Securitization under Asymmetric Information over the Business Cycle

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This version: 04.12.2012

Abstract

Securitization was heavily criticized after the late 2000's financial crisis mainly due to several related agency problems which stem from the asymmetry of information about the quality of sold loans between the issuers of securitized products and their final buyers. However, these problems were not new. Securitization design in practice embedded tools that were supposed to alleviate these problems such as tranche retention schemes or implicit recourse. Despite these tools during the boom preceding the crisis many low quality loans were issued, securitized and sold to the investors, which contributed to the depth of the crisis. In this theoretical model I show that the mentioned tools in particular reputation based implicit recourse can be generally efficient, help to overcome the asymmetry of information and improve the allocation of investment. However, in boom stages of the business cycle this may not be possible. In this model boom stages are characterized by a pooling equilibrium where information remains private and the inefficiency in the investment is accumulating, which then deepens the subsequent downturn.

*I would like to thank for useful comments and suggestions to Sergey Slobodyan, Markus Brunnermeier, Byeongu Jeong, Nobu Kiyotaki, Filip Matějka, Olena Senyuta and participants at Princeton University Student Macroeconomics Workshop. All errors are mine. Correspondence: CERGE-EI, a joint workplace of the Center for Economic Research and Graduate Education, Charles University in Prague, and the Economics Institute of the ASCR, v. v. i. Address: CERGE-EI, P.O. Box 882, Politických vězňů 7, Prague 1, 111 21, Czech Republic

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1 Introduction

Securitization recently attracted a lot of criticism due to its role in the late 2000's financial crisis (e.g. Bernanke 2010). Securitization and in general a market-based system of financial intermediation significantly grew in importance in the decades preceding the crisis (Adrian and Shin 2009). The crisis led to intensified research into the problematic aspects of securitization. New research is often very critical about securitization such as Shleifer and Vishny (2010), who show in a simple one-shot model how banks using securitization and exploiting potential market sentiment of investors create systemic risk and inefficiencies in financial intermediation. Currently regulation of the financial sector on national as well as international level is being redrafted and strengthened. The agency problems related to securitization to which most of the criticism points are, however, not new and securitization design contained tools such as tranche retention schemes or implicit recourse that were supposed to limit these negative aspects of securitization. The question is whether these tools were efficient in the period before the 2000's financial crisis.

In this paper I show that in general reputation concerns allow sponsors of securitized products to credibly signal the quality of the loans by providing implicit recourse and thus limit the problem of private information. However, there are limits to the degree of commitment based on reputation and therefore also to the efficiency of implicit recourse in eliminating the problem of asymmetric information. Following the empirical evidence in Bloom (2009) and Bloom et al. (2011) who find that second moments of firms' TFP in the economy are countercyclical, in this model the relative difference in the projects' (loans') productivity is also countercyclical. As a result it turns out that even though in the steady state provision of implicit recourse helps to achieve a separating equilibrium where qualities of loans become public information, in boom stages of business cycle separation equilibrium would require too high implicit recourse which cannot be credibly promised. Rational investors know that sponsors of securitized products would default with high probability on high implicit recourse enforced only through reputation. Therefore in boom stages of business cycles there are pooling equilibria, in which the information about the quality of loans remains private and the allocation of investment becomes less efficient. This has only very moderate effects as long as the economy stays in the boom. But the accumulated inefficiency becomes more pronounced in the subsequent downturn of the economy, which is thus amplified. Also the longer is the boom, the larger are the inefficiencies in investment accumulated and the deeper will be the subsequent downturn.

The paper is organized in the following way. Chapter 2 reviews the related literature especially literature on securitization and implicit recourse and literature on imperfections in financial intermediation, information frictions and their implications for the dynamics over the business cycle. Chapter 3 introduces the set-up of the model and then separately deals with cases where implicit recourse is not available and when firms can and do provide implicit recourse in equilibrium. The chapter shows the basic behavior of the model, the effect of assumed financial frictions and the effect of implicit recourse. For analytical tractability this chapter focuses on steady state with only idiosyncratic stochasticity and where the aggregate variables turn out to be deterministic. The chapter 4 shows the results from the full-fledged model with aggregate stochasticity and focuses on the switching between the separating and pooling equilibria over the business cycle.

2 Literature review

My research is broadly related to several strands of research. In this chapter I would like to focus on research related to securitization with implicit recourse and to financial intermediation imperfections, information frictions and business cycles.

2.1 Securitization and implicit recourse

Securitization is the process of selling cash flows related to the loans issued by the originator (often called the sponsor). The sale of loans is effectuated in a legally separated entity called a special purpose vehicle (SPV) or special purpose entity (SPE). The entity purchases the right to the cash flows with resources obtained by issuing securities in the capital market. The sponsor and the SPV are "bankruptcy remote" and the sale of loans is officially considered to be complete, i.e., the sponsor should transfer all the risks to the investors of newly emitted securities. The pooled portfolio of loans is usually divided into several tranches ordered by seniority which have a different exposure to risk. Before the crisis securitization was perceived mainly as a means how to disperse credit risk and allocate it to less risk-averse investors who would be compensated by higher returns, while highly risk-averse investors could invest into the most senior tranches with high ratings. Due to the role of securitization played in the late 2000's financial crisis (e.g. Bernanke 2010) securitization attracted a lot of criticism and the attention of researchers turned more to the set of agency problems at different stages of the securitization process (Shin 2009). A detailed review of those agency conflicts has been done, for instance, by Paligorova (2009).

Gorton and Pennachi (1995) were among the first to point to moral hazard problems related to securitization and to address the issue why securitization takes place despite them. Moral hazard problems stem from the fact that if the risk is transferred with the loan from the originator of the loan to the investor, the bank has a reduced incentive to monitor borrowers to increase loan quality. Gorton and Pennachi (1995) argue that before the 1980s securitization was very limited. In the 1980s several regulatory changes took place that effectively increased the cost of deposit funding. One key factor was the imposition of a binding credit requirement for commercial banks.[1] Banks could avoid increased capital requirements by securitization, which moved some of the risky assets off their balance sheet. This view that an important reason for securitization is regulatory arbitrage is shared by many economists (e.g. Gorton and Pennachi 1995, Gertler and Kiyotaki 2010, Gorton and Metrick 2010). Calorimis and Mason (2004) present some evidence suggesting regulatory arbitrage is effectuated by securitizing banks rather to increase efficiency of contracting in the situation where capital requirements are unreasonably high than to abuse the safety net. The moral hazard problems and agency problems in general were then alleviated by the practice of keeping part of the loan in the portfolio on the balance sheet of the originator. Fender and Mitchel (2009) study different tranche retention design and their effect on incentives. But any loan sale, partial or complete, results in lower incentives to monitor borrowers, which of course affects the price investors are willing to pay for the securitized loan. Loan originators have thus incentive to provide implicit recourse.

Implicit recourse is a particular form of implicit guarantee on the quality of the loan. The guarantee cannot be explicit since then it would have to abide to regulations and the loan would have to be kept on the balance sheet of the bank. Nevertheless, much evidence suggests that implicit recourse was frequently used during the securitization process. ("As the saying goes, the only securitization without recourse is the last." (Mason and Rosner 2007, p. 38)) Gorton and Souleles (2006) show in a theoretical model that this mutually implicit collusion between investors and originators of the loans can be an equilibrium result in a repeated game due to the reputation concerns of

[1] "In 1981 regulators announced explicit capital requirements for the first time in U.S. banking history: all banks and bank holding companies were required to hold primary capital of at least 5.5 percent of assets by June 1985." (Gorton and Metrick 2010, p. 10)

the originator who wants to pursue securitization in the future at favorable conditions. Several empirical studies documented concrete cases of implicit recourse or showed indirect evidence of its presence. Higgins and Mason (2004) study 17 discrete recourse events that were directed to an increase in the quality of receivables sponsored by 10 different credit-card banks. The forms of the support provided were for instance adding higher quality accounts to the pool of receivables, removing lower quality accounts, increasing the discount on new receivables, increasing credit enhancement, waiving servicing fee, etc. Higgins and Mason (2004) argue that implicit recourse increases sponsors' stock prices in the short and long run following the recourse. It also improves their long-run operating performance. Recourse may help to signal investors that shocks that made recourse necessary are only transitory.

Another example showing that the risks were not fully transferred during securitization to the SPV is given by Brunnermeier (2009), who argues that when the SPV was subject to liquidity problems which arise from a maturity mismatch between SPV's assets and liabilities and a sudden reduced interest in the instruments emitted by the SPV, the sponsor would grant credit lines to it.

In my model I will concentrate on the relationship between investors and banks, where the latter have better information about the quality of loans, and I will show that due to reputation concerns bank has an incentive to signal this quality. This follows the suggestion by Higgins and Mason (2004) that implicit recourse is used to as a signaling tool.

The implications of securitization with tools similar to implicit recourse were recently studied in Ordognez (2012) who argues that unregulated banking disciplined only by reputation forces may be an efficient by saving on regulatory and bankruptcy costs but seems to be more fragile.

2.2 Financial intermediation imperfections, information frictions and business cycles

In the current financial crisis we could have witnessed important disruptions of financial intermediation. It became clear that frictions in the financial sector are important and should not be omitted from macroeconomic models. The classical papers that endogenize financial frictions on the side of borrowers include Bernanke and Gertler (1989), Bernanke, Gertler and Gilchrist (1999) and Kiyotaki and Moore (1997). These papers introduce an agency problem between borrowers and lenders. The resulting endogenous amplification of the effects of the shocks in the economy and is denoted as the "financial accelerator". From the onset of the crisis, a new generation of literature on financial frictions has been developing which introduces friction on the level of financial intermediates that were omitted in the above mentioned models. [2]

Some of the new macroeconomic models with financial frictions incorporate directly securitization. Brunnermeier and Sannikov (2011) find that securitization enables to share idiosyncratic risks but may be amplifying the systemic risk.

