilde{w}_t}{\tilde{w}_t^o} \right)^{\theta^w} \tilde{L}_t^d + (\omega_w \beta \gamma^{1-\sigma_c}) \left(\frac{\tilde{\pi}_{t+1}}{\tilde{\pi}_t^{\iota_w}} \right)^{\theta^w - 1} \left(\frac{\tilde{w}_{t+1}^o}{\tilde{w}_t^o} \right)^{\theta^w} \tilde{\xi}_{t+1|t} \tilde{f}_{t+1}^{1,w}$$

and

$$\tilde{f}_t^{2,w} = \tilde{w}_t^h \left(\frac{\tilde{w}_t}{\tilde{w}_t^o} \right)^{\theta^w} \tilde{L}_t^d + (\omega_w \beta \gamma^{1-\sigma_c}) \left(\frac{\tilde{\pi}_{t+1}}{\tilde{\pi}_t^{\iota_w}} \right)^{\theta^w} \left(\frac{\tilde{w}_{t+1}^o}{\tilde{w}_t^o} \right)^{\theta^w} \tilde{\xi}_{t+1|t} \tilde{f}_{t+1}^{2,w}$$

2.2 Price NKPC

Firms in the intermediate sector produce a continuum of goods indexed by $i \in [0, 1]$ in a monopolistic competitive environment: each intermediate good is produced by a single firm.

Prices are assumed to be sticky à la Calvo, indeed only a fraction $1 - \omega_p$ of firms can optimally set the price $\tilde{P}_{i,t}^o$ at time t , which is chosen to maximize the expected present discounted value of profits:

$$\max_{\tilde{P}_{i,t}^o} E_t \sum_{j=0}^{\infty} (\omega_p \beta)^j \tilde{\Xi}_{t+j|t} \left[\frac{\tilde{P}_{i,t}^o}{\tilde{P}_{t+j}} \tilde{\Omega}_{t,t+j-1}^p - \frac{\tilde{\mu}_*^p}{\tilde{\mu}_{t+j}^p} \right] \tilde{Y}_{i,t+j}$$

subject to the aggregate demand for good i

$$\tilde{Y}_{i,t+j} = \left[\frac{\tilde{P}_{i,t}^o}{\tilde{P}_{t+j}} \tilde{\Omega}_{t,t+j-1}^p \right]^{-\theta_p} \tilde{Y}_{t+j}$$

where $\tilde{\mu}_t$ is the price mark up and $\theta^p > 1$ is the parameter of the Dixit-Stiglitz aggregator over the j -th firms, i.e.:

$$\tilde{P}_t = \left[\int \tilde{P}_{j,t}^{1-\theta^p} dj \right]^{\frac{1}{1-\theta^p}}$$

Moreover $\iota_p \in [0, 1]$ is the price indexation parameter such that the non optimizing firms could re-adjust their prices to the past inflation:

$$\tilde{\Omega}_{t,t+j-1} = \prod_{j=0}^s \tilde{\pi}_{t+j-1}^{\iota_p}$$

The first order condition, after rearranging, is

$$0 = E_t \sum_{j=0}^{\infty} (\omega_p \beta \gamma^{1-\sigma_c})^j \tilde{\xi}_{t+j|t} (\tilde{p}_t^o)^{-\theta^p} \left(\prod_{k=1}^j \frac{\tilde{\pi}_{t+k-1}^{\iota_p}}{\tilde{\pi}_{t+k}} \right)^{-\theta^p} \tilde{y}_{t+j} \left[\frac{\theta^p - 1}{\theta^p} \tilde{p}_t^o \left(\prod_{k=1}^j \frac{\tilde{\pi}_{t+k-1}^{\iota_p}}{\tilde{\pi}_{t+k}} \right) - \frac{\mu_{\star}^p}{\tilde{\mu}_{t+s}^p} \right]$$

which can be rewritten as

$$\frac{\theta^p - 1}{\theta^p} \tilde{f}_t^{1,p} = \tilde{f}_t^{2,p}$$

where we define

$$\tilde{f}_t^{1,p} = \tilde{y}_t (\tilde{p}_t^o)^{-\theta^p} + \omega_p \beta \gamma^{1-\sigma_c} E_t \left[\left(\frac{\tilde{\pi}_t^{\iota_p}}{\tilde{\pi}_{t+1}} \right)^{1-\theta^p} \left(\frac{\tilde{p}_{i,t}^o}{\tilde{p}_{i,t+1}^o} \right)^{-\theta^p} \tilde{\xi}_{t+1|t} \tilde{f}_{t+1}^{1,p} \right]$$

and

$$\tilde{f}_t^{2,p} = \frac{\tilde{y}_t}{\tilde{\mu}_t^p} (\tilde{p}_t^o)^{-\theta^p-1} + \omega_p \beta \gamma^{1-\sigma_c} E_t \left[\left(\frac{\tilde{\pi}_t^{\iota_p}}{\tilde{\pi}_{t+1}} \right)^{-\theta^p} \left(\frac{\tilde{p}_{i,t}^o}{\tilde{p}_{i,t+1}^o} \right)^{-\theta^p-1} \tilde{\xi}_{t+1|t} \tilde{f}_{t+1}^{2,p} \right]$$

Taylor Rule

The sticky price and wage part of the model is closed by adding the monetary policy reaction function. We assume that the central bank systematically reacts to inflation and to output growth according to the rule

$$\tilde{R}_t = \tilde{R}_{t-1}^{\rho_R} \bar{R}_t^{\rho_R} \exp \epsilon_t^r \quad \bar{R}_t = \left(\frac{\tilde{\pi}_t}{\pi_{\star}} \right)^{\psi_{\pi}} \left(\frac{\tilde{y}_t}{\tilde{y}_{t-1}} \right)^{\psi_y}$$

where ϵ_t^R is a monetary policy shock that captures transitional deviations from the interest rate feedback rule that are unanticipated by the public.

3 Data and Bayesian Estimation

The seven variables used in our analysis are the quarterly data of the log of real GDP per capita (y_t), the log of real consumption per capita (c_t), the log of real investment per capita (i_t), the log of hours per capita (l_t), the log of the GDP deflator (π_t), the log of real wages (w_t), and the federal funds rate (R_t). All the data are obtained from the FRED2 database maintained by the Federal Reserve Bank of St. Louis and all variables, with the exception of hours, federal funds rate and inflation, are taken in first differences (see figure 1), as in SW.

We consider three temporal horizons, that are characterized by different mean and variance for the inflation rate. The first period covers the years 1966 Q1 - 1982 Q4 and it is characterized by high inflation (called Great Inflation). The second interval is from 1983 Q1 to 2004 Q4 and it is characterized by low level and variance of inflation, which represents the Great Moderation. Finally we estimate the full sample, i.e. from 1966 Q1 to 2004 Q4. Table 1 reports mean and variance for the inflation series that characterized the three periods.

| Period | Mean | Variance |
|-------------|------|----------|
| 1966 - 1982 | 6.08 | 0.3183 |
| 1983 - 2004 | 2.43 | 0.0620 |
| 1966 - 2004 | 4.02 | 0.3783 |

Table 1: Mean and variance for the quarterly GDP deflator over the three periods.

In order to investigate the empirical relevance of positive steady state level of inflation we estimate three nested DSGE models. The first one is the medium-sized DSGE presented in the previous section, with a generalized NKPC and partial indexation to past inflation, ι^p and ι^w , for both prices and wages.

The two nested models are recovered as follows: in the first case we set the long run inflation rate at 1, obtaining a medium-sized DSGE model with an hybrid NKPC and partial indexation; in the second case we maintain the hypothesis of positive trend inflation, i.e. $\pi_\star > 1$, but we remove the partial indexation, setting $\iota^p = \iota^w = 0$. This latter case generates a purely forward looking NKPC.

The Bayesian estimation of the two DSGE models is based on the theoretical prescription of An and Schorfheide (2007)⁵. As in SW we fix the depreciation rate of capital, $\delta = 0.025$, the intratemporal elasticity of substitution in the labour market, $\theta^w = 3$, and the steady state exogenous spending-output ratio, $g_y = 0.18$. The inflation level at the steady state is heterogeneous between models and sub periods: it is always assumed equal to 1 for the hybrid DSGE, whereas for the two models with the generalized NKPC we set the steady state rate of inflation at the mean value of the inflation time series over the relative temporal interval, as in table 1. Shocks are taken from SW and are presented in Appendix A. The priors of the parameters are equal to SW and are kept equal across models and periods⁶. In Appendix B are reported the priors

⁵ We set up a MATLAB routine performing a Random Walk Metropolis-Hasting: the algorithm samples using a variance and covariance matrix obtained as the inverse of the Hessian previously computed with Sims' optimization algorithm. In order to obtain a solution to the dynamic system that depend on the time series behavior and structural shocks, the QZ decomposition is performed with Klein's algorithm.

⁶Therefore, we assume that even during the Great Inflation there are no issues of indeterminacy.

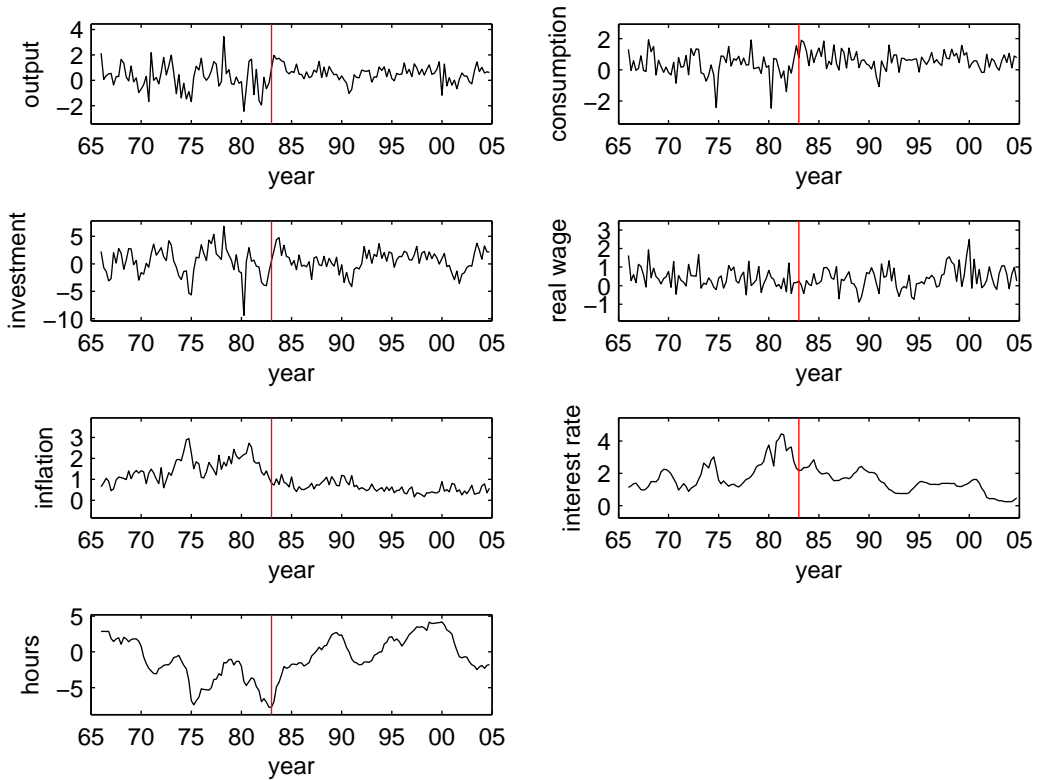


Figure 1: Time series (right to left, upper to bottom): output, consumption, investment, real wage, inflation, interest rate, hours. The red line corresponds to 1983 Q1.

and the posteriors for all the 34 parameters⁷.

Posterior Estimates

Taking into account trend inflation does not seem to affect the posterior distributions of parameters. In fact, the results of the estimation in all the three periods are similar for the standard hybrid model and the one with trend inflation and indexation. Some differences, instead, appear for the model without indexation.

Consider the full period and comparing our results for the hybrid model with SW, we observe that the estimates of the parameters of the shock processes are close. Only some differences appear for the estimations of price and wage shocks, where we obtain higher standard deviations than SW⁸. Given our different choice with respect to SW on modeling monopolistic competition, the coefficients related to stickiness, i.e. $\omega_p, \omega_w, \iota^p$ and ι^w , are slightly higher than in SW. The parameter that largely differs with SW's results is ψ_π , the reaction coefficient of central bank to inflation, that in our model is lower. Our estimate is more in line with Rabanal and Rubio-Ramirez (2005), that

⁷For the model without indexation the estimated parameters are 32.

⁸As in SW the shocks on price and wage are cost push shocks, nevertheless in this analysis they are introduced in a different way with respect to SW. This slight difference could justify the estimations' discrepancy.

obtain an analogous estimation for ψ_π in a similar model over the period 1966-2001⁹. In models with trend inflation, Hornstein and Wolman (2005), Kiley (2007) and Ascari and Ropele (2007) highlight how achieving a unique Rational Expectation Equilibrium at the historical level of inflation requires much stronger response to inflation than anything observed in empirical estimations of central banks' reaction function. Nevertheless, we do not obtain the same conclusion in our estimations. We think the reason is that, differently from us, those authors study small-scale models, as Woodford (2003) or Clarida, Galí and Gertler (1999), and assume that central bank reacts to output gap, whereas we consider a response to the output growth¹⁰. Relevant differences appear between subperiods. Starting with the shock processes, the standard deviations strongly decrease between the Great Inflation and the Great Moderation period, except for the volatility of the wage shock, which presents a mild increase¹¹. The persistence of the processes increases during the Great Moderation and generally the models with trend inflation display higher results than the hybrid one. Considering the structural parameters, in the monetary policy rule we observe a stronger reaction of the central bank to inflation during the Great Moderation, whereas the response to output growth shows only a slightly increase. This result is consistent with Boivin and Giannoni (2006), who find evidence of a more stabilizing monetary policy during the Great Moderation, almost entirely explained by an increasing responsiveness to inflation. Instead, SW observe that the responses to inflation are only marginally higher and the reaction to output gap is lower.