In this paper I will refer often to Kiyotaki and Moore (2012) model of monetary economy with differences in liquidity among different asset classes. Their model features borrowing and re-saleability constraints and stochastic uninsurable arrival of idiosyncratic investment shocks among the market participants. Some of the technical tricks such as logarithmic utility function and constant returns to scale on the individual firm level while decreasing returns at the aggregate level extremely simplify the aggregation across heterogeneous agents in the economy and allows for a relatively tractable treatment. Therefore similar framework is used in other papers such as Kurlat (2011) who studies the sale of projects under asymmetric information and shows how this could lead to the lemons problem and potential market shutdowns.

My model is also related to research about the degree of asymmetric information over the business cycle. While some researchers argue that booms are associate with higher degree of trading and therefore more learning (Veldkamp 2005), others argue that information may be lost in boom periods of business cycles. Gorton and Ordognez (2012) present a model where assets with unknown value can serve as a collateral for borrowing. In booms none of the parties has the incentive to verify the the value of the asset, the economy saves on information acquisition costs and enjoys a "bliss-full ignorance" equilibrium, while in periods with low aggregate productivity lenders have incentives to verify the value of collateral which leads to underinvestment. In my model higher productivity will be also associated with less public information but this would start to create inefficiencies.

3 Model

To allow for maximum tractability the set-up of the model is rather simple. The economy contains a continuum of financial firms which have stochastic investment opportu-

[2] Representatives of those models are e.g. Gertler and Kiyotaki (2010), Curdia and Woodford (2009a, 2009b), Cristiano, Motto and Rostagno (2009), Gilchrist, Ortiz and Zakresjek (2009).

nities. The problem in this model is to transfer resources from firms without investment opportunities or with low quality investment opportunities to firms with the best investment opportunities. The transfer of funds is possible through securitization which is modeled as a sale of cash flows from the funded projects. To keep the model simple I do not model alternative means of transferring funds like debt or equity. It might be, however, interesting to compare these alternative channels similarly as is done in Ordognez (2012).

3.1 Investment projects

There are three types of projects available to financial firms and the allocation of firms to projects is stochastic - i.i.d. shock:

- $(1-\pi)$ have access only to unproductive zero-profit projects,
- $\pi\mu$ firms have access to projects with high gross profit per unit of capital $r_t^h = A_t^h K_t^{\alpha-1}$,
- $\pi(1-\mu)$ firms have access to projects with low gross profit per unit of capital $r_t^l = A_t^l K_t^{\alpha-1}$,

where $A_t^h = \left(A_t + \Delta^h\right)$, $A_t^l = \left(A_t + \Delta^l\right)$, $\Delta^h > \Delta^l$ and $\log A_t = (1-\rho)\log \overline{A} + \rho \log A_{t-1} + u_t$. These shock cannot be insured.

Note that the aggregate component and type-specific component of TFP of projects are additive. This is a crucial assumption that ensures that relative difference in projects' productivity is countercyclical. This is inspired by the empirical results in Bloom (2009) and Bloom et al. (2011).[3]

Similarly as in Kiyotaki and Moore (2012) individual firms face constant returns to scale i.e. they take r_t^h resp. r_t^l as parametric but on the aggregate level there are decreasing returns to scale:

$$Y_t = r_t^h H_t + r_t^l L_t = \left(A_t^h \frac{H_t}{K_t} + A_t^l \frac{L_t}{K_t}\right) K_t^\alpha$$

where $K_t = H_t + L_t$ and H_t, L_t are aggregate holdings of high respectively low quality capital.

[3] Models in Bloom (2009) and Bloom et al. (2011) assume time-varying variance of idiosyncratic TFP shocks and show that higher variance leads to recession, which they also document empirically on the firm level data. I assume a weaker version. While second moments remain constant in my model, the relative difference in projects' productivity is countercyclical similarly as in the mentioned models.

Two core frictions are assumed in the model:

- Investing firms selling securitized loans have to keep a "skin in the game" - $(1-\theta)$ fraction of the investment. For simplicity θ is taken throughout most of the paper as a parameter. But a way how to endogenize it is developed too.
- There is an asymmetry of information about the above described allocation of investment opportunities among firms. Each firm knows the type of the project it is assigned to in the current period but it ignores the allocation of projects among other firms.

Each financial firm maximizes:

$$\max \sum_{s=0}^{\infty} \beta^s \log \left(c_{t+s}^j\right)$$

subject to the following borrowing constraints for firms with access to projects with high, low or zero gross profit per unit of invested capital denoted by superscripts $j = \{h,\ l,\ z\}$ respectively [4]:

$$c_t^h + i_t^h + \left(h_{t+1}^h - i_t^h\right) q_t^h + l_{t+1}^h q_t^l + z_{t+1}^h q_t^z = h_t(r_t^h + \lambda q_t^h) + l_t(r_t^l + \lambda q_t^l)$$

$$c_t^l + i_t^l + \left(l_{t+1}^l - i_t^l\right) q_t^l + h_{t+1}^l q_t^h + z_{t+1}^h q_t^z = h_t(r_t^h + \lambda q_t^h) + l_t(r_t^l + \lambda q_t^l)$$

$$c_t^z + i_t^z + \left(z_{t+1}^z - i_t^z\right) q_t^z + h_{t+1}^z q_t^h + l_{t+1}^z q_t^l = h_t(r_t^h + \lambda q_t^h) + l_t(r_t^l + \lambda q_t^l),$$

where $h,\ l,\ z$ are individual holdings of projects with high, low or zero gross profit per unit of capital, q^h, q^l, q^z are the respective prices, i^j is the investment into new projects of j quality and λ is the share of capital left after depreciation. Similarly as in Kiyotaki and Moore (2012) I assume that subjective discount factor exceeds the share of capital left after depreciation $\beta > \lambda$. This regularity assumption makes the model well-behaved. The maximization problem is also constrained by the "skin in the game" requirements:

[4] Note that these subscripts refer to individual firms of this type, while value of variables might differ within each group, the policy functions remain the same. The subscripts refer to firms' types in period t. So when they appear over the variables in period t+1 this belongs to firms which had this type of investment opportunities in previous period while the type period t+1 can be anything since allocation of types is stochastic i.i.d. each period.

$$h_{t+1}^{h} \geqq (1-\theta)\, i_t^h,\ l_{t+1}^{l} \geqq (1-\theta)\, i_t^l$$

where $r_t^h = \left(A_t + \Delta^h\right) K_t^{\alpha}$, $r_t^l = \left(A_t + \Delta^l\right) K_t^{\alpha}$.

Since utility is logarithmic and budget constraints are linear in individual holdings of assets, the policy functions will be also linear in individual holdings of assets. With logarithmic utility all firms will always consume a constant fraction of their current wealth (for derivation see the appendix).

$$c_t^j = (1-\beta)\left(h_t(r_t^h + \lambda q_t^h) + l_t(r_t^l + \lambda q_t^l)\right)\ \forall j \in \{h,\, l,\, z\}$$

Also there is a continuum of small firms and the arrival of investment opportunities is i.i.d.

Proposition 1. *Due to i.i.d. investment opportunities and linear policy functions, by applying the law of large numbers the aggregate variables and prices do not depend on the distribution of assets across firms.*

The result of Proposition 1 is convenient for achieving the solution, that is why it is popular in the literature. For details see the appendix.

The law of motion for capital is the following:

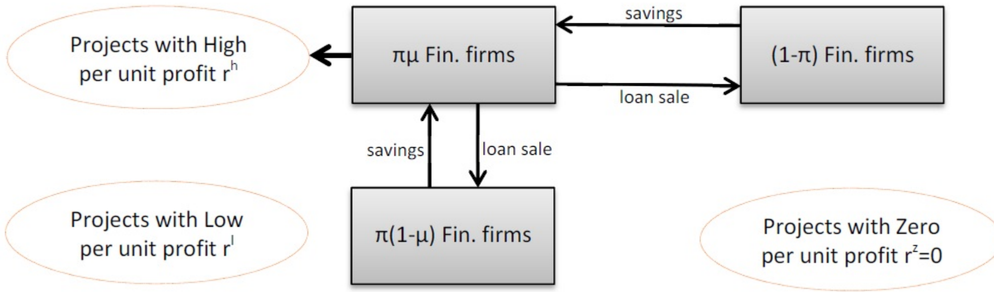
$$K_{t+1} = \lambda K_t + I_t$$

Similar holds for both types of capital (low quality and high quality): $H_{t+1} = \lambda H_t + I_t^h,\ L_{t+1} = \lambda L_t + I_t^l$. And of course both asset markets and goods markets have to clear $Y_t = C_t + I_t$.

3.2 Case with no financial frictions - first best

If none of the two frictions are present i.e. project allocation is public information and firms cannot divert funds necessary for investments to be productive, it is easy to show that only firms with high investment opportunities will invest, securitize the loans and sell them to firms with low or unproductive investment opportunities. Because of competition among firms with high investment opportunities, the price of loans will equal the unit costs of issuing the loan $q^h = 1$. The amount of investment and the allocation is first best.

Figure 3.1: First best case



In the first best case only firms with access to projects with high profit per unit of capital invest and they sell some of these projects to remaining firms.

In the steady state the individual policy functions will be as follows (see the appendix for derivation):

$$c = (1-\beta)\, h \left(r^h + \lambda\right)$$

$$h' = \beta h \left(r^h + \lambda\right).$$

The aggregate variables will be defined by:

$$C = (1-\beta)\, H \left(r^h + \lambda\right)$$

$$I = Y - C = Hr^h - C = (1-\lambda)\, H$$

From this one can derive that net return equals the time preference rate

$$r^h + \lambda = \frac{1}{\beta} \qquad (3.1)$$

i.e. the amount of investment is indeed first best.

3.3 Cases with frictions and without implicit recourse

3.3.1 Introducing the "skin in the game" constraint

From this section on I will consider that firms can divert funds required for loans to be made. They cannot divert all funds though, pretending that the loans were issued and making those castles-in-the-air projects temporarily similar to fully-funded loans requires $\psi < 1$ investment. To eliminate this problem investors can require the issuing firms to retain a sufficiently large skin in the game $(1-\theta)$ to incentivize them to invest fully into the issued loans. A skin in the game constraint is a usual practice observed in securitization contracts. One could also assume other types of incentives such as punishment of cutting the firms from trading completely. I assume it is possible to commit not to buy securitized products from a particular firm but it is not possible to prevent this firm to buy securitized products from others. Not allowing the firm to sell securitized products in an environment with zero profits from securitization is not an effective disciplining tool. It can be used only in case of positive profits as will be shown later.

Let me show how the skin in the game constraint could discipline the firms to invest fully. The incentive compatible constraint requires the return on investing fully to exceed the return on pretending to invest and diverting the investment funds[5].

$$R\left(diverting\,investment\,funds\right) < R\left(properly\,investing\right)$$

When one diverts the funds he uses all his wealth after consumption to invest only the necessary fraction to make the castle-in-the-air projects indistinguishable from real projects and sells the maximum possible of those. Properly investing requires to investing all the wealth after consumption into real projects and sell the maximum share of these. Incentive compatible constraints for firms with high or low investment opportunities $(j=\{h,l\})$ in the case without aggregate stochasticity looks as follows:

$$\frac{\theta\left(q^j-\psi\right)}{(1-\theta)\,\psi} < \frac{r^j+\lambda q^j}{(1-\theta q^j)}\,(1-\theta)$$

This in general would lead to a quadratic equation that points down the minimum level of skin in the game $1-\theta$. For sake for exposition lets consider the case where skin

[5]Note that the results below are derived for the case with no aggregate stochasticity and serve for purely demonstrative purposes. With aggregate stochasticity the firms would consider also the second moments of these returns. Numerical results for this fully-stochastic case are presented in Chapter 4.

in the game constraint is not binding enough and the price of high asset is still equal to one $q^h = 1$. Then the incentive compatible constraint becomes:

$$\theta < \frac{(r^j + \lambda)\,\psi}{(r^j + \lambda)\,\psi + 1 - \psi} \tag{3.2}$$

For firms with nonproductive investment opportunities the incentive compatible constraint is similar, only one compares the situation where they pretend to invest with buying projects issued elsewhere.

$$\frac{\theta\left(q^h - \psi\right)}{(1-\theta)\,\psi} < \frac{r^h + \lambda q^h}{q^h}$$

which simplifies in the case $q^h = 1$ also to (3.2) of high type.

For sufficiently low ψ, skin in the game $1 - \theta$ will be large enough such that $q^h > 1$ and investing firms will be making profit as will be discussed later. Also note that $\partial\theta/\partial r > 0$, which means that in boom stages of business cycles the skin in the game constraint will be less severe.

In the remaining of the exposition of the model I will assume for simplicity a constant skin in the game in order to show some results analytically. Endogenizing the skin in the game does not change the main points of the paper, only make the behavior more realistic. I will come back to this point at the section on the dynamics of the model.

If the costs of building the castles in the air ψ are low enough the skin in the game is binding in the equilibrium. This means that we can rewrite the budget constraint for the investing firm subject to a binding “skin in the game" constraint (e.g. in the case of firms with high investment opportunities):

$$c_t^h + \frac{\left(1 - \theta \hat{q_t^h}\right)}{(1-\theta)} h_{t+1}^h = h_t(r_t^h + \lambda q_t^h) + l_t(r_t^l + \lambda q_t^l),$$

where market price for securitized loans$\hat{q_t^h}$ depends on the information sets of market participants (see below).

Under binding "skin in the game” constraint the prices q_t^h and q_t^l exceed the cost of investment which are equal to 1. Therefore the investing firms are making profits from securitization (sale of projects to firms without investment opportunities) despite the competition among them. Note that for prices exceeding the unit costs of investment the term $\left(1 - \theta q_t^j\right) / (1 - \theta) < 1$ therefore investing firms can acquire assets at more advantageous conditions and therefore they profit from securitization.

The problem can have the following recursive formulation:

$$V(l,h;K,\omega,A)=\pi\left(\mu V^h(l,h;K,\omega,A)+(1-\mu)V^l(l,h;K,\omega,A)\right)+(1-\pi)V^s(l,h;K,\omega,A),$$

where for $j=\{h,l,z\}$:

$$V^j(l,h;K,\omega,A)=\max_{c,i,h',l'}\left[\log(c)+\beta EV(l',h';K,\omega,A)\right]$$

subject to the respective borrowing constraint stipulated above.

Definition 1. A recursive competitive equilibrium consists of prices $\{q^h(\bar{S}),\ q^l(\bar{S}),$ $q(\bar{S})\}$ and gross profit per unit of capital $\left\{r(\bar{S})\right\}$, individual decision rules $\{c^j(\bar{s};\bar{S}), h^{j\prime}(\bar{s};\bar{S}),$ $l^{j\prime}(\bar{s};\bar{S}),\ z^{j\prime}(\bar{s};\bar{S})\}$ value functions $\{V(\bar{s};\bar{S}), V^s(\bar{s};\bar{S}), V^h(\bar{s};\bar{S}), V^l(\bar{s};\bar{S})\}$ and law of motion for $\bar{S}=\{K,\omega,A,\Sigma\}$ such that: (i) $\{c^j(\bar{s};\bar{S}), h^{j\prime}(\bar{s};\bar{S}), l^{j\prime}(\bar{s};\bar{S}), z^{j\prime}(\bar{s};\bar{S})\}$ and $\{V(\bar{s};\bar{S}), V^s(\bar{s};\bar{S}), V^h(\bar{s};\bar{S}), V^l(\bar{s};\bar{S})\}$ solve the each firms' problem given the available information set and taking $\{q^h(\bar{S}),\ q^l(\bar{S}),\ q(\bar{S})\}$, $\left\{r(\bar{S})\right\}$ and law of motion for $\bar{S}=\{K,\omega,A\}$ as given; (ii) high quality and low quality asset markets and good markets clear and (iii) the law of motion for $\bar{S}=\{K,\omega,A\}$ is consistent with the individual firms' decisions.

Note that $\bar{S}$ is the set of aggregate state variables and $\bar{s}=\{h,l\}$ is the set of individual firm state variables. Σ represents the allocation of investment opportunities across firms.

3.3.2 Public information case without implicit recourse

In equilibrium investment will generally be allocated to both high and low investment projects but in the public information case where the allocation of projects to firms is observable share of high quality projects will exceed the rate of arrival of these projects to financial firms (μ).

Compared to the first-best case where the "skin in the game" constraint is not binding the allocation is inefficient and the aggregate level of output and capital will be lower in the steady state.

Under binding "skin in the game" constraint the aggregate investment into respective types of assets will be:

$$I_t^H = \pi\mu \frac{\beta \left(H_t \left(\left(A_t + \Delta^h \right) K_t^{\alpha-1} + \lambda q_t^h \right) + L_t \left(\left(A_t + \Delta^l \right) K_t^{\alpha-1} + \lambda q_t^l \right) \right)}{\left(1 - \theta q_t^h \right)} \tag{3.3}$$

$$I_t^L = \pi(1-\mu) \frac{\beta \left(H_t \left(\left(A_t + \Delta^h \right) K_t^{\alpha-1} + \lambda q_t^h \right) + L_t \left(\left(A_t + \Delta^l \right) K_t^{\alpha-1} + \lambda q_t^l \right) \right)}{\left(1 - \theta q_t^l \right)}. \tag{3.4}$$

Prices of particular assets will be determined from Euler equations of saving firms that make them in equilibrium marginally indifferent between investing in high or low projects:

$$E_t \left[\frac{\frac{r_{t+1}^h + \lambda q_{t+1}^h}{q_t^h}}{\left(\omega_{t+1} \frac{r_{t+1}^h + \lambda q_{t+1}^h}{q_t^h} + (1 - \omega_{t+1}) \frac{r_{t+1}^l + \lambda q_{t+1}^l}{q_t^l} \right)} \right] = 1 \tag{3.5}$$

$$E_t \left[\frac{\frac{r_{t+1}^l + \lambda q_{t+1}^l}{q_t^l}}{\left(\omega_{t+1} \frac{r_{t+1}^h + \lambda q_{t+1}^h}{q_t^h} + (1 - \omega_{t+1}) \frac{r_{t+1}^l + \lambda q_{t+1}^l}{q_t^l} \right)} \right] = 1. \tag{3.6}$$

The derivation of these conditions can be found in the appendix.

Finally goods market clearing condition has to hold too:

$$Y_t = C_t + I_t. \tag{3.7}$$

For simplicity and tractability I will show some qualitative properties of the model using the steady state conditions where the aggregate productivity is constant $A_t = A$. The way how to solve fully stochastic steady state is described in the appendix, but the results are only numerical. The numerical results of the fully stochastic model are presented at the end of this paper.

Proposition 2. *If skin in the game is sufficiently large i.e. θ is sufficiently low to satisfy*

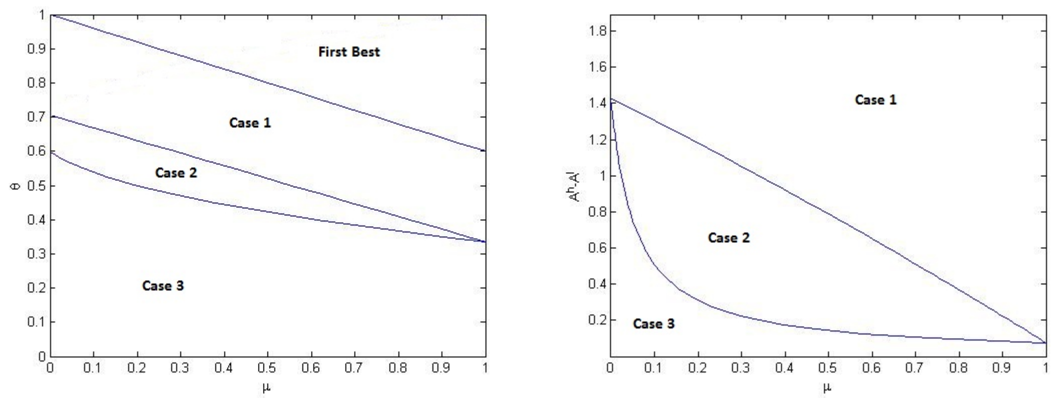
$$(1 - \lambda)(1 - \pi\mu) > (\pi\mu\lambda + (1 - \lambda)\theta),$$

then in the deterministic steady state:

(i) the price of high quality assets (Tobin's q of high type) q^h exceeds 1;

(ii) the steady state level of output and capital is lower then in the first best case.