The estimates for the two models with indexation are very close in all the periods analyzed, whereas the model without indexation present a low value for ψ_π in the full period¹². A significant increase in persistence is observed in the coefficient ρ^R , that relates the past nominal interest rate to the actual one in the monetary policy function. It is interesting to note that the degree of price stickiness increases during the Great Moderation period, whereas the degree of wage stickiness presents a slight reduction¹³. This result is consistent with works like Blanchard and Galí (2007) or Blanchard and Riggi (2009), which found an overall reduction in the degree of real wage rigidity, while it is at odds with the estimations of SW where during the Great Moderation ω_w increases.

The degree of indexation on past inflation, for both prices and wages, decreases during the Great Moderation, and it takes similar values in both models. Cogley and Sbordone (2008) show in a VAR that taking into account a time-varying shifts of trend inflation there is no need to include indexation. In our medium scale DSGE, and similarly in

⁹While SW assume a Taylor function with output gap, in our model the central bank reacts to output growth.

¹⁰The idea of implementing the Taylor rule with a response to output growth goes back to Erceg and Levin (2003) and was supported by Walsh (2003) and Orphanides and Williams (2006). Coibion and Gorodnichenko (2011) show that responding to output growth can help restore determinacy for plausible inflation responses, while Ascari and Ropele (2007) emphasize the potentially destabilizing role of responding to the output gap under positive level of inflation.

¹¹This result is supported by Heathcote, Storesletten and Violante (2010) show a rising instability of US male earnings for recent decades.

¹²Schorfheide (2005) show how a model without indexation and without regime switching over the period 1960 Q1 - 1997 Q4 implies a value for $\psi_\pi \in [1, 1.3]$, i.e. the parameter is close to the indeterminacy region.

¹³It is tempting to compare our estimates with the microeconomic evidence on the average duration of prices, like Bils and Klenow (2004) or Nakamura and Steinsson (2006). However the comparison is difficult because we have partial indexation.

| Parameter | Hybrid | Trend | Trend No Index |
|------------|--------|-------|----------------|
| 1966-2004 | | | |
| ι^p | 0.32 | 0.31 | - |
| ι^w | 0.60 | 0.46 | - |
| ω_p | 0.87 | 0.87 | 0.87 |
| ω_w | 0.81 | 0.84 | 0.89 |
| ψ_π | 1.37 | 1.26 | 1.13 |
| ψ_y | 0.20 | 0.20 | 0.19 |
| ρ_r | 0.27 | 0.28 | 0.29 |
| 1966-1982 | | | |
| ι^p | 0.43 | 0.40 | - |
| ι^w | 0.61 | 0.60 | - |
| ω_p | 0.82 | 0.80 | 0.80 |
| ω_w | 0.78 | 0.77 | 0.78 |
| ψ_π | 1.4 | 1.44 | 1.38 |
| ψ_y | 0.14 | 0.13 | 0.13 |
| ρ_r | 0.23 | 0.24 | 0.23 |
| 1983-2004 | | | |
| ι^p | 0.18 | 0.17 | - |
| ι^w | 0.55 | 0.54 | - |
| ω_p | 0.87 | 0.86 | 0.86 |
| ω_w | 0.74 | 0.74 | 0.75 |
| ψ_π | 1.83 | 1.83 | 1.86 |
| ψ_y | 0.18 | 0.18 | 0.18 |
| ρ_r | 0.45 | 0.46 | 0.44 |

Table 2: Posterior means for the three models in the three periods.

Aruoba and Schorfheide (2011), we do not observe such situation: this can be due to the fact that we keep time-invariant level of trend inflation or, more probably, to the presence of more frictions¹⁴.

Moreover, given that the two models with indexation show similar results for the parameter related to wage indexation, it seems that the presence of trend inflation does not contribute to account for the stickiness of the labour market.

Considering the joint distributions of (ι^p, ω_p) and (ι^w, ω_w) , we better understand the different relationship between price, or wage, rigidity and long run inflation¹⁵. During the Great Inflation period we observe a positive correlation between ι^p and ω_p , whereas for the Great Moderation the correlation goes to zero, as in figure (2).

It is interesting to note that the information on the link between indexation and stickiness is coherent between the two models if we consider subperiod with homogeneous behavior for inflation (i.e. Great Moderation or Great Inflation), whereas when we study the full sample a positive correlation is evident only for the model with trend inflation. This result emphasizes the idea suggested by Galì and Gertler (1999): with

¹⁴In a preliminary version of this work we have analyzed the basic trivariate Woodford (2003) model, without wage rigidity, and we were able to recover the result on zero indexation: for the Great Moderation period we estimate ι^p equal to 0.2 and 0.0025, for the hybrid and the generalized NKPC model respectively. See Appendix E.

¹⁵Under the assumption that the posterior of the estimated parameters are normally distributed, the joint distribution is a bivariate normal. Thus, the contour gives us informations on the correlation between the distributions of the two parameters. The relationship between coefficients is due to the use of the inverse Hessian in the Random Walk Metropolis Hastings.

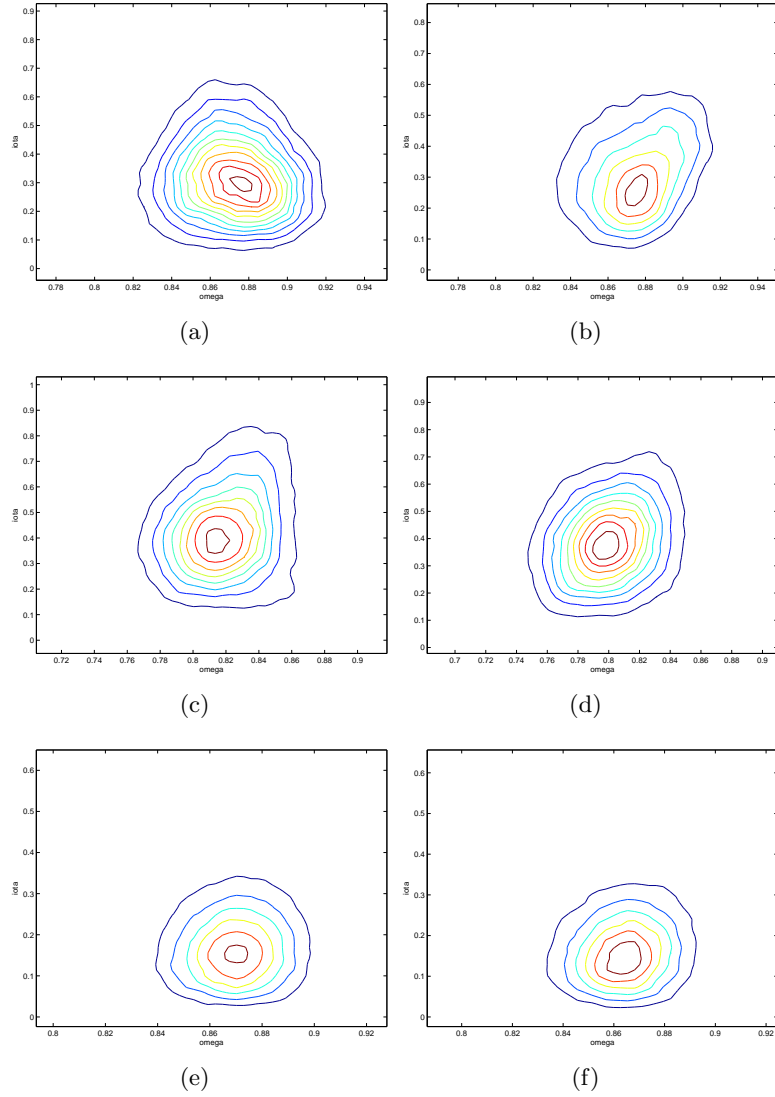


Figure 2: Contour plot for the joint distribution (ι^p, ω_p) dividing by periods and models: Full sample (1st line), Great Inflation (2nd line) and Great Moderation (3rd line). Considering the two models: hybrid NKPC (left column) and generalized NKPC (right column).

high and volatile inflation the average price duration decreases, because of the higher cost of not adjusting. Therefore in order to hold down this cost, when the stickiness increases, price indexation to past inflation must increase more than proportionally. In particular, since the presence of a positive level of inflation is not enough to account for price stickiness, this positive relationship is more evident for the full period, suggesting the necessity of studying separately the periods, characterized by different levels of stickiness.

Considering the joint distribution for the labour market parameters, (ι^w, ω_w) , we observe a different relationship between the level of inflation and their correlation structure: in periods with low and stable inflation there is a negative correlation between ι^w and ω_w , whereas during the Great Inflation period the correlation is close to zero.

Short run dynamics

As we have seen in the previous section, the inclusion of trend inflation does not seem to affect dramatically the estimates of the structural parameters of the model, apart for the model without indexation. Here, instead, we want to check if there is an effect on the short run dynamics of the model, following a monetary policy shock. We concentrate on this shock because DSGE models are used extensively by central banks to evaluate their policy and we want to see if taking into account trend inflation can lead to different considerations.

In Appendix C there are the IRFs to a negative monetary policy shock, i.e. an unexpected increase in interest rate, computed at the posterior mean, for the three DSGE models in each period considered. Generally we can see that the two models with indexation present close IRFs¹⁶. For the Great Inflation period some differences appear in the responses of the model without indexation, in particular for inflation, which recovers the steady state level faster than in the other two models, and for work hours, where the effect of the shock is more persistent. For the Great Moderation period the three models present exactly the same responses for all the variables. Considering the results for the full period, we notice that the only main difference between the two models with indexation is for the worked hours, where the generalized model shows a more persistent response. The model without indexation presents almost always a stronger reaction associated with a faster recovery of the steady state level.

These observations on the IRFs lead us to two conclusions. The first is that, whenever there is indexation to past inflation, taking into account trend inflation does not influence dramatically the short run dynamic of the economy. The second is that a model without indexation is similar to the other models only in period with low volatility.

Finally we consider the cross-correlations generated by the models. In period of high volatility for inflation and interest rate, the cross-correlations of all models decrease faster than in the other periods and present larger credible intervals. Analyzing the different declinations of the NKPC, for the Great Moderation period the cross-correlations are very similar across the models, whereas for more volatile periods we observe that for the correlation involving π and R the hybrid NKPC generate lower correlations. The model without indexation presents the highest correlations associated with larger credible band.

Forecasting

One of the features that makes DSGE models attractive to central banks is their ability to produce reliable forecasts. The literature on the assessment of DSGE model forecasts has focused on point forecasts, predominantly evaluated with the root-mean-squared error (RMSE) measures. SW show that DSGE model forecasts for U.S. data in terms of RMSEs are competitive with forecasts generated by a Bayesian VAR.

We consider an in-sample forecast: the prediction's horizon for Great Inflation period is from 1978 Q4 to 1980 Q4, whereas for both the Great Moderation period and the

¹⁶We found that short run dynamic is not affected by trend inflation if there is partial indexation also in a preliminary analysis we have conducted using the trivariate textbook model proposed by Woodford (2003), differently from the conclusions of Ascari and Ropele (2007). For more details see Appendix E.

full sample, the forecast horizon is from 2000 Q4 to 2002 Q4. In tables 3 and 4 there are the RMSEs forecast, i.e.:

$$RMSE = \frac{1}{H} \sqrt{\sum_{h=1}^H (y_{t+h} - \hat{y}_{t+h|t})^2}$$

where the forecast horizon is $H = 8$. As general observation, the three models generate similar RMSEs and the forecasts obtained for period with low volatility, as in table 3, have smaller errors than those for period with high volatility, as in table 4.

Let us consider the forecast horizon 2000 Q4 - 2002 Q4. Using the parameters estimated over the full period, the preferred model is the one without indexation. Indeed, given the data characteristics, it is better a model that does not anchor the forecast to a volatile past. On the other hand, for the Great Moderation period, the two models with indexation are equally preferred. Considering the second forecast interval, i.e. 1978 Q4 - 1980 Q4, for R and π the chosen model is the one with trend inflation and indexation, whereas for output, i.e. the variable over the three considered which is the less related to indexation, the preferred model is the one with trend inflation but without indexation.

| Forecast Period: 2000 Q4 - 2002 Q4 | | Hybrid | Trend | Trend No Index |
|---|-------|--------|--------|----------------|
| Estimation Period: 1966 - 2004 | Y | 0.2405 | 0.2238 | 0.2032 |
| | π | 0.0806 | 0.0700 | 0.0590 |
| | R | 0.1563 | 0.1490 | 0.1348 |
| Estimation Period: 1983 - 2004 | Y | 0.2095 | 0.2099 | 0.2234 |
| | π | 0.0668 | 0.0674 | 0.0690 |
| | R | 0.1267 | 0.1265 | 0.1644 |

Table 3: In sample forecast RMSEs for output, inflation and interest rate using the parameters' posterior mean for the subperiods: 1966 - 2004 and 1983 - 2004.

| Forecast Period: 1978 Q4 - 1980 Q4 | | Hybrid | Trend | Trend No Index |
|---|-------|--------|--------|----------------|
| Estimation Period: 1966 - 1982 | Y | 0.3778 | 0.3671 | 0.3548 |
| | π | 0.2095 | 0.1582 | 0.2061 |
| | R | 0.2739 | 0.2576 | 0.3016 |

Table 4: In sample forecast RMSEs for output, inflation and interest rate using the parameters' posterior mean for the period, 1966 - 1982.

4 Loss function analysis: Theory

In this section we want to deepen our analysis using the quantitative evaluation procedure of Schorfheide (2000), which proceeds in three steps.