Figure 3.2: Type of deterministic steady state depending on selected parameter values



The above proposition does not suffice for the characterization of the model's steady state.

Proposition 3. *Suppose that the condition from Proposition 1 holds, then depending on parameter values deterministic steady state is characterized by one of the following case:*

Case 1: only firm with access to high quality loans issues credit and securitizes ($q^l < 1$);

Case 2: firms with access to low quality loans use mix strategy and issue credit with probability φ, ($q^l = 1$);

Case 3: all firms with access to high and low quality loans issue credit and securitize ($q^l > 1$).

The cases are ranked from the lest restricted ($q^l < 1$), where output and capital levels are relatively the closest to first best case, to the most restricted ($q^l > 1$), where output is the lowest:

$$K_H > K_M > K_B,$$

where subscript H denotes Case 1 with only high projects financed, subscript M denotes Case 2 with mixed strategy of firms with access to low quality investment and subscript B denotes Case 3 where both firms with access to low and high quality projects issue credit to the limit of the skin in the game.

Proofs of the above propositions are in the appendix.

3.3.3 Private information case without implicit recourse

Let's consider more interesting case where the allocation of projects among firms is private information. Since the values of Δ^h, Δ^l, A_t are public information the uncertainty about the quality of a particular newly financed project is resolved in the next period. I assume that past projects are not anonymous therefore the quality of all existing projects becomes public information in the period following their securitization. Of course one might consider anonymous assets or more persistent uncertainty and in the first case there might be an interesting lemons market problem on the secondary market similarly as in Kurlat (2011). This is, however not the focus of this paper.

Since there is no way to distinguish between the projects firms would have to invest in both high and low projects. To maximally diversify the portfolio the rate of investment into different types of projects would be equal to the probability of arrival of these investment opportunities i.e. μ fraction of investment is allocated into high quality projects and $1-\mu$ fraction to low quality projects.

Even when skin in the game constraint is not binding enough to influence aggregate quantities and prices the capital and output levels are lower then in the first best case due to inefficient allocation of capital. When the skin in the game constraint is not binding,

$$\bar{r} = \mu r^h + (1-\mu) r^l = \frac{1}{\beta} - \lambda.$$

The level of capital K_P is determined by:

$$K_P = \left[\frac{1}{\mu A^h + (1-\mu) A^l}\left(\frac{1}{\beta} - \lambda\right)\right]^{\frac{1}{\alpha-1}} < \left[\frac{1}{A^h}\left(\frac{1}{\beta} - \lambda\right)\right]^{\frac{1}{\alpha-1}} = K_{FB}.$$

Suppose $(1-\pi)(1-\lambda) > \pi\lambda + (1-\lambda)\theta$ then the skin in the game starts to bind in this case of private information. The deterministic steady state conditions then collapse to the two following equations in (K, q):

$$(1-\lambda)(1-\theta q) = \pi\beta\left(\mu r^h + (1-\mu) r^l + \lambda q\right)$$

$$\mu r^h + (1-\mu) r^l = (1-\lambda) + (1-\beta)\left(\mu r^h + (1-\mu) r^l + \lambda q\right)$$

where $q = \mu q^h + (1-\mu) q^l$ from this we can easily derive:

$$q = \frac{(1-\pi)(1-\lambda)}{\pi\lambda + (1-\lambda)\theta}$$

$$K = \left[\frac{(1-\lambda) + (1-\beta)\lambda q}{\beta\left(\mu A^h + (1-\mu)A^l\right)}\right]^{\frac{1}{\alpha-1}}.$$

Note that $\omega = {}^{H}/_{K} = \mu$ and that here we can see conditions for the skin in the game constraint to be binding.

Proposition 4. *Compared to public information case the allocation of capital is less efficient, therefore the capital is less productive and in the steady state the amount of capital and output is lower.*

For proof see the appendix.

3.4 Implicit recourse and reputation equilibrium case

3.4.1 Introducing implicit recourse

The outcome of private information case is clearly inefficient compared to public information case. Firms with high quality investment opportunities have incentives to distinguish themselves from low quality investment firms and one way to do it while not restricting their investment potential is by providing **implicit recourse**. Under this strategy they will promise minimum gross profit per unit of capital r_t^G to the investors and should the true gross profits in the following period fall below this one, the issuing bank would reimburse the difference. This promise is not enforced by any explicit contract, rather it is a result of a collusion between issuers of loans and the buyers of projects. It can be enforced in a reputation equilibrium where securitizing firms try to keep reputation of sticking to the promise and firms buying securitized projects with implicit recourse keep reputation of punishing firms that did not full-fill the promise.

The problem of firms can be then written recursively:

$$V^{ND}\left(\bar{s}, w - cir; \bar{S}\right) = \pi\left(\mu V^{ND,h}\left(\bar{s}, w - cir; \bar{S}\right) + (1-\mu)V^{ND,l}\left(\bar{s}, w - cir; \bar{S}\right)\right) + (1-\pi)V^{ND,z}\left(\bar{s}, w - cir; \bar{S}\right)$$

$$V^{D}\left(\bar{s}, w; \bar{S}\right) = \pi\left(\mu V^{D,h}\left(\bar{s}, w; \bar{S}\right) + (1-\mu)V^{D,l}\left(\bar{s}, w; \bar{S}\right)\right) + (1-\pi)V^{D,z}\left(\bar{s}, w; \bar{S}\right)$$

$$V^{ND,j}\left(\bar{s},w;\bar{S}\right)=\max_{c,i,h',l',r\{G\}'}\left[\log\left(c\right)+\beta E\left[\max\left(V^{ND}\left(\bar{s}',w'-cir';\bar{S}'\right),V^{D}\left(\bar{s}',w';\bar{S}'\right)\right)\right]\right]$$

$$V^{D,j}\left(\bar{s},w;\bar{S}\right)=\max_{c,i,h',l'}\left[\log\left(c\right)+\beta EV^{D}\left(\bar{s}',w';\bar{S}'\right)\right]$$

subject to the budget constraints which take the following form for investing firms for which "skin in the game" constraint is binding (e.g. in case of firms with high investment opportunities):

$$c_t^h+\frac{\left(1-\theta q_t^{\hat{G},h}\right)}{(1-\theta)}h_{t+1}^h+cir_t=h_t^S(r_t^h+\lambda q_t^h)+l_t^S(r_t^l+\lambda q_t^l)+h_t^P(r_t^{\hat{G},h}+\lambda q_t^h)+l_t^P(r_t^{\hat{G},l}+\lambda q_t^l).$$

The incentive compatible constraints for non-defaulting are the following:

$$V^{ND}\left(\bar{s},w-cir;\bar{S}\right)>V^{D}\left(\bar{s},w;\bar{S}\right) \tag{3.8}$$

$$V^{P}\left(\bar{s};\bar{S}\right)>V^{NP}\left(\bar{s};\bar{S}\right), \tag{3.9}$$

where $V^{ND}, V^{D}, V^{P}, V^{NP}$ are the value functions if firm, never defaulted, when firm defaulted, when firm always punished a default on a promise on gross profits and when firm failed to punished respectively. w is individual wealth level before deducting cir, which are costs of providing implicit recourse, $\bar{s}=\{h,l,h^p,l^p\}$ is a vector of other individual state variables, where P, S superscripts denote assets sold in the previous period on the primary market which potentially bear implicit guarantee or on the secondary market respectively, $\bar{S}=\{K,\omega,A\}$ is a vector of aggregate state variables, $r_t^{\hat{G},h}$ is the return received from securitized assets with implicit recourse conditional on potential default and $q_t^{\hat{G},j}$ is the price of securitized loans of type j depending on the information structure. One can imagine different punishment rules such as punishment which says that after a default on implicit recourse the financial firm would not be able sell loans in several following periods. Here I assume that punishment has a permanent effect which also gives highest possible credibility to implicit recourse.

Definition 2. A recursive competitive equilibrium consists of prices $\{q^h\left(\bar{S}\right),\ q^l\left(\bar{S}\right),\ q^{G,h}\left(\bar{S}\right),\ q^{G,l}\left(\bar{S}\right),\ q^{G}\left(\bar{S}\right)\}$ and gross profit per unit of capital $\left\{r\left(\bar{S}\right)\right\}$, individual decision rules $\{c^j\left(\bar{s};\bar{S}\right),\ h^{j\prime}\left(\bar{s};\bar{S}\right),\ l^{j\prime}\left(\bar{s};\bar{S}\right),\ z^{j\prime}\left(\bar{s};\bar{S}\right),\ r_t^{G,h}\left(\bar{s};\bar{S}\right),\ r_t^{G,l}\left(\bar{s};\bar{S}\right)\}$, value functions $\{V^{ND}\left(\bar{s};\bar{S}\right)$, $V^{ND,s}\left(\bar{s};\bar{S}\right)$, $V^{ND,h}\left(\bar{s};\bar{S}\right)$, $V^{ND,l}\left(\bar{s};\bar{S}\right)$, $V^{D}\left(\bar{s};\bar{S}\right)$, $V^{D,s}\left(\bar{s};\bar{S}\right)$, $V^{D,h}\left(\bar{s};\bar{S}\right)$, $V\left(\bar{s};\bar{S}\right)\}$ and law of motion for $\bar{S}=\{K,\omega,A,\Sigma\}$ such that: (i) $\{c^{j}\left(\bar{s};\bar{S}\right)$, $h^{j\prime}\left(\bar{s};\bar{S}\right)$, $l^{j\prime}\left(\bar{s};\bar{S}\right)$, $z^{j\prime}\left(\bar{s};\bar{S}\right)$, $r_t^{G,h}\left(\bar{s};\bar{S}\right)$, $r_t^{G,l}\left(\bar{s};\bar{S}\right)\}$ and $\{V^{ND}\left(\bar{s};\bar{S}\right)$, $V^{ND,s}\left(\bar{s};\bar{S}\right)$, $V^{ND,h}\left(\bar{s};\bar{S}\right)$, $V^{ND,l}\left(\bar{s};\bar{S}\right)$, $V^{D}\left(\bar{s};\bar{S}\right)$, $V^{D,s}\left(\bar{s};\bar{S}\right)$, $V^{D,h}\left(\bar{s};\bar{S}\right)$, $V\left(\bar{s};\bar{S}\right)\}$ solve the each firms' problem given the available information set and taking $\{q^{h}\left(\bar{S}\right)$, $q^{l}\left(\bar{S}\right)$, $q^{G,h}\left(\bar{S}\right)$, $q^{G,l}\left(\bar{S}\right)$, $q^{G}\left(\bar{S}\right)\}$, $\left\{r\left(\bar{S}\right)\right\}$ and law of motion for $\bar{S}=\{K,\omega,A\}$ as given; (ii) both primary and secondary markets for high quality and low quality assets and good markets clear and (iii) the law of motion for $\bar{S}=\{K,\omega,A\}$ is consistent with the individual firms' decisions.