In the **first step** we compute the posterior distributions $p(\theta_{(i)}|Y_T, \mathcal{M}_i)$ for model parameters $\theta_{(i)}$ and the posterior model probabilities:

$$\pi_{i,T} = \frac{\pi_{i,0} p(Y_T | \mathcal{M}_i)}{\sum_{i=0}^2 \pi_{i,0} p(Y_T | \mathcal{M}_i)}$$

where $p(Y_T|\mathcal{M}_i)$ is the marginal data density:

$$p(Y_T|\mathcal{M}_i) = \int p(Y_T|\theta_{(i)}, \mathcal{M}_i)p(\theta_{(i)}|\mathcal{M}_i)d\theta_{(i)}$$

under model \mathcal{M}_i , where the labels are:

| i | Model |
|---|--|
| 0 | DSGE with Hybrid NKPC |
| 1 | DSGE with trend inflation and partial indexation |
| 2 | DSGE with trend inflation, without indexation |
| 3 | VAR(1) |
| 4 | VAR(2) |
| 5 | VAR(3) |
| 6 | VAR(4) |

This approach takes into account the potential misspecifications of the candidate model, since neither population moments nor IRFs are directly observable in the data. Therefore, a probabilistic representation of the data that serves as benchmark for DSGE models' comparison has to be constructed. Note that to implement the procedure, we include in our analysis also a structural VAR, because it is more densely parametrized than the DSGE models and this can avoid dynamic misspecifications¹⁷. In order to compute the marginal data density for the DSGE models we need to choose a numerical approximation approach: taking into account the results of Schorfheide (2000) and An and Schorfheide (2007), we decide to approximate the marginal data density with Geweke (1999) modified harmonic mean estimator, as presented in Appendix D. Whereas for the Bayesian VAR we can recover the marginal data density in closed-form solution, since we adopt a natural conjugate prior.

In the **second step** we compute the population characteristics ϕ , which are a function $f(\theta_{(i)})$ of the model parameters $\theta_{(i)}$. Based on the posterior distribution of $\theta_{(i)}$, one can obtain a posterior for ϕ conditional on model \mathcal{M}_i , denoted by $p(\phi|Y_T, \mathcal{M}_i)$. Since we are considering six different models, the overall posterior of ϕ is given by the mixture:

$$p(\phi|Y_T) = \sum_{i=0}^6 p(\phi|Y_T, \mathcal{M}_i)\pi_{i,T}$$

where the posterior probabilities $\pi_{i,T}$, computed at the previous step, determine the weights of the densities $p(\phi|Y_T, \mathcal{M}_i)$.

In the **third step** we introduce a loss function in order to assess the ability of the DSGE models to replicate patterns of co-movements among key macroeconomic variables and impulse responses to structural shocks. The loss function is a function that penalizes deviations of models' moments from the population characteristics, which were computed in the previous two steps. Given a specific definition of loss function, we need to provide a measure of how well model \mathcal{M}_i reproduces the populations characteristics ϕ , i.e. we want to compare the three DSGE models according to a posterior risk of deviating from the populations characteristic. We will discuss two types of loss functions, based on different ideas of divergence from the population characteristics.

¹⁷For a detailed presentation of the VAR estimation and identification see Appendix D.

Loss function 1: point distance

The first loss function we present, $L^1(\phi, \hat{\phi})$, penalizes deviations of DSGE model predictions $\hat{\phi}$ from population characteristics ϕ . The prediction from DSGE model \mathcal{M}_i is obtained as follows: suppose a decision maker bases decisions exclusively on DSGE model \mathcal{M}_i , thus the optimal predictor is

$$\hat{\phi}_i = \arg \min_{\tilde{\phi} \in \mathbb{R}^m} \int L^1(\phi, \tilde{\phi}) p(\phi|Y_T, \mathcal{M}_i) d\phi$$

The loss function we use is taken from Schorfheide (2000) and it is defined as

$$L^1(\phi, \tilde{\phi}) = \mathbb{I} \left\{ p(\phi|Y^T) > p(\tilde{\phi}|Y^T) \right\}$$

Indeed it penalizes point predictions that lie in regions of low posterior density, i.e. L^1 identifies a model's distribution close to the population characteristic's distribution if the two relative modes are close, under the assumption that the distributions are unimodal.

Loss function 2: distribution distance

While the first loss function reduces to a comparison between two single points, we propose a different idea of loss function that allows us to use all the informations held in the posterior distributions, i.e. $p(\phi|Y_T)$ and $p(\phi|Y_T, \mathcal{M}_i)$.

In particular, when we study the distributions of the characteristics generated by the models, we observe that they can be asymmetric. Therefore the simple comparison of summary statistics, like the mode in the previous example of $L^1(\phi, \hat{\phi})$, could induce a biased conclusion.

Departing from this observation we decide to consider the entire distribution, i.e. $p(\phi|Y^T, \mathcal{M}_i)$. For this purpose our loss function is inspired to the generalized entropy proposed by Ullah (1996). A divergence measure can be derived in terms of the ratio

$$\lambda \equiv \frac{p(\phi|Y^T, \mathcal{M}_i)}{p(\phi|Y^T)}$$

such that the difference in the distributions is large when $p(\phi|Y^T, \mathcal{M}_i)$ is far from $p(\phi|Y^T)$ and is equal to 1 if and only if $p(\phi|Y^T, \mathcal{M}_i) = p(\phi|Y^T)$. Therefore an alternative measure of divergence can be developed in terms of the information, or entropy, content in λ . Let us consider a convex function $g(\lambda)$ such that $g(1) = 0$. The information content in $p(\phi|Y^T, \mathcal{M}_i)$ with respect to $p(\phi|Y^T)$, or divergence of $p(\phi|Y^T, \mathcal{M}_i)$ with respect to $p(\phi|Y^T)$, is then:

$$H_g(p(\phi|Y^T, \mathcal{M}_i), p(\phi|Y^T)) = g\left(\frac{p(\phi|Y^T, \mathcal{M}_i)}{p(\phi|Y^T)}\right)$$

This divergence measure can be considered as an extension of the entropy functional. In particular, we consider the family of functions:

$$g_{\bar{\alpha}}(\lambda) = \begin{cases} \frac{1}{\bar{\alpha}-1} [1 - \lambda^{\bar{\alpha}-1}] & \text{if } \bar{\alpha} > 0 \text{ and } \bar{\alpha} \neq 1 \\ -\log \lambda & \text{if } \bar{\alpha} = 1 \end{cases}$$

where $g_{\bar{\alpha}}(\lambda)$ has two characteristics: $g_{\bar{\alpha}}(1) = 0$ and $g_{\bar{\alpha}}(\lambda)$ is monotonic. Therefore the loss function is:

$$L_{\bar{\alpha}}^2(\phi, \phi_{\mathcal{M}_i}) = \begin{cases} \frac{1}{\bar{\alpha}-1} \left[1 - \left(\frac{p(\phi|Y^T, \mathcal{M}_i)}{p(\phi|Y^T)} \right)^{\bar{\alpha}-1} \right] & \text{if } \bar{\alpha} > 0 \text{ and } \bar{\alpha} \neq 1 \\ -\log \left(\frac{p(\phi|Y^T, \mathcal{M}_i)}{p(\phi|Y^T)} \right) & \text{if } \bar{\alpha} = 1 \end{cases}$$

A drawback of this approach based on entropy is the role of the distribution's support. Since one of the interpretation of entropy is the information that the distribution $p(\phi|Y^T, \mathcal{M}_i)$ carries about the benchmark distribution $p(\phi|Y^T)$, the support of the first has to lie in the support of the latter. Indeed, the part of the compared distribution which lie outside the support of the benchmark distribution embodies no informations about the benchmark one¹⁸. Since in our analysis the distribution of the population characteristics always shows a bigger variance than the compared models, the support of the first is always bigger than the support of the latter and we do not suffer this drawback.

Risk function

Let us now define the posterior risk related to the two types of loss function. In particular, under the first definition of loss the DSGE models are judged according to the expected loss of $\hat{\phi}_i$ under the overall posterior distribution $p(\phi|Y_T)$, with the posterior risk function:

$$\mathcal{R}^1(\hat{\phi}_i|Y_T) = \int L^1(\phi, \hat{\phi}_i)p(\phi|Y_T)d\phi$$

where $\mathcal{R}^1(\hat{\phi}_i|Y_T) \in [0, 1]$. The model i^{th} is preferred to the model j^{th} if

$$\mathcal{R}^1(\hat{\phi}_i|Y_T) < \mathcal{R}^1(\hat{\phi}_j|Y_T)$$

Instead, according to the second definition of loss function, we need to summarize the information relative to the distance between the kernel distributions of the characteristics for the population and the i -th model. Since the loss function takes both positive and negative values, we propose two functions in order to summarize the posterior risk, i.e. the sum of the square values or the sum of the absolute values:

$$\mathcal{R}_{\bar{\alpha}}^S(\phi_{\mathcal{M}_i}|Y_T) = \sum [L_{\bar{\alpha}}^2(\phi, \phi_{\mathcal{M}_i})p(\phi|Y^T)]^2$$

or

$$\mathcal{R}_{\bar{\alpha}}^A(\phi_{\mathcal{M}_i}|Y_T) = \sum |L_{\bar{\alpha}}^2(\phi, \phi_{\mathcal{M}_i})p(\phi|Y^T)|$$

The final measure of the goodness of fit of one DSGE model with respect to another is given by the ratio of the posterior risks associated with model (\mathcal{M}_i) and the model (\mathcal{M}_j) :

$$\begin{aligned} \text{Ratio}_{\bar{\alpha}}^S &= \frac{\mathcal{R}_{\bar{\alpha}}^S(\phi_{\mathcal{M}_i}|Y_T)}{\mathcal{R}_{\bar{\alpha}}^S(\phi_{\mathcal{M}_j}|Y_T)} \\ \text{Ratio}_{\bar{\alpha}}^A &= \frac{\mathcal{R}_{\bar{\alpha}}^A(\phi_{\mathcal{M}_i}|Y_T)}{\mathcal{R}_{\bar{\alpha}}^A(\phi_{\mathcal{M}_j}|Y_T)} \end{aligned}$$

¹⁸This drawback is particularly severe when $\bar{\alpha} = 1$. In this case, the comparison can be made only when the two distributions have the same support. Since the implied loss function involves a logarithmic function, if the support of $p(\phi|Y^T, \mathcal{M}_i)$ is bigger, then we have $L_1^2(\phi, \phi_{\mathcal{M}_i}) = -\log(\infty)$, or if the support of $p(\phi|Y^T)$ is bigger, then $L_1^2(\phi, \phi_{\mathcal{M}_i}) = -\log(0)$.

When the ratio is smaller than 1, the \mathcal{M}_i is preferred to \mathcal{M}_j , whereas the converse is true when the ratio is bigger than 1. We take into account the following ratios:

$$\frac{\mathcal{M}_1}{\mathcal{M}_0} \quad \frac{\mathcal{M}_2}{\mathcal{M}_0} \quad \frac{\mathcal{M}_2}{\mathcal{M}_1}$$

5 Loss function analysis: Results

The model with the highest value for the marginal data density is the VAR(4) in all periods, as shown in tables 5, 6, and 7. Comparing the three DSGE during the Great Inflation period, the model with generalized NKPC and indexation, i.e. \mathcal{M}_1 , shows the highest marginal data density among the DSGE models. During the Great Moderation, instead, the results show a slight preference for the hybrid model. Finally, when we consider the full period the hybrid model seems to describe better the data than the models with trend inflation. This result can be due to the fact the model with a time invariant trend inflation can suffer the change of regime in inflation, that some authors collocate at the beginning of the 80's. The model without indexation, i.e. \mathcal{M}_2 , shows always the worst fit, confirming the important role of indexation.

| Model | Prior Prob. $\pi_{i,0}$ | $\ln p(Y_T^* Y_*, \mathcal{M}_i)$ | Harmonic Mean | Post Prob. $\pi_{i,T}$ |
|-------|-------------------------|-----------------------------------|---------------|------------------------|
| 0 | 1/7 | - | -741.5273 | $1 e^{-08}$ |
| 1 | 1/7 | - | -745.9096 | $2 e^{-10}$ |
| 2 | 1/7 | - | -756.0987 | $8 e^{-15}$ |
| 3 | 1/7 | -773.5461 | - | $2 e^{-22}$ |
| 4 | 1/7 | -744.7351 | - | $7 e^{-10}$ |
| 5 | 1/7 | -736.2191 | - | $3 e^{-06}$ |
| 6 | 1/7 | -723.6860 | - | ~ 1 |

Table 5: Results for the first step, full sample (1966-2004).

| Model | Prior Prob. $\pi_{i,0}$ | $\ln p(Y_T^* Y_*, \mathcal{M}_i)$ | Harmonic Mean | Post Prob. $\pi_{i,T}$ |
|-------|-------------------------|-----------------------------------|---------------|------------------------|
| 0 | 1/7 | - | -349.7733 | $1 e^{-23}$ |
| 1 | 1/7 | - | -347.5604 | $1 e^{-21}$ |
| 2 | 1/7 | - | -357.0488 | $2 e^{-28}$ |
| 3 | 1/7 | -346.0916 | - | $3 e^{-24}$ |
| 4 | 1/7 | -321.9535 | - | $6 e^{-14}$ |
| 5 | 1/7 | -309.6899 | - | $1 e^{-07}$ |
| 6 | 1/7 | -296.4994 | - | 0,9999 |

Table 6: Results for the first step, Great Inflation (1966-1982).

In table 8, 9 and 10 there are the results for the comparisons considering the correlations between output, inflation and interest rate in terms of $R^1(\hat{\phi}_i|Y_T)$ and $Ratio_2^A$, computed up to lag 12 ($h = 0, 1, \dots, 12$). In each cell there are three rows with the percentage of cases in which a model is preferred to another, according to our two risk functions¹⁹.

¹⁹According to the first loss function, model $i - th$ is preferred to model $j - th$ if $R^1(\hat{\phi}_i|Y_T) < R^1(\hat{\phi}_j|Y_T)$. Under the loss function based on entropy the criterion is $Ratio_2^A < 1$.

| Model | Prior Prob. $\pi_{i,0}$ | $\ln p(Y_T^* Y_*, \mathcal{M}_i)$ | Harmonic Mean | Post Prob. $\pi_{i,T}$ |
|-------|-------------------------|-----------------------------------|---------------|------------------------|
| 0 | 1/7 | - | -212.9269 | $2 e^{-05}$ |
| 1 | 1/7 | - | -213.6370 | $1 e^{-05}$ |
| 2 | 1/7 | - | -217.3644 | $2 e^{-07}$ |
| 3 | 1/7 | -237.1459 | - | $6 e^{-16}$ |
| 4 | 1/7 | -209.9751 | - | $4 e^{-04}$ |
| 5 | 1/7 | -207.3185 | - | 0.0061 |
| 6 | 1/7 | -202.2180 | - | 0.9935 |

Table 7: Results for the first step, Great Moderation 1983 - 2004.

| | | \mathcal{R}^1 | | | $Ratio_2^A$ | | | |
|-------------------------------------|---------|-------------------------------------|-------------|-----------|-------------|-------------|-----------|------|
| | | Y_{t-h} | π_{t-h} | R_{t-h} | Y_{t-h} | π_{t-h} | R_{t-h} | |
| $\mathcal{M}_1 \succ \mathcal{M}_0$ | Y_t | 0.15 | 0.08 | 0.08 | 0.15 | 0.23 | 0.15 | |
| | | $\mathcal{M}_2 \succ \mathcal{M}_0$ | 0.15 | 0.77 | 0.85 | 0.23 | 0.62 | 0.85 |
| | | $\mathcal{M}_2 \succ \mathcal{M}_1$ | 0.31 | 0.77 | 0.85 | 0.31 | 0.62 | 0.85 |
| $\mathcal{M}_1 \succ \mathcal{M}_0$ | π_t | 0.46 | 0 | 0.08 | 0.15 | 0.08 | 0 | |
| | | $\mathcal{M}_2 \succ \mathcal{M}_0$ | 0.54 | 0 | 0.08 | 0.08 | 0.08 | 0 |
| | | $\mathcal{M}_2 \succ \mathcal{M}_1$ | 0.69 | 0.08 | 0.23 | 0.15 | 0.08 | 0 |
| $\mathcal{M}_1 \succ \mathcal{M}_0$ | R_t | 0.62 | 0 | 0.08 | 0.31 | 0 | 0.15 | |
| | | $\mathcal{M}_2 \succ \mathcal{M}_0$ | 0.15 | 0 | 0.08 | 0 | 0 | 0.08 |
| | | $\mathcal{M}_2 \succ \mathcal{M}_1$ | 0.15 | 0.08 | 0.08 | 0 | 0 | 0.08 |

Table 8: Percentage of cases in which $\mathcal{M}_i \succ \mathcal{M}_j$, considering the correlations between t and $t-h$ ($h = 0, \dots, 12$). \mathcal{R}^1 indicates the first type of risk function and $Ratio_2^A$ the ratio of the second type of posterior risk. Period 1966-2004.

| | | \mathcal{R}^1 | | | $Ratio_2^A$ | | | |
|-------------------------------------|---------|-------------------------------------|-------------|-----------|-------------|-------------|-----------|------|
| | | Y_{t-h} | π_{t-h} | R_{t-h} | Y_{t-h} | π_{t-h} | R_{t-h} | |
| $\mathcal{M}_1 \succ \mathcal{M}_0$ | Y_t | 0.62 | 0.23 | 0.23 | 0.46 | 0.38 | 0.23 | |
| | | $\mathcal{M}_2 \succ \mathcal{M}_0$ | 0.08 | 0.53 | 0.31 | 0.31 | 0.46 | 0.15 |
| | | $\mathcal{M}_2 \succ \mathcal{M}_1$ | 0.15 | 0.69 | 0.69 | 0.46 | 0.54 | 0.62 |
| $\mathcal{M}_1 \succ \mathcal{M}_0$ | π_t | 0.77 | 0.38 | 0.92 | 0.62 | 0.38 | 1.0 | |
| | | $\mathcal{M}_2 \succ \mathcal{M}_0$ | 0.62 | 0.38 | 0.23 | 0.85 | 0.08 | 0 |
| | | $\mathcal{M}_2 \succ \mathcal{M}_1$ | 0.23 | 0.69 | 0.08 | 0.77 | 0.15 | 0 |
| $\mathcal{M}_1 \succ \mathcal{M}_0$ | R_t | 0.23 | 0.92 | 0.85 | 0 | 1.0 | 0.92 | |
| | | $\mathcal{M}_2 \succ \mathcal{M}_0$ | 0.54 | 0.92 | 0.54 | 0.62 | 0.23 | 0.23 |
| | | $\mathcal{M}_2 \succ \mathcal{M}_1$ | 0.62 | 0.31 | 0.15 | 1.0 | 0 | 0.23 |

Table 9: Percentage of cases in which $\mathcal{M}_i \succ \mathcal{M}_j$, considering the correlations between t and $t-h$ ($h = 0, \dots, 12$). \mathcal{R}^1 indicates the first type of risk function and $Ratio_2^A$ the ratio of the second type of posterior risk. Period 1966-1982.

We do not find a clear evidence that a particular model should always be preferred to the others, nor across different periods, neither inside each period. With high volatility of inflation, i.e. in the full sample, the hybrid model is preferred by both types of loss functions. Instead, in a period with high level of inflation but a relative low associated volatility, i.e. the Great Inflation, the model that seems to better replicate the population characteristics under the first definition of loss is the DSGE with Generalized NKPC and indexation. Considering the second type of loss function the conclusion is ambiguous. For period characterized by low level and volatility of inflation, i.e. the Great Moderation, the two models with trend inflation seems to

| | | \mathcal{R}^1 | | | $Ratio_2^A$ | | |
|-------------------------------------|---------|-----------------|-------------|-----------|-------------|-------------|-----------|
| | | Y_{t-h} | π_{t-h} | R_{t-h} | Y_{t-h} | π_{t-h} | R_{t-h} |
| $\mathcal{M}_1 \succ \mathcal{M}_0$ | Y_t | 0.15 | 0.69 | 0.92 | 0.85 | 0.69 | 0.92 |
| $\mathcal{M}_2 \succ \mathcal{M}_0$ | | 0.15 | 0.77 | 0.85 | 0.54 | 0.62 | 0.77 |
| $\mathcal{M}_2 \succ \mathcal{M}_1$ | | 0.23 | 0.85 | 0.77 | 0.38 | 0.62 | 0.77 |
| $\mathcal{M}_1 \succ \mathcal{M}_0$ | π_t | 0.46 | 0.85 | 1.0 | 0.92 | 0.92 | 0.92 |
| $\mathcal{M}_2 \succ \mathcal{M}_0$ | | 0.54 | 0.08 | 0.08 | 0.46 | 0.15 | 0 |
| $\mathcal{M}_2 \succ \mathcal{M}_1$ | | 0.54 | 0.15 | 0 | 0.31 | 0.15 | 0 |
| $\mathcal{M}_1 \succ \mathcal{M}_0$ | R_t | 0.46 | 0.69 | 0.85 | 0.92 | 1.0 | 0.69 |
| $\mathcal{M}_2 \succ \mathcal{M}_0$ | | 0.54 | 0.08 | 0.15 | 1.0 | 0.23 | 0.08 |
| $\mathcal{M}_2 \succ \mathcal{M}_1$ | | 0.62 | 0.08 | 0.23 | 0.69 | 0.15 | 0.15 |

Table 10: Percentage of cases in which $\mathcal{M}_i \succ \mathcal{M}_j$, considering the correlations between t and $t-h$ ($h = 0, \dots, 12$). \mathcal{R}^1 indicates the first type of risk function and $Ratio_2^A$ the ratio of the second type of posterior risk. Period 1983-2004.

predict better the population characteristics. The models with trend inflation have poor precision in presence of an underlying change of regime that is not explicitly modeled.

We want to highlight the differences between the two loss functions and the associated posterior risk. For example, let us consider the correlation between Y_t and π_{t-h} for the period 1966-1983. Under the first definition of loss, the associated posterior risk suggests that preferences are $\mathcal{M}_2 \succ \mathcal{M}_0 \succ \mathcal{M}_1$, whereas under the second definition of loss the ratio between posterior risks leads to $\mathcal{M}_0 \succ \mathcal{M}_2 \succ \mathcal{M}_1$. The result for the first type of loss function is driven by the nearness between the population characteristic's mode and the mode of the model without indexation, even if the distribution of the correlation implied by the hybrid model is closer to the population characteristic's distribution than the ones generated by the other two DSGE models, as shown by the box plots in figure 3. It is important to consider this information on distribution too, because there are no restrictions that guarantee the normality of the distributions, and all the implied properties, for the correlations.

The same exercise is repeated for cumulated impulse response functions over ten periods for a negative monetary policy shock. In table 11 we summarize the results for the first type of posterior risk, i.e. \mathcal{R}^1 , whereas in table 12 there are the results for the second formulation of loss function, considering the sum of the absolute values with $\alpha = 2$, i.e. $Ratio_2^A$.

| Period | Models | Output | Inflation | Interest Rate |
|-------------|-----------------|--------|-----------|---------------|
| 1966 - 2004 | \mathcal{M}_0 | 0.60 | 0.16 | 0.42 |
| | \mathcal{M}_1 | 0.53 | 0.26 | 0.39 |
| | \mathcal{M}_2 | 0.49 | 0.31 | 0.18 |
| 1966 - 1982 | \mathcal{M}_0 | 0.01 | 0.80 | 0.04 |
| | \mathcal{M}_1 | 0.30 | 0.68 | 0.15 |
| | \mathcal{M}_2 | 0.27 | 0.35 | 0.03 |
| 1983 - 2004 | \mathcal{M}_0 | 1 | 0.20 | 0.9997 |
| | \mathcal{M}_1 | 0.9999 | 0.16 | 0.9997 |
| | \mathcal{M}_2 | 0.32 | 0.17 | 0.67 |

Table 11: Results under \mathcal{R}^1 for the IRFs to a monetary shock considering the three periods and the three DSGE models.

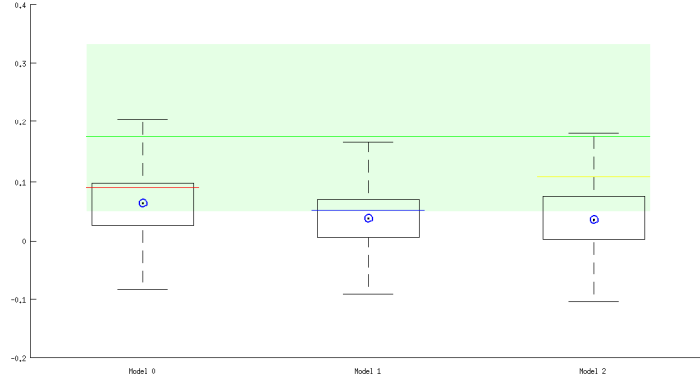


Figure 3: Box plots for the distribution of $\Gamma_{Y_t, \pi_{t-4}}$ obtained by the three DSGE models: hybrid (\mathcal{M}_0 in red), generalized with indexation (\mathcal{M}_1 in blue) and generalized without indexation (\mathcal{M}_2 in yellow). The solid lines are the associated modes. The shaded green area corresponds to the interval $[0.25; 0.75]$ for the population characteristic and the solid green line is the relative mode.

| Period | Models | Output | Inflation | Interest Rate |
|-------------|-------------------------------------|--------|-----------|---------------|
| 1966 - 2004 | $\mathcal{M}_1 \succ \mathcal{M}_0$ | 2.99 | 19.23 | 1.05 |
| | $\mathcal{M}_2 \succ \mathcal{M}_0$ | 0.75 | 11.41 | 0.54 |
| | $\mathcal{M}_2 \succ \mathcal{M}_1$ | 0.25 | 0.59 | 0.52 |
| 1966 - 1982 | $\mathcal{M}_1 \succ \mathcal{M}_0$ | 0.63 | 3.85 | 0.55 |
| | $\mathcal{M}_2 \succ \mathcal{M}_0$ | 0 | 0.22 | 0.44 |
| | $\mathcal{M}_2 \succ \mathcal{M}_1$ | 0 | 0.06 | 0.80 |
| 1983 - 2004 | $\mathcal{M}_1 \succ \mathcal{M}_0$ | 2.23 | 0.81 | 1.30 |
| | $\mathcal{M}_2 \succ \mathcal{M}_0$ | 0.20 | 0.50 | 0.01 |
| | $\mathcal{M}_2 \succ \mathcal{M}_1$ | 0.09 | 0.62 | 0.007 |

Table 12: Results under $Ratio_2^A$ for the IRFs to a monetary shock considering the three periods and the three DSGE models.

We have already observed that the IRFs to a negative monetary policy shock are very similar for the two models with indexation, i.e. \mathcal{M}_0 and \mathcal{M}_1 : it is emblematic that, considering the Great Moderation period, the risk implied by the first type of loss function for interest rates is exactly the same for the two models. In general, does not appear also here any superiority of one model with respect to another.

6 Conclusion

Assuming positive levels of steady state inflation in a basic New Keynesian model shrinks the determinacy region and affects the short run dynamic with respect to the standard textbook model approximated around a zero inflation steady state. Nevertheless, the empirical relevance of trend inflation in estimating a DSGE is not clear, since most of the analyses are performed on calibrated models. In this analysis we estimate, via Bayesian techniques, three declinations of a medium-scale New Keynesian

DSGE model with real and nominal frictions and we study the quantitative implications of trend inflation over different temporal horizons, which are characterized by different average levels of inflation. The posterior estimates for the structural parameters present some differences among the models, whereas the short run dynamic seems to be not affected by the inclusion of trend inflation. The models' comparison is built up applying two loss functions: the first based on a point distance criteria, whereas the second on the idea of entropy.

Comparing the ability to fit the data of the three DSGE models, via marginal data density, we observe that for the Great Inflation period the preferred model across the DSGE is the one with generalized NKPC and indexation, whereas for the Great Moderation period there is a slight preference for the model with hybrid NKPC.

The results for the cross-correlations and IRF based on loss functions do not present a clear evidence that a model should always be preferred to another, nor across different periods, neither inside each period.

Finally, in the forecast analysis the three models generate similar forecast RMSEs. Nevertheless, using the parameters estimated over a period with high volatility of inflation, the preferred model is the one without indexation, suggesting that given the data characteristics it is better a model that does not anchor the forecast to a volatile past. On the contrary, when the parameters are estimated on the Great Moderation period, the two models with indexation are equally preferred. It is interesting that for forecasting in period with high level of inflation, but relative low volatility, the model with trend inflation and indexation is preferred.