Note that $\bar{S}$ is the set of aggregate state variables and $\bar{s}=\left\{h^{S},l^{S},h^{P},l^{P}\right\}$ is the set of individual firm state variables. Σ represents the allocation of investment opportunities across firms.

Since the asymmetry of information concerns the quality of the loan and not the aggregate productivity guaranteeing the aggregate productivity where issuers of both types of projects have the same advantage is not efficient if investor in high quality projects wants to distinguish himself from the low type. The highest chance to avoid mimicking by low type is to guarantee the relative performance of the loan which maximizes the difference in costs of implicit recourse provision between the two types. This is especially important since there is a limit to how much an issuer of the loan can promise. Also buyers of securitized loan want to limit the defaulting on implicit recourse since then they do not get any benefit except maybe the information about project allocations. From this perspective it is also more efficient to condition the guarantee on the aggregate TFP (A_{t+1}):

$$r_{t+1}^{G}\left(A_{t+1}\right)=\left(A_{t+1}+\triangle_t^{G}\right)K_{t+1}^{\alpha-1}$$

The costs of implicit recourse are then given by:

$$cir_{t+1}=\theta i_t K_{t+1}^{\alpha-1}\left(\Delta_t^{G}-\Delta_t^{h/l}\right)$$

The promise of guaranteed gross profits is only on the primary market by which I mean the sale of loans by their issuer not further resale and trading with the securitized loans. The assets are not anonymous therefore once a particular project is sold with informative implicit recourse i.e. the type becomes public information, it remains public information in the future. Recall that the allocation of investment opportunities to firms is changing every period but once the project is funded it keeps its type forever. Funded projects only depreciate over time. The uncertainty about project quality lasts only for one period in this set-up even if implicit recourse is not provided. For simplicity and tractability I will also restrict the guarantee to the performance of the loans to one period after the issuance.

Assumption: To simplify the solution even further I assume that firms which default and loose reputation can liquidate the firm and start a new one with no previous record of defaulting but the liquidation and transfer is costly. In particular the firm can transfer only $(1-\phi)$ fraction of assets into the new firm. I guess and verify that such option will be used in equilibrium

$$V^{ND}\left(\bar{s},(1-\phi)w;\bar{S}\right) > V^{D}\left(\bar{s},w;\bar{S}\right). \tag{3.10}$$

Using this simplification the firm, when supposed to fulfill the implicit recourse promise, compares the cost of fulfilling the implicit recourse with costs of loosing a fraction of wealth. Then the incentive compatible constraint for non-defaulting simplifies to

$$V^{ND}\left(\bar{s},w-cir;\bar{S}\right) > V^{ND}\left(\bar{s},(1-\phi)w;\bar{S}\right).$$

$$\phi\left(\left(1-\theta\right)i_t\left(r^i_{t+1}+\lambda q^i_{t+1}\right)\right) > cir_{t+1} = \theta i_t K^{\alpha-1}_{t+1}\left(\Delta^G_t-\Delta^i_t\right) \tag{3.11}$$

This defines and upper bound on the Δ^G_t that can be credibly promised $\left(\Delta^{G,cred}_t\right)$.

3.4.2 Public information case with implicit recourse

Although one might think that public information case is uninteresting, it is an important benchmark. The fact that there is competition among the issuing firms means that implicit recourse will be provided by profit maximizing issuers even in this case, where it does not serve as a tool that would distinguish the firm type. If the firms could coordinate they wouldn't be providing implicit recourse in this case since it only lowers their profit. Due to competition firms tend to out-bet each other. If the promises would be credible they would end up promising such a level that they would not make any extra profit from securitization.

The optimal level of implicit recourse when not constrained by credibility will be determined by the following F.O.C. (note that individual firm ignores the effects of this choice on aggregate variables):

$$\frac{\partial V^{ND}}{\partial \Delta^{G,j}} = \frac{\partial V^{ND'}}{\partial \left(w' - cir'\right)} \frac{\partial \left(w' - cir'\right)}{\partial \Delta^{G,j}} = 0$$

after substituting in this case with constant aggregate productivity $q^{j,IR} = \frac{\left(A+\Delta^G\right)K^{\alpha-1}+\lambda q^j}{\left(A+\Delta^j\right)K^{\alpha-1}+\lambda q^j} q^j$ this condition implies that

$$q^j = 1,$$

which means that as far as there are positive profits from securitization, the competition will drive the level of implicit recourse high to the level where extra profits from securitization are zero. For details on the derivation see the appendix. However, when extra profits from securitization are zero, the punishment has zero costs and the original non-defaulting incentive compatible constraint would not hold (3.8). Therefore the maximum credible level of promise is equal the type quality $\Delta^{Gcred,j} = \Delta^j$.

If the condition (3.10) holds, then the maximum level of promise possible is determined by the simplified non-defaulting incentive compatible constraint (3.11) which becomes

$$\Delta^{Gcred,j} = \Delta^j + \frac{\phi\left(1-\theta\right)}{\theta K^{\alpha-1}} \left(r^j + \lambda q^j\right). \tag{3.12}$$

Proposition 5. *Suppose that the condition from Proposition 1 holds, then depending on parameter values deterministic steady state is characterized by one of the following cases:*

Case 1: only firm with access to high quality loans issues credit, securitizes loans and provides implicit recourse $\Delta^{Gcred,h}$ *(*$q^h > 1,\ q^l < 1, \Delta^{Gcred,h} \geq \Delta^h$*);*

Case 2: firm with access to high quality loans issues credit, securitizes loans and provides implicit recourse $\Delta^{Gcred,h}$ *, firms with access to low quality loans use a mix strategy and issue credit with probability* φ *and provide implicit recourse equal to the type quality (*$q^h > 1,\ q^l = 1,\ \Delta^{Gcred,h} \geq \Delta^h,\ \Delta^{Gcred,l} = \Delta^l$*);*

Case 3: all firms with access to high and low quality loans issue credit, securitize and provide implicit recourse ($q^h > 1,\ q^l > 1,\ \Delta^{Gcred,h} \geq \Delta^h,\ \Delta^{Gcred,l} \geq \Delta^l$*).*

As I will discuss later the model in the next chapter is calibrated such that the steady state will be characterized by Case 1.

Proposition 6. *Compared to the public information case without implicit recourse the* *amount of capital and output is higher in the case with implicit recourse, the allocation of capital is in favor of high quality projects and the wealth is less distributed towards the firms with investment opportunities. This holds in all cases except when the provided implicit recourse has no value (*$\Delta^{Gcred,h} = \Delta^h$*) and the two cases are identical.*

3.4.3 Private information case with implicit recourse

This is the most interesting case where providing implicit recourse can signal the type of the the investment opportunity. Due to signaling there is a multiplicity of Perfect Bayesian Equilibria generally both pooling and separating. I will use the intuitive criterion (Cho and Kreps 1987) as a refinement to eliminate the dominated equilibria with unreasonable out of equilibrium beliefs.

Pooling Equilibria: In pooling equilibria both firms will choose to promise the same level of implicit recourse given beliefs of investors. Under no aggregate stochasticity there are several candidates for the pooling Perfect Bayesian Equilibria (PBE):

Case 1: Both firms will select $\Delta^G = \Delta^{Gcred,l}$. Investors believe that when observing implicit recourse $\Delta^G > \Delta^{Gcred,l}$ then $Pr(j = h) = 0$ and when observing $\Delta^G < \Delta^{Gcred,l}$ then $Pr(j = h) \leq \mu$. In this equilibrium none of the firms defaults. None of the firms has incentive to unilaterally decrease implicit recourse or increase it.

Case 2: Both firms select $\min\left(\Delta^{Gminsep}, \Delta^{Gub,h}, \Delta^{Gcred,h}\right) \geq \Delta^G \geq \Delta^{Glb,h}$. Investors believe that when observing implicit recourse above Δ^G then $Pr(j = h) = 0$ and when observing implicit recourse belowΔ^G then $Pr(j = h) \leq \mu$.

$\Delta^{Gminsep}$ is the minimum level of implicit recourse which the low types would not mimic under any beliefs (see definition later). Note that choosing $\Delta^G < \Delta^{Gcred,l}$ is not an equilibrium since both types will have incentives to increase implicit recourse to $\Delta^G = \Delta^{Gcred,l}$ due to competition no matter what are the beliefs of investors since both types would fulfill the implicit recourse promise. Similarly choosing $\Delta^{Gcred,l} < \Delta^G < \Delta^{Glb,h}$ is not an equilibrium since both types will have incentives to decrease implicit recourse to $\Delta^G = \Delta^{Gcred,l}$. This is due to the fact that in the mentioned interval the fact that firms with low investment opportunities default on implicit recourse which bring investors lower utility then when $\Delta^G = \Delta^{Gcred,l}$. And this negative effect on price is larger together with potentially higher costs of higher implicit recourse (when $\Delta^G > \Delta^h$) outweigh the effect of higher implicit recourse on the price of sold assets. Note that for some parameter values it is possible that $\Delta^{Gcred,h} < \Delta^{Glb,h}$, then there is only one pooling PBE (Case 1).