Concluding, we do not find evidence that a model with trend inflation should be always preferred. In all our various comparisons the presence of trend inflation does not give results strongly different from the classical model, apart for the case in which we suppress indexation to past inflation. The major differences appear in the full sample where the standard hybrid model seems to describe better the data. Since this period is characterized by the highest volatility of inflation, we argue that the models with trend inflation seems to have poorer precision than the standard hybrid model in the presence of an underlying change of regime that is not explicitly modeled.

Nevertheless, we want to highlight that the model with trend inflation could be useful for the analysis of the Great Inflation. In fact, for this period, it shows the better fit with the data in terms of marginal data density and the lowest forecast RMSE, in particular for the inflation series.

Appendix A: the model

In this section we present the log-linearized equations that characterized the model. With respect to SW the main difference lies in the fact that we consider a steady state level of gross inflation, π_* , greater than one. Given that, we should consider in particular that price and wage dispersion affect the dynamic also up the first order approximation.

The log-linearized aggregate resource constraint of this closed economy is given by

$$y_t = c_y c_t + i_y i_t + z_y z_t + \epsilon_t^g$$

where y_t is real GDP, absorbed by real private consumption c_t , real private investments i_t , capital utilization rate z_t and exogenous government spending ϵ_t^g . The parameter

c_y is the steady-state consumption-output ratio and i_y is the steady-state investment-output ratio, where

$$c_y = 1 - g_y - i_y$$

and g_y is the steady-state exogenous spending-output ratio. The steady-state investment-output ratio is determined by

$$i_y = (\gamma - 1 + \delta)k_y$$

where k_y is the steady-state capital-output ratio, γ is the steady-state labour-augmenting growth rate, and δ is the depreciation rate of capital; the parameter z_y is equal to $r_k^* k_y$, where $k_y = \frac{k^*}{y^*}$.

The dynamics of consumption follows from the consumption Euler equation given by

$$c_t = c_1 c_{t-1} + (1 - c_1) E_t c_{t+1} + c_2 (l_t - E_t l_{t+1}) - c_3 (r_t - E_t \pi_{t+1} + \epsilon_t^b)$$

where l_t is hours worked, r_t is the nominal interest rate and the coefficients are:

$$\begin{aligned} c_1 &= \frac{\lambda}{\gamma} \left(1 + \frac{\lambda}{\gamma} \right) \\ c_2 &= \left[(\sigma_c - 1) \frac{w_*^h L_*}{c_*} \right] \frac{1}{\sigma_c \left(1 + \frac{\lambda}{\gamma} \right)} \\ c_3 &= \left(1 - \frac{\lambda}{\gamma} \right) \frac{1}{\sigma_c \left(1 + \frac{\lambda}{\gamma} \right)} \end{aligned}$$

where λ measures external habit formation, σ_c is the inverse of the elasticity of intertemporal substitution for constant labour, while $\frac{w_*^h L_*}{c_*}$ is the steady-state hourly real wage bill to consumption ratio²⁰.

The log-linearized investment Euler equation is given by

$$i_t = i_1 i_{t-1} + (1 - i_1) E_t i_{t+1} + i_2 q_t + \epsilon_t^i$$

where q_t is the real value of the existing capital stock, while ϵ_t^i is an exogenous investment specific technology variable. The parameters are given by

$$\begin{aligned} i_1 &= \frac{1}{1 + \beta \gamma^{1 - \sigma_c}} \\ i_2 &= \frac{1}{1 + \beta \gamma^{1 - \sigma_c} \gamma^2 \phi} \end{aligned}$$

where β is the discount factor used by households, and ϕ is the steady-state elasticity of the capital adjustment cost function.

The dynamic equation for the value of the capital stock is

$$q_t = q_1 E_t q_{t+1} + (1 - q_1) E_t r_{t+1}^k - (r_t - E_t \pi_{t+1} + \epsilon_t^b)$$

where r_t^k is the rental price of capital. The parameter q_1 is given by

$$q_1 = \beta \gamma^{-\sigma_c} (1 - \delta)$$

²⁰ If $\sigma_c = 1$ (log-utility) and $\lambda = 1$ (no external habit) then the above equation reduces to the familiar purely forward looking consumption Euler equation

Turning to the supply-side of the economy, the log-linearized aggregate production function can be expressed as

$$s_t^p + y_t = \alpha k_t^s + (1 - \alpha) l_t^d + \epsilon_t^a$$

where k_t^s is capital services used in production, l_t^d represents labour demand and ϵ_t^a an exogenous total factor productivity variable, the parameter α reflects the share of capital in production, while s_t^p is the relative price dispersion evolution due to the Dixit-Stiglitz aggregator:

$$s_t^p = \theta^p (\pi_\star^{1-\iota_p} - 1) \frac{\omega_p \pi_\star^{(\theta^p-1)(1-\iota_p)}}{1 - \omega_p \pi_\star^{(\theta^p-1)(1-\iota_p)}} (\pi_t - \iota_p \pi_{t-1}) + \omega_p \pi_\star^{\theta_p(1-\iota_p)} s_{t-1}^p$$

where s_t has a lower bound equal to 1 and π_\star is the inflation at the steady state. From the Calvo pricing mechanism, $1 - \omega_p$ is the probability that a firm can reoptimize its price at time t , whereas $\theta^p > 1$ is the parameter of the Dixit-Stiglitz aggregator over the j -th firms:

$$\tilde{P}_t = \left[\int \tilde{P}_{j,t}^{1-\theta^p} dj \right]^{\frac{1}{1-\theta^p}}$$

Moreover $\iota_p \in [0, 1]$ is the price index such that the non optimizing firms could re-adjust their prices to the past inflation:

$$\begin{aligned} \tilde{Y}_{i,t+j} &= \left[\frac{\tilde{P}_{i,t}^o}{\tilde{P}_{t+j}} \tilde{\Omega}_{t,t+j-1} \right]^{-\theta^p} \tilde{Y}_{t+j} \\ \tilde{\Omega}_{t,t+j-1} &= \prod_{j=0}^s \tilde{\pi}_{t+j-1}^{\iota_p} \end{aligned}$$

The capital services variable is used to reflect that newly installed capital only becomes effective with a one period lag. This means that

$$k_t^s = k_{t-1} + z_t$$

where k_t is the installed capital. The degree of capital utilization is determined from cost minimization of the households, who provide capital services, and it is therefore a positive function of the rental rate of capital. Specifically,

$$z_t = z_1 r_t^k$$

where

$$z_1 = \frac{1 - \psi}{\psi}$$

and ψ is a positive function of the elasticity of the capital adjustment cost function and normalized to be between 0 and 1.

The log-linearized equation that specifies the development of installed capital is

$$k_t = k_1 k_{t-1} + (1 - k_1) i_t + k_2 \epsilon_t^i$$

The two parameters are given by

$$\begin{aligned} k_1 &= \frac{1 - \delta}{\gamma} \\ k_2 &= \left(1 - \frac{1 - \delta}{\gamma} \right) (1 + \beta \gamma^{1-\sigma_c}) \gamma^2 \phi \end{aligned}$$

From the monopolistically competitive goods market, the price markup μ_t^p is equal to minus

$$\mu_t^p = \alpha(k_t^s - l_t^d) + \epsilon_t^a - w_t$$

where the real wage is given by w_t .

Cost minimization of firms also implies that the rental rate of capital is related to the capital-labour ratio and to the real wage, according to

$$r_t^k = -(k_t - l_t^d) + w_t$$

In the monopolistically competitive labour market the wage markup is equal to the difference between the real wage and the marginal rate of substitution between labour and consumption

$$\mu_t^w = w_t - \left[\sigma_l l_t + \frac{c_t - \frac{\lambda}{\gamma} c_{t-1}}{1 + \frac{\lambda}{\gamma}} \right]$$

where σ_l is the elasticity of labour supply with respect to the real wage. Market clearing on the labour market implies

$$\begin{aligned} L_t &= \int_0^1 \left(\frac{W_t(j)}{W_t} \right)^{\theta_w} L_t^d dj \\ &= \tilde{s}_t^w L_t^d \Rightarrow l_t = s_t^w + l_t^d \end{aligned}$$

where \tilde{s}_t^w is the relative wage dispersion, characterized by the log linearized dynamics:

$$\begin{aligned} s_t^w &= -\theta^w (1 - \omega_w) \pi_\star^{(1-\iota_w)\theta^w} (w_t^o - w_t) + \omega_w \pi_\star^{(1-\iota_w)\theta^w} + \\ & s_{t-1}^w + \theta^w (\pi_t - \iota_w \pi_{t-1}) - \theta^w (w_{t-1} - w_t) \end{aligned}$$

The loglinearization of the Generalized NKPC derived in the main text is given by

$$f_t^{1,p} = f_t^{2,p}$$

where

$$\begin{aligned} f_t^{1,p} &= (1 - A_1^p) [\theta^p p_{i,t}^o + y_t] + A_1^p \left[\frac{\iota^p}{\theta^p + 1} \pi_t + \theta^p p_{i,t}^o - \theta^p \pi_{t+1} - \theta^p p_{i,t+1}^o + f_{t+1}^{1,p} + \xi_{t+1|t} \right] \\ f_t^{2,p} &= (1 - A_2^p) [(\theta^p - 1) p_{i,t}^o + y_t - \mu_t^p] + \\ & A_2^p \left[\iota^p \theta^p \pi_t - (\theta^p + 1) p_{i,t}^o + (1 - \theta^p) \pi_{t+1} + (1 - \theta^p) p_{i,t+1}^o + f_{t+1}^{2,p} + \xi_{t+1|t} \right] \end{aligned}$$

and the coefficients are

$$\begin{aligned} A_1^p &= \omega_p \beta \gamma^{1-\sigma_c} \left[\frac{1}{\pi_\star} \right]^{(1-\iota^p)(\theta^p+1)} \\ A_2^p &= \omega_p \beta \gamma^{1-\sigma_c} \left[\frac{1}{\pi_\star} \right]^{(1-\iota^p)\theta^p} \end{aligned}$$

whereas $p_{i,t}^o$, the optimal price, evolves according to:

$$p_{i,t}^o = \frac{\omega_p \pi_\star^{(\theta^p-1)(1-\iota_p)}}{1 - \omega_p \pi_\star^{(\theta^p-1)(1-\iota_p)}} (\pi_t - \iota_p \pi_{t-1})$$

Imposing $\pi_\star = 1$, it is possible to recover the standard hybrid NKPC:

$$\pi_t = \pi_1 \pi_{t-1} + \pi_2 E_t \pi_{t+1} - \pi_3 \mu_t^p$$

where

$$\pi_1 = \frac{\iota^p}{1 + \beta\gamma^{1-\sigma_c}\iota^p} \quad \pi_2 = \frac{\beta\gamma^{1-\sigma_c}}{1 + \beta\gamma^{1-\sigma_c}\iota^p} \quad \pi_3 = \frac{(1 - \beta\gamma^{1-\sigma_c}\omega_p)(1 - \omega_p)}{(1 + \beta\gamma^{1-\sigma_c}\iota^p)\omega_p}$$

Whereas the loglinearized wage equation with trend inflation, previously derived, is given by

$$f_t^{1,w} + w_t^o = f_t^{2,w}$$

where

$$f_t^{1,w} = (1 - A_1^w) \left[\theta^w (w_t - w_t^o) + L_t^d \right] + A_1^w \left[(\theta^w - 1) (\pi_{t+1} - \iota_w \pi_t) + \theta^w (w_{t+1}^o - w_t^o) + \xi_{t+1|t} + f_{t+1}^{1,w} \right]$$

and

$$f_t^{2,w} = (1 - A_2^w) \left[\theta^w (w_t - w_t^o) + L_t^d + w_t - \mu_t^w \right] + A_2^w \left[\theta^w (\pi_{t+1} - \iota_w \pi_t) + \theta^w (w_{t+1}^o - w_t^o) + \xi_{t+1|t} + f_{t+1}^{2,w} \right]$$

and the coefficients are:

$$A_1^w = \omega_w \beta \gamma^{1-\sigma_c} \pi_\star^{(1-\iota_w)(\theta^w-1)}$$

$$A_2^w = \omega_w \beta \gamma^{1-\sigma_c} \pi_\star^{(1-\iota_w)\theta^w}$$

whereas the equation for the optimal wage w_t^o is:

$$w_t^o = \frac{w_\star}{w_\star(1 - \omega_w)} w_t - \frac{\omega_w}{1 - \omega_w} \pi_\star^{(1-\iota_w)(\theta^w-1)} \frac{w_\star}{w_\star^o} [w_{t-1} + \iota_w \pi_{t-1} - \pi_t]$$

Imposing $\pi_\star = 1$, it is possible to recover the standard wage equation:

$$w_t = w_1 w_{t-1} + (1 - w_1)(E_t w_{t+1} + E_t \pi_{t+1}) - w_2 \pi_t + w_3 \pi_{t-1} - w_4 \mu_t^w$$

where

$$w_1 = \frac{1}{1 + \beta\gamma^{1-\sigma_c}} \quad w_2 = \frac{1 + \beta\gamma^{1-\sigma_c}\iota^w}{1 + \beta\gamma^{1-\sigma_c}}$$

$$w_3 = \frac{\iota^w}{1 + \beta\gamma^{1-\sigma_c}} \quad w_4 = \frac{(1 - \beta\gamma^{1-\sigma_c}\omega_w)(1 - \omega_w)}{(1 + \beta\gamma^{1-\sigma_c}\iota^p)\omega_w}$$

The sticky price and wage part of the model is closed by adding the monetary policy reaction function

$$R_t = \rho_R R_{t-1} + (1 - \rho_R) [\psi_\pi \pi_t + \psi_y (y_t - y_{t-1})] + \epsilon_t^r$$

There are 7 exogenous processes in the Smets and Wouters (2007) model. These are generally modeled as AR(1) process with the exception of the exogenous spending process (where the process depends on both the exogenous spending shock η_t^g and the total factor productivity shock η_t^a) and the exogenous price and wage markup processes, which are treated as ARMA(1,1) processes. Therefore we have

- government spending shock: $\epsilon_t^g = \rho^g \epsilon_{t-1}^g + \sigma^g \eta_t^g + \rho_{ga} \sigma^a \eta_t^a$
- investment shock: $\epsilon_t^i = \rho^i \epsilon_{t-1}^i + \sigma^i \eta_t^i$
- preference shock: $\epsilon_t^b = \rho^b \epsilon_{t-1}^b + \sigma^b \eta_t^b$
- total factor productivity shock: $\epsilon_t^a = \rho^a \sigma^a \epsilon_{t-1}^a + \sigma^a \eta_t^a$
- inflation shock: $\epsilon_t^p = \rho^p \epsilon_{t-1}^p + \sigma^p \eta_t^p - \mu_p \sigma^p \eta_{t-1}^p$
- wage shock: $\epsilon_t^w = \rho^w \epsilon_{t-1}^w + \sigma^w \eta_t^w - \mu_w \sigma^w \eta_{t-1}^w$
- interest rate shock: $\epsilon_t^r = \rho^r \epsilon_{t-1}^r + \sigma^r \eta_t^r$

The shocks $\eta_t^j \sim N(0, 1)$, for $j = \{a, b, g, i, p, r, w\}$.

Appendix B

In table (13) there are the priors, whereas in the next are reported the posterior mode and 95 probability intervals, considering all the sample (tab. 14), the Great Inflation period (tab. 15) and the Great Moderation period (tab. 16) for the three models.

| Param. | Prior | Distribution | |
|----------------|---------------|--------------|--------|
| | Dist. | Mean | St.Dev |
| σ^i | InvGamma | 0.1 | 2.0 |
| σ^b | InvGamma | 0.1 | 2.0 |
| σ^a | InvGamma | 0.1 | 2.0 |
| σ^g | InvGamma | 0.1 | 2.0 |
| σ^p | InvGamma | 0.1 | 2.0 |
| σ^w | InvGamma | 0.1 | 2.0 |
| σ^r | InvGamma | 0.1 | 2.0 |
| ρ_i | Beta | 0.5 | 0.2 |
| ρ_b | Beta | 0.5 | 0.2 |
| ρ_a | Beta | 0.5 | 0.2 |
| ρ_g | Beta | 0.5 | 0.2 |
| ρ_p | Beta | 0.5 | 0.2 |
| ρ_w | Beta | 0.5 | 0.2 |
| ρ_r | Beta | 0.5 | 0.2 |
| ρ_{ag} | Beta | 0.5 | 0.2 |
| μ_p | Beta | 0.5 | 0.2 |
| μ_w | Beta | 0.5 | 0.2 |
| $\bar{\gamma}$ | Trunc. Normal | 0.4 | 0.1 |
| $\bar{\beta}$ | Gamma | 0.25 | 0.1 |
| σ_c | Trunc. Normal | 1.5 | 0.1 |
| ϕ | Trunc. Normal | 4 | 0.5 |
| λ | Beta | 0.7 | 0.1 |
| θ^p | Trunc. Normal | 5.0 | 0.5 |
| ι^p | Beta | 0.5 | 0.2 |
| ω_p | Beta | 0.5 | 0.1 |
| ι^w | Beta | 0.5 | 0.2 |
| ω_w | Beta | 0.5 | 0.1 |
| ρ | Beta | 0.75 | 0.1 |
| ψ_π | Trunc. Normal | 1.5 | 0.25 |
| ψ_y | Trunc. Normal | 0.12 | 0.05 |
| α | Trunc. Normal | 0.3 | 0.05 |
| σ_l | Trunc. Normal | 2 | 0.75 |
| ψ | Beta | 0.5 | 0.15 |
| \bar{l} | Normal | 0.0 | 2.00 |

Table 13: Priors.

where

$$\bar{\gamma} = 100(\gamma - 1) \quad \bar{\beta} = 100(\beta^{-1} - 1)$$

and \bar{l} , the steady state hours worked, is normalized to be equal to zero.

Posteriors: Full sample

| Param | Hybrid | | | Trend | | | Trend no index | | |
|-------------|----------|----------|---------|----------|----------|---------|----------------|----------|---------|
| | 5 perc. | Mean | 95 perc | 5 perc. | Mean | 95 perc | 5 perc. | Mean | 95 perc |
| σ_i | 0.3088 | 0.36927 | 0.44309 | 0.3098 | 0.38135 | 0.47237 | 0.33466 | 0.42582 | 0.53109 |
| σ_b | 0.23364 | 0.26921 | 0.30505 | 0.23356 | 0.26955 | 0.30671 | 0.22656 | 0.26304 | 0.29942 |
| σ_a | 0.41903 | 0.46918 | 0.52291 | 0.42786 | 0.47753 | 0.53223 | 0.44112 | 0.49357 | 0.55178 |
| σ_g | 0.50864 | 0.55931 | 0.61423 | 0.50455 | 0.55573 | 0.61145 | 0.4996 | 0.55408 | 0.61198 |
| σ_p | 0.22846 | 0.28202 | 0.33862 | 0.21813 | 0.27455 | 0.32987 | 0.16436 | 0.23594 | 0.29027 |
| σ_w | 0.38422 | 0.44368 | 0.50618 | 0.38895 | 0.44614 | 0.50608 | 0.40513 | 0.45976 | 0.51771 |
| σ_r | 0.23602 | 0.25991 | 0.28669 | 0.23635 | 0.2604 | 0.28689 | 0.23399 | 0.25758 | 0.28431 |
| ρ_i | 0.7061 | 0.8038 | 0.89194 | 0.64529 | 0.76797 | 0.87348 | 0.5792 | 0.68986 | 0.80962 |
| ρ_b | 0.035003 | 0.11706 | 0.22491 | 0.029503 | 0.10911 | 0.21236 | 0.038875 | 0.1215 | 0.23241 |
| ρ_a | 0.88962 | 0.93953 | 0.99043 | 0.88944 | 0.94594 | 0.9928 | 0.93528 | 0.97668 | 0.99692 |
| ρ_g | 0.97458 | 0.9896 | 0.9977 | 0.96876 | 0.98934 | 0.99799 | 0.94324 | 0.97624 | 0.99639 |
| ρ_p | 0.88102 | 0.92243 | 0.96407 | 0.8177 | 0.9068 | 0.96447 | 0.7998 | 0.89666 | 0.96905 |
| ρ_w | 0.95032 | 0.96941 | 0.98686 | 0.33317 | 0.86409 | 0.98819 | 0.037753 | 0.69411 | 0.99316 |
| ρ_r | 0.15568 | 0.27475 | 0.39738 | 0.15839 | 0.27747 | 0.39671 | 0.171 | 0.29091 | 0.41283 |
| ρ_{ag} | 0.17482 | 0.33545 | 0.49864 | 0.17898 | 0.33686 | 0.50213 | 0.17041 | 0.34269 | 0.52956 |
| μ_p | 0.33693 | 0.51178 | 0.67318 | 0.33877 | 0.512 | 0.66113 | 0.18278 | 0.35853 | 0.53857 |
| μ_w | 0.60832 | 0.73755 | 0.86769 | 0.38385 | 0.70558 | 0.8826 | 0.17849 | 0.67501 | 0.92525 |
| γ | 0.36462 | 0.3975 | 0.43265 | 0.33379 | 0.38791 | 0.43055 | 0.30015 | 0.33941 | 0.38895 |
| β | 0.13409 | 0.21944 | 0.33136 | 0.13117 | 0.2106 | 0.31138 | 0.1333 | 0.21843 | 0.32894 |
| σ_c | 1.02 | 1.1333 | 1.2987 | 1.0322 | 1.1952 | 1.4934 | 1.0895 | 1.288 | 1.514 |
| ϕ | 4.4477 | 5.1342 | 5.8388 | 4.4698 | 5.149 | 5.842 | 4.5994 | 5.2704 | 5.9667 |
| λ | 0.79147 | 0.836 | 0.87043 | 0.78472 | 0.83049 | 0.86793 | 0.80544 | 0.8389 | 0.86982 |
| θ^p | 1.6039 | 1.7811 | 2.0009 | 1.6473 | 1.8556 | 2.0992 | 1.6415 | 1.9647 | 2.495 |
| ι_p | 0.13069 | 0.32397 | 0.56265 | 0.12224 | 0.31813 | 0.54775 | - | - | - |
| ω_p | 0.83617 | 0.87172 | 0.90541 | 0.83813 | 0.8744 | 0.90739 | 0.84073 | 0.8705 | 0.90038 |
| ι_w | 0.33429 | 0.60353 | 0.8551 | 0.10136 | 0.45988 | 0.78316 | - | - | - |
| ω_w | 0.76041 | 0.80975 | 0.86443 | 0.76336 | 0.83834 | 0.94452 | 0.80152 | 0.89474 | 0.93673 |
| ρ | 0.67533 | 0.73177 | 0.78214 | 0.66415 | 0.71953 | 0.77097 | 0.65556 | 0.71515 | 0.76858 |
| ψ_π | 1.1412 | 1.374 | 1.6649 | 1.0746 | 1.2596 | 1.5737 | 1.0413 | 1.1376 | 1.3236 |
| ψ_y | 0.13067 | 0.20355 | 0.27794 | 0.13045 | 0.20096 | 0.27397 | 0.11438 | 0.18769 | 0.26719 |
| α | 0.1493 | 0.18419 | 0.21977 | 0.12654 | 0.16694 | 0.20577 | 0.12311 | 0.15412 | 0.18865 |
| σ_l | 1.0416 | 1.4715 | 2.1413 | 1.0289 | 1.6072 | 2.6947 | 1.0421 | 1.5807 | 2.553 |
| ψ | 0.51982 | 0.70453 | 0.86247 | 0.424 | 0.65633 | 0.84811 | 0.3402 | 0.57498 | 0.7967 |
| \bar{l} | -2.6358 | -0.47038 | 1.8631 | -2.5519 | -0.30474 | 2.0232 | -2.4042 | -0.21312 | 2.0303 |

Table 14: Posteriors for the models with $\pi_\star = 1$ (hybrid NKPC), with $\pi_\star = 1 + (4.02/400)$ (generalized NKPC) with or without Indexation, **period 1966 - 2004**.

Posteriors: Great Inflation

| Param | Hybrid | | | Trend | | | Trend | no | index |
|----------------|----------|---------|----------|----------|---------|-----------|----------|---------|---------|
| | 5 perc. | Mean | 95 perc | 5 perc. | Mean | 95 perc | 5 perc. | Mean | 95 perc |
| σ_i | 0.32977 | 0.42355 | 0.55303 | 0.34388 | 0.4446 | 0.5808 | 0.3067 | 0.41762 | 0.58281 |
| σ_b | 0.24244 | 0.30997 | 0.38055 | 0.23675 | 0.30375 | 0.37311 | 0.2428 | 0.30675 | 0.37393 |
| σ_a | 0.50185 | 0.60537 | 0.72583 | 0.50557 | 0.61413 | 0.73947 | 0.48661 | 0.61837 | 0.75683 |
| σ_g | 0.52609 | 0.62101 | 0.73217 | 0.52693 | 0.62628 | 0.73959 | 0.52601 | 0.62146 | 0.73128 |
| σ_p | 0.25343 | 0.34269 | 0.42475 | 0.21062 | 0.35108 | 0.4449 | 0.20992 | 0.31419 | 0.40076 |
| σ_w | 0.3039 | 0.36698 | 0.43833 | 0.30898 | 0.37162 | 0.4429 | 0.30832 | 0.36777 | 0.43637 |
| σ_r | 0.31294 | 0.36253 | 0.42061 | 0.31134 | 0.36039 | 0.41711 | 0.31159 | 0.35975 | 0.41603 |
| ρ_i | 0.63701 | 0.85068 | 0.9601 | 0.60551 | 0.81422 | 0.93835 | 0.54734 | 0.79582 | 0.95031 |
| ρ_b | 0.077741 | 0.22145 | 0.39837 | 0.075859 | 0.22407 | 0.40248 | 0.070686 | 0.21694 | 0.38909 |
| ρ_a | 0.68513 | 0.79318 | 0.89556 | 0.68346 | 0.80512 | 0.92069 | 0.6666 | 0.7866 | 0.89387 |
| ρ_g | 0.46349 | 0.71475 | 0.93687 | 0.48841 | 0.75676 | 0.94451 | 0.48232 | 0.75464 | 0.95501 |
| ρ_p | 0.34019 | 0.80564 | 0.9648 | 0.65264 | 0.86908 | 0.966 | 0.83652 | 0.92038 | 0.98108 |
| ρ_w | 0.8786 | 0.93034 | 0.96841 | 0.86204 | 0.92326 | 0.96888 | 0.90372 | 0.94474 | 0.98587 |
| ρ_r | 0.091475 | 0.23359 | 0.40016 | 0.094205 | 0.2396 | 0.41027 | 0.091467 | 0.22894 | 0.38362 |
| ρ_{ag} | 0.29013 | 0.50044 | 0.71554 | 0.29138 | 0.50154 | 0.7185 | 0.30017 | 0.52043 | 0.75154 |
| μ_p | 0.26858 | 0.49681 | 0.6976 | 0.26476 | 0.49058 | 0.69312 | 0.18317 | 0.38261 | 0.5842 |
| μ_w | 0.42305 | 0.59076 | 0.7816 | 0.42402 | 0.59775 | 0.80958 | 0.46469 | 0.6483 | 0.83786 |
| $\bar{\gamma}$ | 0.23572 | 0.29716 | 0.36493 | 0.22842 | 0.29695 | 0.39783 | 0.23198 | 0.30695 | 0.4077 |
| $\bar{\beta}$ | 0.12781 | 0.2151 | 0.33889 | 0.13365 | 0.23257 | 0.37086 | 0.12884 | 0.22165 | 0.3543 |
| σ_c | 1.2243 | 1.391 | 1.5597 | 1.1874 | 1.3919 | 1.572 | 1.2659 | 1.4269 | 1.5899 |
| ϕ | 3.4267 | 4.1948 | 4.9705 | 3.4289 | 4.2033 | 4.9964 | 3.4623 | 4.2146 | 4.9826 |
| λ | 0.65454 | 0.73256 | 0.80027 | 0.64697 | 0.72648 | 0.79993 | 0.66557 | 0.7356 | 0.79816 |
| θ^p | 1.7751 | 2.174 | 2.706 | 1.7897 | 2.2339 | 2.9968 | 1.7684 | 2.2461 | 2.9113 |
| ι_p | 0.17686 | 0.4327 | 0.7431 | 0.1777 | 0.39869 | 0.65472 | - | - | - |
| ω_p | 0.77836 | 0.81887 | 0.85922 | 0.75989 | 0.80057 | 0.84184 | 0.76202 | 0.80853 | 0.86082 |
| ι_w | 0.31955 | 0.60625 | 0.86593 | 0.32686 | 0.60443 | 0.85446 | - | - | - |
| ω_w | 0.71439 | 0.78115 | 0.84715 | 0.70811 | 0.77738 | 0.84646 | 0.71093 | 0.7765 | 0.83657 |
| ρ | 0.59837 | 0.69763 | 0.77655 | 0.58767 | 0.68496 | 0.764 | 0.59514 | 0.68447 | 0.76066 |
| ψ_π | 1.0567 | 1.3921 | 1.7009 | 1.049 | 1.4451 | 1.7839 | 1.0341 | 1.3784 | 1.8026 |
| ψ_y | 0.056046 | 0.13625 | 0.21384 | 0.053767 | 0.1338 | 0.21324 | 0.061199 | 0.13515 | 0.21166 |
| α | 0.15594 | 0.19876 | 0.24402 | 0.15309 | 0.20148 | 0.25083 | 0.15291 | 0.19733 | 0.24415 |
| σ_l | 1.0191 | 1.3896 | 2.1225 | 1.0132 | 1.3644 | 2.0772 | 1.0138 | 1.3226 | 1.9726 |
| ψ | 0.22673 | 0.42203 | 0.66291 | 0.21774 | 0.42642 | 0.68759 | 0.19845 | 0.41443 | 0.69474 |
| \bar{l} | -4.8558 | -2.9362 | -0.35722 | -4.9721 | -2.6822 | -0.013897 | -4.5969 | -2.1145 | 1.0014 |