Consistent with the description above let me define $\Delta^{Glb,h}$:

$$R^h\left(\Delta^{Glb,h}\right) = R^h\left(\Delta^{Gcred,l}\right),$$

where $\Delta^{Glb,h} > \Delta^{Gcred,l}$. When $\Delta^{Glb,h} < \Delta^h$ this reduces to:

$$\frac{\left(\left(A+\Delta^h\right)K^{\alpha-1}+\lambda q^h\right)(1-\theta)}{1-\theta\frac{\left(A+\mu\Delta^{Glb,h}+(1-\mu)\Delta^l\right)K^{\alpha-1}+\lambda q^h}{\left(A+\Delta^h\right)K^{\alpha-1}+\lambda q^h}q^h} > \frac{\left(\left(A+\Delta^h\right)K^{\alpha-1}+\lambda q^h\right)(1-\theta)}{1-\theta\frac{\left(A+\Delta^{Gcred,l}\right)K^{\alpha-1}+\lambda q^h}{\left(A+\Delta^h\right)K^{\alpha-1}+\lambda q^h}q^h}$$

$$\mu\Delta^{Glb,h} + (1-\mu)\Delta^l = \Delta^{Gcred,l},$$

which when using the equation (3.12) becomes:

$$\Delta^{Glb,h} = \Delta^l + \frac{\phi(1-\theta)}{\mu\theta K^{\alpha-1}}\left(r^h+\lambda q^h\right).$$

When $\Delta^{Glb,h} > \Delta^h$ this expression is more complicated.

$\Delta^{Gub,h}$ is used in the above formula to control for cases where firms with high investment opportunities would optimally choose $\Delta^G < \Delta^{Gcred,h}$ due to the effect of defaulting firms with low investment opportunities. Suppose $\Delta^{Gub,h} > \Delta^{Gcred,h}$ then we can obtain it by F.O.C.:

$$\frac{\partial V^{ND}}{\partial\Delta^{G,h}} = \frac{\partial V^{ND'}}{\partial\left(w'-cir'\right)}\frac{\partial\left(w'-cir'\right)}{\partial\Delta^{G,h}} = 0$$

$$\frac{\partial}{\partial\Delta^{G,h}}\frac{\left(\left(A+\Delta^h-\frac{\theta}{1-\theta}\left(\Delta^{G,h}-\Delta^h\right)\right)K^{\alpha-1}+\lambda q^h\right)(1-\theta)}{1-\theta\frac{\left(A+\mu\Delta^{G,h}+(1-\mu)\Delta^l\right)K^{\alpha-1}+\lambda q^h}{\left(A+\Delta^h\right)K^{\alpha-1}+\lambda q^h}q^h} = 0$$

$$\Delta^{Gub,h} = \frac{\left(q^h-1\right)\left(r^h+\lambda q^h\right)}{\theta(1-\mu)q^h} + \Delta^l - \lambda\left(q^h-q^l\right).$$

Separation Equilibria: There is potentially a continuum of separation equilibria where firm with access to low quality investments saves and buys securitized assets from firms with high investment opportunities. Firms with access to high quality investments invest, securitize and provide implicit recourse $\Delta^{G,h} \in \left(\Delta^{Gminsep}, \Delta^{Gcred,h}\right)$, where $\Delta^{Gminsep}$ is the minimum implicit recourse which prevents mimicking by firms with low investment opportunities. Out of equilibrium beliefs for $\Delta^{G,h} < \Delta^{Gcred,h}$ are when observed $\Delta^G > \Delta^{G,h}$ then $Pr(j=h) = 0$.

Applying Intuitive Criterion: If a separating equilibrium exists, then all pooling

Figure 3.3: Case where Intuitive Criterion selects Separating Equilibrium

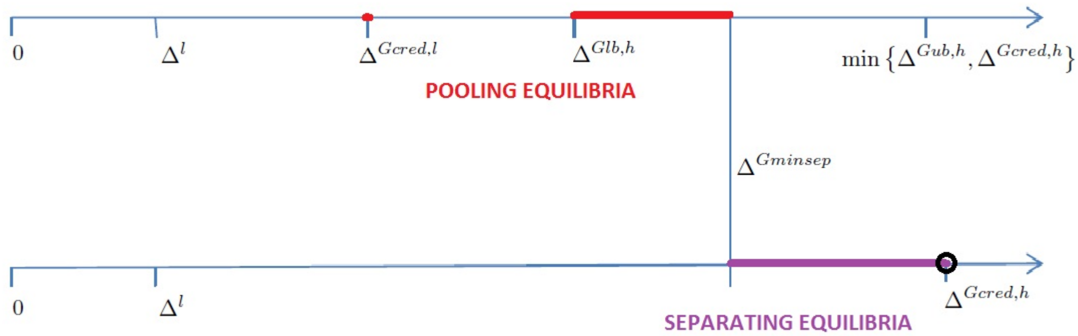
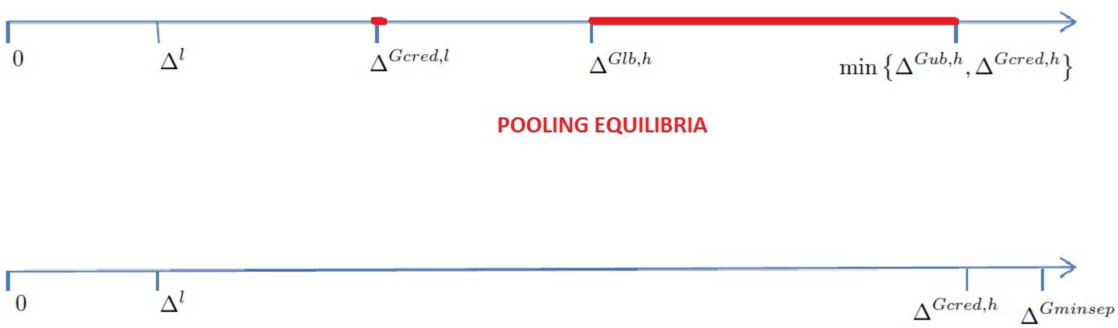


Figure 3.4: Case where there is no Separating Equilibrium



equilibria are dominated and therefore fail the Intuitive Criterion. In particular Intuitive Criterion selects only one separation equilibrium, where firms with access to high quality investments invest, securitize and provide implicit recourse $\Delta^{G,h} = \Delta^{Gcred,h}$. So after application of Intuitive Criterion there is either one unique separating equilibrium left or one or multiple pooling equilibria.

The conditions for existence of separating equilibrium are the following:

The minimum level of implicit recourse needed to achieve separation $\Delta^{Gminsep}$ can be derived from the following conditions (under equality). Implicit recourse $\Delta^{G,h}$ has to be high enough such that the firm with low investment opportunity would not have incentives to mimic high investment firm. This condition can be in a case of no-aggregate uncertainty written as:

$$R^l\,(mimicking) < R^l\,(investing)$$

$$\frac{\left(A+\Delta^{l}-\frac{\theta}{1-\theta}\max\left(\Delta^{G,h}-\Delta^{l},0\right)\right)K_{t+1}^{\alpha-1}+\lambda q^{h}}{\frac{1-\theta q^{h,IR}}{1-\theta}}<\frac{\left(A+\Delta^{l}-\frac{\theta}{1-\theta}\max\left(\Delta^{G,l}-\Delta^{l},0\right)\right)K_{t+1}^{\alpha-1}+\lambda q^{l}}{\frac{1-\theta q^{l,IR}}{1-\theta}}$$

which under no-default condition[6] simplifies to

$$\lambda\left(q^{h}-q^{l}\right)<\frac{\left(\Delta^{G,h}-\Delta^{l}\right)}{1-\theta}K^{\alpha-1}\left(1-q^{l,IR}\right) \tag{3.13}$$

As long as $q^{l,IR}>1$ firms with lower quality projects will want to mimic the high quality project firms. So $\Delta^{G,h}$ has to be high enough to bring $q_t^{l,IR}$ below 1.

When $q^l<q_t^{l,IR}<1$ it is not profitable for low types to issue loans ans securitize them. The promise $\Delta^{G,h}$ have to be high enough to satisfy also (again for now under the simplification without aggregate uncertainty):

$$R^{l}\left(mimicking\right)<R^{l}\left(buying\ high\ loans\right)$$

$$\frac{\left(A+\Delta^{l}-\frac{\theta}{1-\theta}\max\left(\Delta^{G}-\Delta^{l},0\right)\right)r+\lambda q^{h}}{\frac{1-\theta q^{h,IR}}{1-\theta}}<\frac{\left(A+\Delta^{h}\right)r+\lambda q^{h}}{q^{h}}$$

which reduces under no default condition (under default condition still holds but is no longer sufficient) to the following equation:

$$\left(r^{l}+\lambda q^{l}\right)\left(q^{l}-1\right)<\theta\left(1-q^{l}\right)\Delta^{G}K^{\alpha-1} \tag{3.14}$$

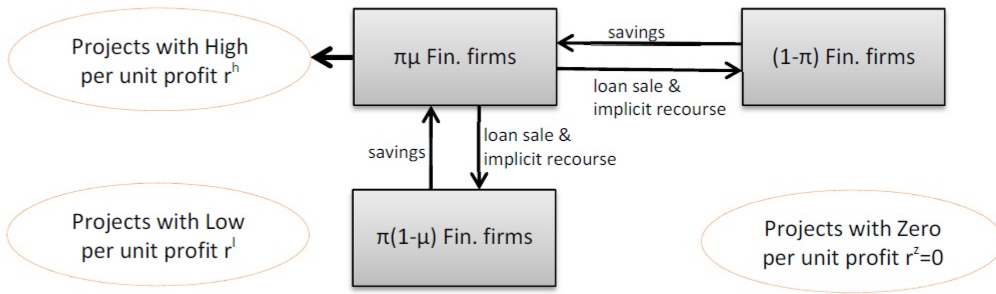
which implies a necessary condition for separation $q^l<1$.