Table 15: Posteriors for the models with $\pi_\star = 1$ (hybrid NKPC), with $\pi_\star = 1 + (6.08/400)$ (generalized NKPC) with or without Indexation, **period 1966 - 1982**.

Posteriors: Great Moderation

| Param | Hybrid | | | Trend | | | Trend | no | index |
|----------------|----------|---------|----------|----------|---------|----------|---------|---------|----------|
| | 5 perc. | Mean | 95 perc | 5 perc. | Mean | 95 perc | 5 perc. | Mean | 95 perc |
| σ_i | 0.37798 | 0.49039 | 0.62423 | 0.37885 | 0.49081 | 0.62477 | 0.38259 | 0.49413 | 0.62459 |
| σ_b | 0.12358 | 0.1869 | 0.23872 | 0.12065 | 0.18675 | 0.23784 | 0.12993 | 0.19132 | 0.23982 |
| σ_a | 0.33902 | 0.39027 | 0.44711 | 0.339 | 0.38951 | 0.44707 | 0.34115 | 0.39299 | 0.45242 |
| σ_g | 0.35805 | 0.40561 | 0.45921 | 0.3579 | 0.40493 | 0.4589 | 0.35585 | 0.40373 | 0.45759 |
| σ_p | 0.17774 | 0.21019 | 0.24812 | 0.17649 | 0.2093 | 0.2481 | 0.17329 | 0.20266 | 0.23668 |
| σ_w | 0.31452 | 0.38808 | 0.47404 | 0.31855 | 0.39076 | 0.4726 | 0.31758 | 0.38949 | 0.468 |
| σ_r | 0.11235 | 0.12873 | 0.14736 | 0.1128 | 0.12942 | 0.14843 | 0.11347 | 0.13026 | 0.14908 |
| ρ_i | 0.41625 | 0.56567 | 0.70344 | 0.41585 | 0.5647 | 0.70262 | 0.41766 | 0.56008 | 0.69516 |
| ρ_b | 0.073406 | 0.26795 | 0.553 | 0.069994 | 0.26288 | 0.56136 | 0.06455 | 0.24387 | 0.52191 |
| ρ_a | 0.87333 | 0.92584 | 0.96356 | 0.87659 | 0.92735 | 0.9635 | 0.87685 | 0.92852 | 0.96474 |
| ρ_g | 0.94218 | 0.96806 | 0.98777 | 0.94097 | 0.96756 | 0.98798 | 0.9414 | 0.96871 | 0.98867 |
| ρ_p | 0.77873 | 0.85544 | 0.9113 | 0.77649 | 0.85331 | 0.90961 | 0.77969 | 0.85168 | 0.90589 |
| ρ_w | 0.95611 | 0.96921 | 0.98012 | 0.9563 | 0.96933 | 0.98004 | 0.95675 | 0.96967 | 0.98026 |
| ρ_r | 0.33022 | 0.45501 | 0.57552 | 0.34091 | 0.4647 | 0.58437 | 0.31628 | 0.43976 | 0.5572 |
| ρ_{ag} | 0.19732 | 0.37138 | 0.55264 | 0.20805 | 0.37965 | 0.56171 | 0.21609 | 0.3851 | 0.56175 |
| μ_p | 0.20099 | 0.37847 | 0.55157 | 0.19389 | 0.37207 | 0.54798 | 0.14452 | 0.29962 | 0.45749 |
| μ_w | 0.47582 | 0.63334 | 0.77032 | 0.47895 | 0.63989 | 0.78023 | 0.50807 | 0.66813 | 0.80744 |
| $\bar{\gamma}$ | 0.31051 | 0.35358 | 0.39667 | 0.30726 | 0.35234 | 0.3963 | 0.30387 | 0.34961 | 0.39514 |
| $\bar{\beta}$ | 0.15046 | 0.26043 | 0.40264 | 0.14675 | 0.25015 | 0.38627 | 0.14838 | 0.25275 | 0.3903 |
| σ_c | 1.1911 | 1.3317 | 1.4803 | 1.1826 | 1.3291 | 1.4816 | 1.1884 | 1.3324 | 1.482 |
| ϕ | 4.1184 | 4.8301 | 5.5555 | 4.1002 | 4.8253 | 5.5742 | 4.1225 | 4.8327 | 5.5562 |
| λ | 0.66898 | 0.75396 | 0.81642 | 0.66712 | 0.75539 | 0.81743 | 0.68374 | 0.76381 | 0.82256 |
| θ^p | 1.8565 | 2.0566 | 2.2884 | 1.8643 | 2.0761 | 2.3198 | 1.8597 | 2.0687 | 2.3088 |
| ι_p | 0.056969 | 0.17721 | 0.32918 | 0.054248 | 0.17204 | 0.32136 | - | - | - |
| ω_p | 0.84389 | 0.86961 | 0.8943 | 0.83808 | 0.86385 | 0.88848 | 0.83948 | 0.86425 | 0.88804 |
| ι_w | 0.23045 | 0.5479 | 0.84997 | 0.22032 | 0.53724 | 0.8365 | - | - | - |
| ω_w | 0.66342 | 0.74018 | 0.80979 | 0.66283 | 0.74049 | 0.81276 | 0.67542 | 0.75349 | 0.82666 |
| ρ | 0.75714 | 0.79925 | 0.83646 | 0.75246 | 0.79598 | 0.83504 | 0.75556 | 0.79679 | 0.83347 |
| ψ_π | 1.5911 | 1.829 | 2.0893 | 1.5926 | 1.8318 | 2.0972 | 1.627 | 1.8613 | 2.1186 |
| ψ_y | 0.11076 | 0.18506 | 0.26099 | 0.10865 | 0.1837 | 0.25904 | 0.10812 | 0.18191 | 0.25636 |
| α | 0.1657 | 0.20455 | 0.24233 | 0.1655 | 0.20413 | 0.24195 | 0.16773 | 0.20523 | 0.24301 |
| σ_l | 1.1105 | 1.9509 | 2.9088 | 1.1437 | 1.9373 | 2.8776 | 1.1017 | 1.8097 | 2.7063 |
| ψ | 0.68245 | 0.80872 | 0.91664 | 0.67629 | 0.8081 | 0.91732 | 0.67735 | 0.81196 | 0.92106 |
| \bar{l} | -4.3283 | -2.3806 | -0.36137 | -4.2788 | -2.3287 | -0.32871 | -4.3154 | -2.3392 | -0.31948 |

Table 16: Posteriors for the models with $\pi_* = 1$ (hybrid NKPC), with $\pi_* = 1 + (2.43/400)$ (generalized NKPC) with or without Indexation, **period 1983 - 2004**.

Appendix C

IRFs

In the following figures there are the IRFs to a negative monetary policy shock, considering the three periods: Full sample (fig.4), Great Inflation (fig.5) and Great Moderation (fig.6). The blue line is the hybrid model, the green line is the model with trend inflation and indexation and the red line is the model with trend inflation but without indexation.

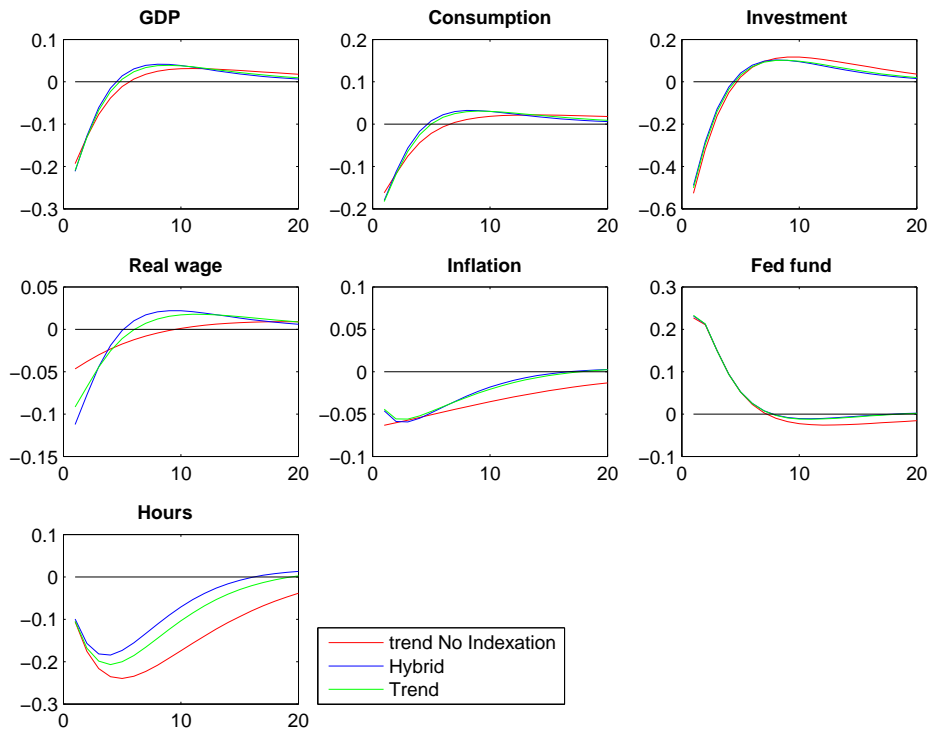


Figure 4: IRFs to a monetary policy shock for the full sample, 1966 - 2004.

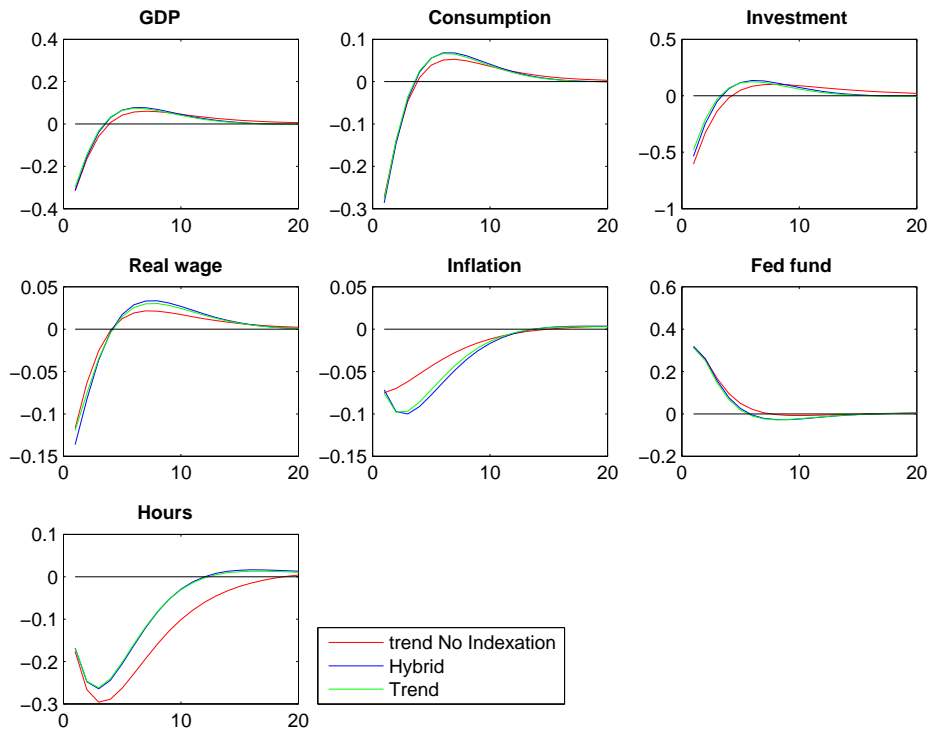


Figure 5: IRFs to a monetary policy shock for the Great Inflation period, 1966 - 1982.

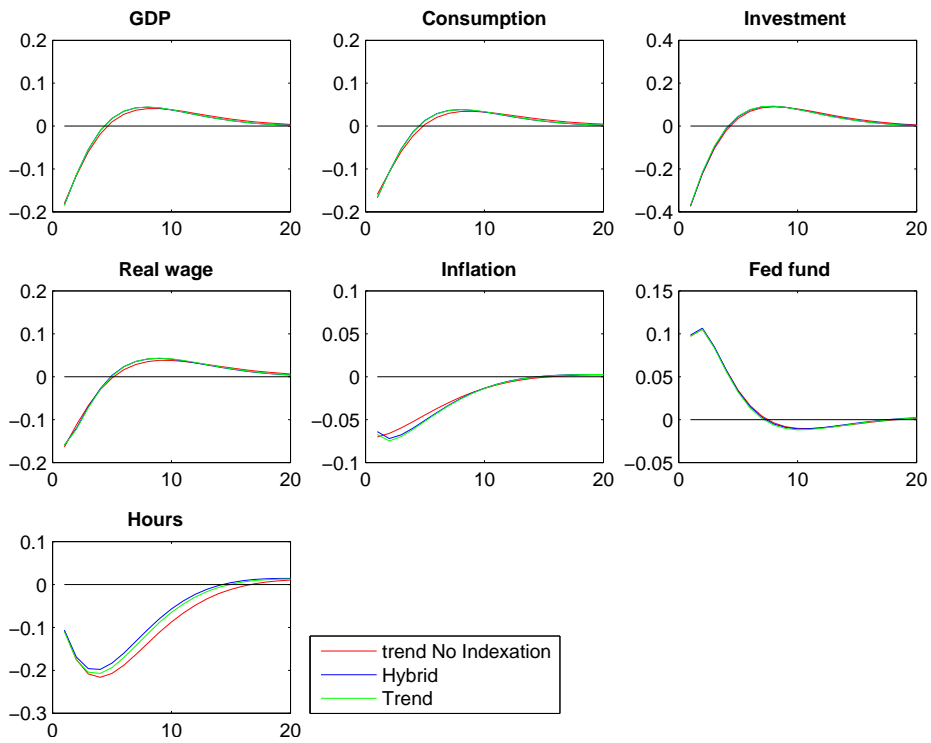


Figure 6: IRFs to a monetary policy shock for the Great Moderation period, 1983 - 2004.

Appendix D

Bayesian Structural VAR

We obtain the estimates and the IRFs for the structural VAR following the two steps procedure of Koop (1992). In the first step, following Koop and Korobilis (2009), we estimate the VAR reduced form via Bayesian technique with a natural conjugate prior, whereas in the second step we recover the structural form. Given the uncertainty about the right lag length, we consider all the possible lags from 1 to 4.

In the first step we follow Koop and Korobilis (2009), considering the multivariate version of the Wold decomposition theorem, which states that any covariance stationary $m \times 1$ vector time series, y_t , can be rewritten as a possibly infinitely ordered vector moving average:

$$y_t = \sum_{j=1}^{\infty} A_j y_{t-j} + \varepsilon_t$$

where y_t , for $t = 1, \dots, T$, is a (7×1) vector containing observations of the seven time series, ε_t is a (7×1) vector of errors and A_j is a (7×7) matrix of coefficients. We assume ε_t to be i.i.d. $N(0, \Sigma)$ and since there is uncertainty with respect to the appropriate lag length p of the VAR, we use a mixture of four vector autoregressions.

In order to estimate this VAR we use the Bayesian technique with the natural conjugate prior. We rewrite our VAR(p) as:

$$y_{mt} = z'_{mt} \beta_m + \varepsilon_{mt}$$

where $m = 1, \dots, 7$ variables. Stacking all equations into vectors/matrices, i.e. $y_t = (y_{1t}, \dots, y_{7t})'$, and defining

$$y = \begin{pmatrix} y_1 \\ \vdots \\ y_T \end{pmatrix}$$

we can rewrite:

$$y = Z\beta + \varepsilon$$

where ε is a $N(0, I \otimes \Sigma)$. As prior for this model we use the independent Normal-Wishart:

$$\begin{aligned} \beta &\sim N(\underline{\beta}, \Sigma \otimes \underline{V}) \\ \Sigma^{-1} &\sim W(\underline{S}^{-1}, \underline{\nu}) \end{aligned}$$

With this technique we obtain the estimations for \hat{A}_j , where $j = 1, \dots, p$, and $\hat{\Sigma}$. Since we are interested in the structural IRFs, in the second step we rewrite our VAR as:

$$y_t = \sum_{j=0}^p C_j e_{t-j}$$

where e_t is a structural error. The two representations are related by noting that $C_j = A_j C_0$, where $\Sigma = C_0 C_0'$. However, because Σ is a symmetric matrix, an estimation of Σ is not enough to obtain C_0 . In particular we identify the response to a negative monetary policy shock via sign restrictions: following Uhlig (2005), we impose for the first 4 observations a positive response of the interest rate and a negative reactions

for inflation, output and investment. In papers like Mountford and Uhlig (2009) the authors are agnostic on the response of output. Nevertheless, we decide to impose a negative restriction, considering the results obtained for the DSGE models.

In figure 7 there are the IRFs to a monetary shock for the 7 observed variables for a SVAR(4) on the full sample (1966-2004): the black line is the posterior mean, whereas the blue area denotes the 90% confidence interval.

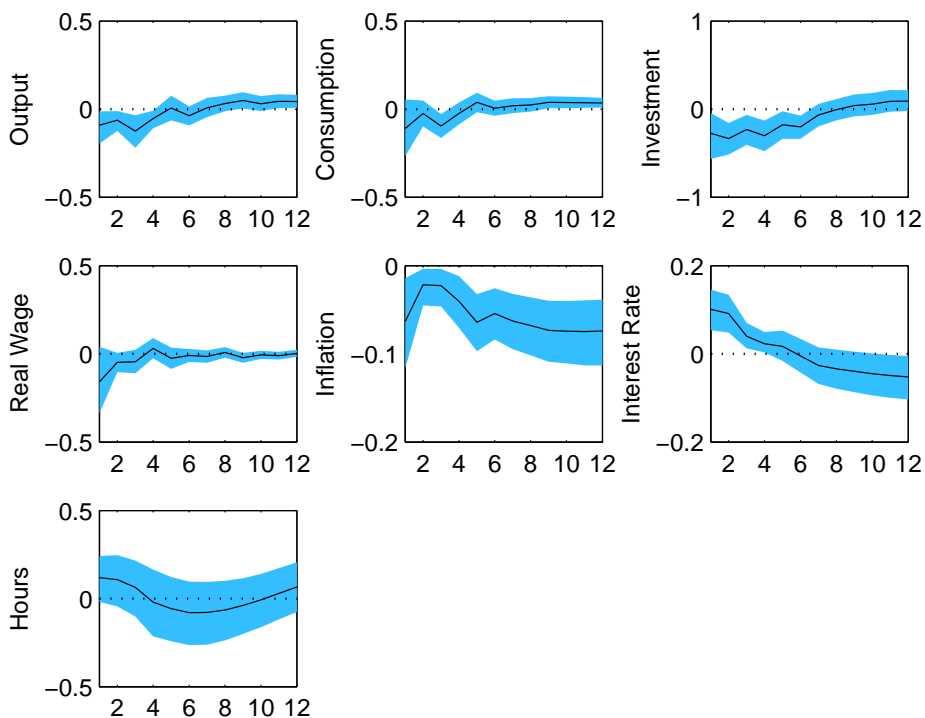


Figure 7: IRFs of SVAR(4) to monetary policy shock (right to left, upper to bottom): output, consumption, investment, real wage, inflation, interest rate, hours. Period: 1966 - 2004.

Geweke modified harmonic mean

The harmonic mean estimators are based on the identity

$$\frac{1}{p(Y)} = \int \frac{f(\theta)}{\mathcal{L}(\theta|Y)p(\theta)} p(\theta|Y) d\theta$$

where $f(\theta)$ has the property that $\int f(\theta) d\theta = 1$. Conditional on the choice of $f(\theta)$ an estimator is

$$\hat{p}(Y) = \left[\frac{1}{n_{sim} - n_{burn}} \sum_{s=s_{burn}+1}^{n_{sim}} \frac{f(\theta^{(s)})}{\mathcal{L}(\theta^s|Y)p(\theta^{(s)})} \right]^{-1}$$

where $\theta^{(s)}$ is drawn from the posterior $p(\theta|Y)$. To make the numerical approximation efficient, $f(\theta)$ should be chosen so that the summands are of equal magnitude. Geweke

(1999) proposed to use the density of a truncated multivariate normal distribution

$$f(\theta) = \tau^{-1}(2\pi)^{-d/2}|\bar{V}_\theta|^{-1/2} \exp\left\{-\frac{1}{2}(\theta - \bar{\theta})'\bar{V}_\theta^{-1}(\theta - \bar{\theta})\right\} \\ \times \mathbb{I}\left\{(\theta - \bar{\theta})'\bar{V}_\theta^{-1}(\theta - \bar{\theta}) \leq F_{\chi_d^2}^{-1}(\tau)\right\}$$

Here $\bar{\theta}$ and \bar{V} are the posterior mean and covariance matrix computed from the output of the posterior simulator, d is the dimension of the parameter vector, $F_{\chi_d^2}$ is the cumulative density function of a χ^2 random variable with d degrees of freedom, and $\tau \in (0, 1)$.

Appendix E

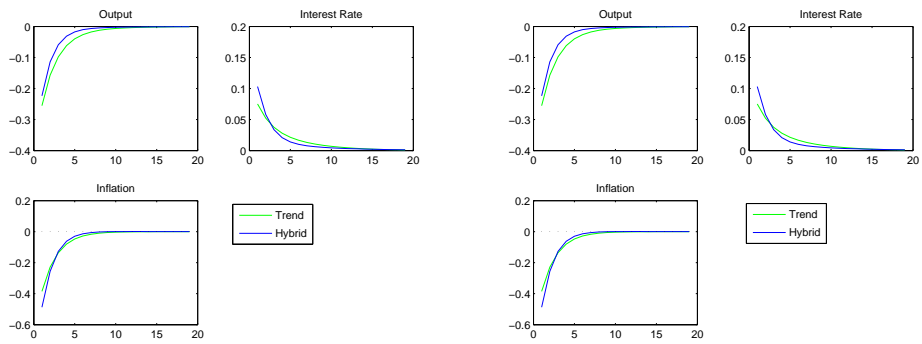
We consider the textbook model in Woodford (2003) with sticky prices a la Calvo and capital in a Cobb-Douglas production function. Time is discrete and continues forever. In the economy there are a continuum of infinitely-lived agents of three types: households, intermediate good producers and retailers. The three observed variables are output, interest rate and inflation. The estimations for the parameters of interest are in table 17, considering a model with hybrid NKPC and a model with generalized NKPC.

| | Hybrid | | | Trend | | |
|------------------|----------|---------|----------|------------|----------|----------|
| | 5 perc. | mean | 95 perc. | 5 perc. | mean | 95 perc. |
| 1966-2004 | | | | | | |
| ι^p | 0.037197 | 0.22497 | 0.4198 | 1.1321e-06 | 0.076736 | 0.021297 |
| ω_p | 0.65239 | 0.66871 | 0.68489 | 0.65026 | 0.67052 | 0.66593 |
| ψ_π | 1.9646 | 2.2081 | 2.478 | 1.9627 | 2.3014 | 2.2112 |
| ψ_y | 1.0953 | 1.2749 | 1.4749 | 1.1363 | 1.3766 | 1.3134 |
| ρ_r | 0.34882 | 0.49808 | 0.62686 | 0.28202 | 0.47115 | 0.4312 |
| 1966-1982 | | | | | | |
| ι^p | 0.019503 | 0.30928 | 0.64003 | 8.7759e-07 | 0.091131 | 0.30351 |
| ω_p | 0.64365 | 0.66002 | 0.67606 | 0.64419 | 0.66016 | 0.67595 |
| ψ_π | 1.9462 | 2.2507 | 2.595 | 2.0431 | 2.3791 | 2.7673 |
| ψ_y | 0.90492 | 1.1022 | 1.332 | 0.97579 | 1.1881 | 1.4335 |
| ρ_r | 0.24535 | 0.38716 | 0.5289 | 0.22691 | 0.36367 | 0.50367 |
| 1983-2004 | | | | | | |
| ι^p | 0.08974 | 0.28607 | 0.50601 | 0.044841 | 0.149 | 0.27379 |
| ω_p | 0.64247 | 0.65869 | 0.67475 | 0.63869 | 0.65481 | 0.67092 |
| ψ_π | 1.6953 | 1.8479 | 2.0094 | 1.5843 | 1.7535 | 1.938 |
| ψ_y | 0.90672 | 1.0593 | 1.225 | 0.76091 | 0.91933 | 1.0789 |
| ρ_r | 0.6372 | 0.70534 | 0.76231 | 0.64847 | 0.75013 | 0.81183 |

Table 17: Results for the Woodford textbook model with hybrid NKPC or generalized NKPC with indexation.

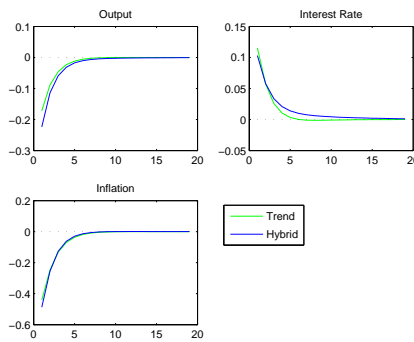
In figure 8 there are the IRFs to a negative monetary policy shock for the three observables. We found that short run dynamic is not affected by trend inflation if there is partial indexation. This result is at odds with Ascari and Ropele (2007). We suppose that this difference is due to two reasons: firstly, Ascari and Ropele obtain the IRFs keeping the calibrated parameters constant across different levels of trend inflation²¹. Secondly, Ascari and Ropele observed the main differences in IRFs when they compare the zero trend model with values for trend inflation equal to 8% or 10%. Nevertheless these levels are never observed as inflation mean on the subperiods we have studied.

²¹In particular Fernández-Villaverde and Rubio-Ramírez (2007), studying a time varying model, claim that the parameters $\{\iota^p, \iota^w, \omega_p, \omega_p\}$, that generate the nominal rigidity in the economy, vary over time and have a tenuous link with the microeconomic foundations. Moreover in their analysis these parameters display an high relationship with the time varying level of inflation. Therefore calibrated values could induce misleading conclusions.



(a)

(b)



(c)

Figure 8: IRFs to a negative monetary policy shock for the three periods: full sample 8(a), Great Inflation 8(b) and Great Moderation 8(c).

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