Proposition 7. *A necessary condition for existence of a separation equilibrium is that* $q^{l,IR}<1$ *(which means also*$q^l<1$*). This implies that in a separating equilibrium firms with access to low quality investments saves and buys securitized assets from firms with high investment opportunities.*

Separating steady state is more efficient from aggregate perspective since level of capital and output are higher because resources are better allocated. Pooling is less efficient since the allocation of capital is equal to the rate of arrival of the investment opportunities. But still pooling with implicit recourse is better then pooling equilibrium

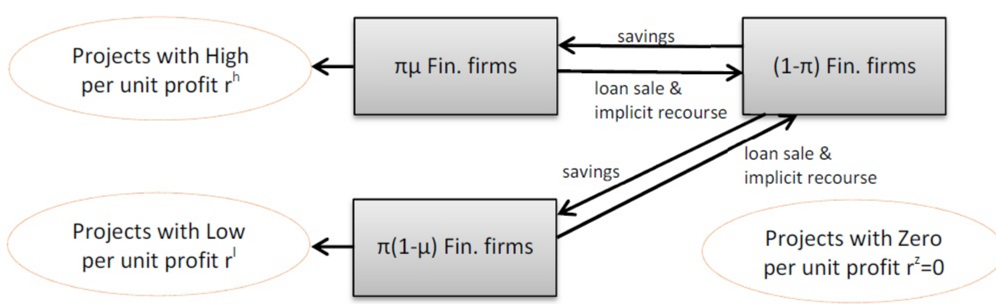
[6] Under default on implicit recourse while mimicking LHS of the inequality would be higher so the original condition would become a necessary but not sufficient condition.

Figure 3.5: Private information case with implicit recourse: Separating equilibrium



In the separating equilibrium the implicit recourse provided by the firms with access to projects with high per unit profits is high enough so that it is not profitable for firms with access to projects with low per unit profits to mimic them. They are better off buying the projects with high per unit profits from the issuing firms.

Figure 3.6: Private information case with implicit recourse: Pooling equilibrium



In the pooling equilibrium both firms with access to project with high per unit profits and low per unit profits provide the same level of implicit recourse. They are indistinguishable and therefore both firms invest into projects and sell them to firms with no investment opportunities.

without this option since there is more investment made in aggregate even though its allocation is equally bad.

4 Dynamics and numerical examples

In this chapter I show the solution of the fully stochastic version of the model under private information and with possibility of implicit recourse. Recall that we there is one stochastic process that allocates investment opportunities among firms independently of their characteristics and the aggregate productivity is the following stochastic process:

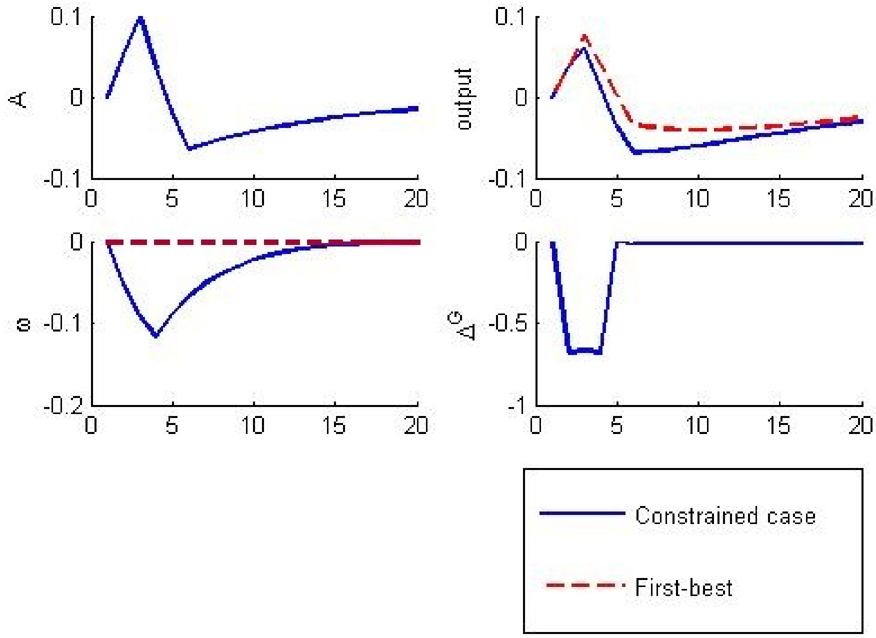
$$\log A_t = (1-\rho)\log \overline{A} + \rho \log A_{t-1} + u_t.$$

For simplicity I assume that u_t has a binomial distribution. With probability $p = 0.5$: $u_t = \epsilon$ and with probability $(1-p)$: $u_t = -\epsilon$. This assumption simplifies the solution but is not crucial for the results and the model could be generalized with $u \sim N(0, \sigma)$. Having a limited size of the shock to productivity can result under proper parametrization of the model to no-defaults on implicit recourse at all.

In the analysis of the dynamic properties of the model I focus on the switching between the separation and pooling equilibria over the business cycle. I show that even though in steady state there may be a separating equilibrium when the aggregate productivity increases and the economy is in boom stage of the business cycle the separating equilibrium is no longer sustainable and the economy switches to the pooling equilibrium where both type of firms provide the same level of implicit guarantees and both will invest. One can demonstrate this analytically on a simplified example as shown in the appendix, but the full model can be solved only numerically. The intuition behind the result is the following. As the aggregate productivity increases the relative difference in productivity of the two nonzero profit project types is reduced. Therefore a higher promise is needed to satisfy the separation conditions (3.13,3.14,**??**). Intuitively for instance the first condition says that $q^l < 1$ for separation but in boom even the quality of low type projects is relatively high and therefore one has to promise high implicit recourse to drive the prices of low projects below zero. At some point the required level of implicit recourse to achieve separation exceeds the incentive compatible limits and the economy switches to the pooling equilibrium.

The parameters of the model are not calibrated yet. For illustration of qualitative properties I solved the model for the following parameter values: $\alpha = 0.5$, $\beta = 0.95$,

Figure 4.1: Impulse responses



$\mu = 0.8$, $\pi = 0.1$, $\lambda = 0.75$, $\theta = 0.6$, $\phi = 0.1$, $\epsilon = 0.05$, $\rho = 0.9$, $\bar{A} = 2.4$, $\Delta^h = 1$, $\Delta^l = 0$.

In the Figure 1 I show how the economy behaves in a particular episode of a two positive shocks followed by three negative productivity shocks Recall u_t has a binomial distribution with limited size within the period. The point of this exercise is to show the switch from separating equilibrium to pooling and back and its effects on the output. The impulse responses start from a steady state to which they converge after a long period of zero productivity shocks and then I introduce the described sequence of productivity shock after which the shocks are zero again. On the graph I report for comparison impulse responses of the constrained model under private information and with implicit recourse provision and the efficient first-best case. Note that the graph depicts deviations from each model's steady state (here meaning state after a long sequence of zero shocks). So even though on the graph both first-best case and constrained case start at the same point the absolute level of these variables is mostly different. In particular the first-best case is characterized by higher output and capital levels.

You can see on the figure that as the constrained economy moves to the boom stage of the business cycle it also switches from the separating equilibrium to pooling equilibrium (ω decreases). While the share of high quality projects(ω) remains constant in the first best case at 100% (0 on the graph because this represents deviations from the steady state). Lower share of high quality projects in the constrained case slows slightly the growth of output and accumulation of capital but the effect is small since in boom stage the difference in the two qualities is rather small. But the inefficiency in allocation of capital keeps accumulating. As the economy exogenously moves to a recession one can see that the accumulated inefficiency in the allocation of capital is more pronounced as now the difference is qualities is more important. So we can see that booms have almost the same relative size as in the first-best case but busts following a boom stage are more deep in constrained case then in the first best case. It can be shown that the difference in the depth of a recession compared to the first best case is larger the larger are the inefficiencies accumulated in the pooling equilibrium during the preceding boom period.

5 Conclusions

In this paper I show that in general reputation concerns allow sponsors of securitized products to signal the quality of the loans by providing implicit recourse and thus they limit the problem of private information typical for securitization. However, there are limits to the efficiency of this particular reputation based tools, which become more pronounced in boom stages of the business cycles. The costs of sufficiently high implicit recourse that would avoid mimicking by firms with investment projects of lower quality exceed the limit which can still be credibly promised. In the resulting pooling equilibrium the information about the quality of loans is lost and the investment allocation becomes more inefficient. Due to this mechanism large inefficiencies in the allocation of capital can be accumulated in the boom stage of the business cycle. The accumulated inefficiencies can then amplify the subsequent downturn of the economy. This mechanism can contribute to the our understanding of the recent financial crisis where in the period preceding it many inefficient investments whose exact quality was unknown were undertaken. While this was not a problem as long as the economy was performing these low quality loans and their large amount in the economy contributed to the depth of the financial crisis.

6 Appendix

6.1 Proofs

6.1.1 Proof of Proposition 1

All firms policy functions have linear functions in their individual state variables and the allocation of investment opportunities is independent on individual holdings of assets. This allows easy aggregation and results in an important result that aggregate quantities and prices do not depend on the distribution of assets across individual firms. Aggregate level of high and low assets H, L does not depend on the distribution of the assets, therefore so does not r and neither prices q^h, q^{IR}, q^l.

6.1.2 Proof of Proposition 2

In the first best allocation $q^h = 1$. Should the skin in the game be binding the $q^h > 1$. Let's consider the least restrictive case where still only the firm with access to high quality loans is issuing credit and securitizes these loans and the skin in the game is not high enough to allow firm with access to low quality investment opportunities to profitably issue loans $q^l < 1$.

Case 1: Only firms with access to high quality projects gives credit and securitizes: Steady state conditions are the following:

$$(1-\lambda)\left(1-\theta q^h\right) = \pi\mu\beta\left(r^h + \lambda q^h\right)$$

$$\frac{A^h}{q^h} = \frac{A^l}{q^l}$$

$$r^h = (1-\lambda) + (1-\beta)\left(r^h + \lambda q^h\right).$$

Combining these equations we can obtain

$$q_H^h = \frac{(1-\lambda)(1-\pi\mu)}{(1-\lambda)\theta + \pi\mu\lambda}$$

$$K_H = \left[\frac{(1-\lambda) + \frac{(1-\beta)\lambda(1-\lambda)(1-\pi\mu)}{(1-\lambda)\theta+\pi\mu\lambda}}{\beta A^h}\right]^{\frac{1}{\alpha-1}}.$$

As long as $q^h = 1$, we would obtain $K_H = \left[\frac{1}{A^h}\left(\frac{1}{\beta} - \lambda\right)\right]^{\frac{1}{\alpha-1}}$ which is the first best optimal level of capital (compare with (3.1)). If $(1-\lambda)(1-\pi\mu) > (1-\lambda)\theta + \pi\mu\lambda$ then $q^h > 1$. Deterministic steady state level of capital is then lower then in the first best case:

$$K_H = \left[\frac{(1-\lambda) + (1-\beta)\lambda q_H^h}{\beta A^h}\right]^{\frac{1}{\alpha-1}} < \left[\frac{(1-\lambda) + (1-\beta)\lambda}{\beta A^h}\right]^{\frac{1}{\alpha-1}} = K_{FB}.$$

6.1.3 Proof of Proposition 3

Proposition 2 claims that there are three possible types of steady state depending on the parameter values. In the proof of Proposition 1 above I described already the least restricted case where only firm with access to high quality projects will be issuing and securitizing loans. By continuing to tighten the skin in the game constraint we will increase the price of low quality asset to 1 ($q^l = 1$). At this point the firms with access to low quality loans will be indifferent between buying high quality securitized assets or issue and securitize their own loans. Credit to low quality projects counterweights the effect of tightening skin in the game constraint and therefore the price stay at the same levels ($q^l = 1$, $q^h = A^h/A^l$). For an interval of θ there will be an steady state in which firms with access to low quality investment will play a mixed strategy when giving credit with probability φ. As θ decreases (skin in the game rises), φ increases all the way up to 1, where a third type of steady state takes place. In this firms with access to both high and low quality projects will be all issuing credit and securitizing always.

Case 2: Firm with access to low quality projects issues credit with probability φ: Steady state conditions are the following:

$$(1-\lambda)\left(1-\theta q^h\right)\omega = \pi\mu\beta\left(\omega\left(r^h + \lambda q^h\right) + (1-\omega)\left(r^l + \lambda q^l\right)\right) \tag{6.1}$$

$$(1-\lambda)\left(1-\theta q^l\right)(1-\omega) = \pi(1-\mu)\varphi\beta\left(\omega\left(r^h + \lambda q^h\right) + (1-\omega)\left(r^l + \lambda q^l\right)\right) \tag{6.2}$$

$$\frac{A^h}{q^h} = \frac{A^l}{q^l} \tag{6.3}$$

$$q^l = 1 \tag{6.4}$$

$$\omega r^h + (1-\omega) r^l = (1-\lambda) + (1-\beta)\left(\omega\left(r^h + \lambda q^h\right) + (1-\omega)\left(r^l + \lambda q^l\right)\right). \tag{6.5}$$

Let's define

$$q \equiv \frac{q^h}{A^h} = \frac{q^l}{A^l} \tag{6.6}$$

and

$$D \equiv \omega A^h + (1-\omega) A^l. \tag{6.7}$$

Using (6.6), (6.7) and combining equations (6.1), (6.2) and (6.3):

$$(1-\lambda)(1-\theta q D) = \pi(\mu + \varphi(1-\mu))\beta D\left(K^{\alpha-1} + \lambda q\right)$$

$$(1-\lambda) - \pi(\mu + \varphi(1-\mu))\beta D K^{\alpha-1} = qD\left[(1-\lambda)\theta + \pi(\mu + \varphi(1-\mu))\beta\lambda\right] \tag{6.8}$$

We can also rewrite (6.5):

$$\beta D K^{\alpha-1} = 1 - \lambda + (1-\beta) D\lambda q \tag{6.9}$$

Combining (6.8), (6.9) we get

$$q_M = \frac{(1-\lambda)(1-\pi(\mu+\varphi(1-\mu)))}{(1-\lambda)\theta + \pi(\mu+\varphi(1-\mu))\lambda}\frac{1}{D} \tag{6.10}$$

Substituting (6.10) back into (6.9) we get:

$$K_M = \left[\frac{(1-\lambda) + \frac{(1-\beta)\lambda(1-\lambda)(1-\pi(\mu+\varphi(1-\mu)))}{(1-\lambda)\theta+\pi(\mu+\varphi(1-\mu))\lambda}}{\beta D}\right]^{\frac{1}{\alpha-1}} \tag{6.11}$$

Case 3: Firms with access to both high and low quality projects are always giving credit: Deterministic steady state is defined by:

$$\left(1-\lambda\right)\left(1-\theta q^{h}\right)\omega=\pi\mu\beta\left(\omega\left(r^{h}+\lambda q^{h}\right)+\left(1-\omega\right)\left(r^{l}+\lambda q^{l}\right)\right) \tag{6.12}$$

$$\left(1-\lambda\right)\left(1-\theta q^{l}\right)\left(1-\omega\right)=\pi(1-\mu)\beta\left(\omega\left(r^{h}+\lambda q^{h}\right)+\left(1-\omega\right)\left(r^{l}+\lambda q^{l}\right)\right) \tag{6.13}$$

$$\frac{A^{h}}{q^{h}}=\frac{A^{l}}{q^{l}} \tag{6.14}$$

$$\omega r^{h}+\left(1-\omega\right)r^{l}=\left(1-\lambda\right)+\left(1-\beta\right)\left(\omega\left(r^{h}+\lambda q^{h}\right)+\left(1-\omega\right)\left(r^{l}+\lambda q^{l}\right)\right). \tag{6.15}$$

Using (6.6), (6.7) and combining equations (6.12), (6.13) and (6.14):

$$\left(1-\lambda\right)\left(1-\theta qD\right)=\pi\beta D\left(K^{\alpha-1}+\lambda q\right)$$

$$\left(1-\lambda\right)-\pi\beta DK^{\alpha-1}=qD\left[\left(1-\lambda\right)\theta+\pi\beta\lambda\right] \tag{6.16}$$

We can also rewrite (6.15):

$$\beta DK^{\alpha-1}=1-\lambda+\left(1-\beta\right)D\lambda q \tag{6.17}$$

Combining (6.16), (6.17) we get

$$q_{B}=\frac{\left(1-\lambda\right)\left(1-\pi\right)}{\left(1-\lambda\right)\theta+\pi\lambda}\frac{1}{D} \tag{6.18}$$

Substituting (6.18) back into (6.17) we get:

$$K_{B}=\left[\frac{\left(1-\lambda\right)+\frac{(1-\beta)\lambda(1-\lambda)(1-\pi)}{(1-\lambda)\theta+\pi\lambda}}{\beta D}\right]^{\frac{1}{\alpha-1}} \tag{6.19}$$

Second part of proposition claims that $K_H > K_M > K_B$.

To show this lets first focus on the in the brackets part of the formulas for capital: Since in Case 1 $q_H^l < 1$ then $q_H^h < \frac{A^h}{A^l}$. And since $q_M^l = 1$ then $\frac{(1-\lambda)(1-\pi(\mu+\varphi(1-\mu)))}{(1-\lambda)\theta+\pi(\mu+\varphi(1-\mu))\lambda} =$

$\frac{D_M}{A^l}$. The following inequality then holds

$$\frac{(1-\lambda)+(1-\beta)\,\lambda q_H^h}{\beta A^h} < \frac{(1-\lambda)}{\beta A^h}+(1-\beta)\,\lambda\frac{1}{\beta A^l} < \frac{(1-\lambda)}{\beta D_M}+(1-\beta)\,\lambda\frac{1}{\beta A^l} = \frac{(1-\lambda)+\frac{(1-\beta)\lambda(1-\lambda)(1-\pi(\mu+\varphi(1-\mu)))}{(1-\lambda)\theta+\pi(\mu+\varphi(1-\mu))\lambda}}{\beta D_M}.$$

This implies that

$$K_H = \left[\frac{(1-\lambda)+(1-\beta)\,\lambda q_H^h}{\beta A^h}\right]^{\frac{1}{\alpha-1}} > \left[\frac{(1-\lambda)+\frac{(1-\beta)\lambda(1-\lambda)(1-\pi(\mu+\varphi(1-\mu)))}{(1-\lambda)\theta+\pi(\mu+\varphi(1-\mu))\lambda}}{\beta D_M}\right]^{\frac{1}{\alpha-1}} = K_M.$$

Similarly we can show that $K_P > K_B$. Since $w_B < w_P$ then $D_B < D_P$. Also $q_B^l > 1$ then $\frac{(1-\lambda)(1-\pi)}{(1-\lambda)\theta+\pi\lambda} > \frac{D_B}{A^l}$. This implies that

$$\frac{(1-\lambda)+\frac{(1-\beta)\lambda(1-\lambda)(1-\pi(\mu+\varphi(1-\mu)))}{(1-\lambda)\theta+\pi(\mu+\varphi(1-\mu))\lambda}}{\beta D_M} = \frac{(1-\lambda)}{\beta D_M}+(1-\beta)\,\lambda\frac{1}{\beta A^l} < \frac{(1-\lambda)}{\beta D_B}+(1-\beta)\,\lambda\frac{1}{\beta A^l} < \frac{(1-\lambda)+\frac{(1-\beta)\lambda(1-\lambda)(1-\pi)}{(1-\lambda)\theta+\pi\lambda}}{\beta D_B},$$

$$K_M = \left[\frac{(1-\lambda)+\frac{(1-\beta)\lambda(1-\lambda)(1-\pi(\mu+\varphi(1-\mu)))}{(1-\lambda)\theta+\pi(\mu+\varphi(1-\mu))\lambda}}{\beta D_M}\right]^{\frac{1}{\alpha-1}} > \left[\frac{(1-\lambda)+\frac{(1-\beta)\lambda(1-\lambda)(1-\pi)}{(1-\lambda)\theta+\pi\lambda}}{\beta D_B}\right]^{\frac{1}{\alpha-1}} = K_B$$

6.1.4 Proof of Proposition 4

In the proof of Proposition 1 and 2 we already proved that $K_{FB} > K_H > K_M > K_B$. To prove Proposition 3 it suffices to prove that $K_B > K_{private}$, where $K_{private}$ is the level of capital under private information about the allocation of investment opportunities. To obtain $K_B > K_{private}$, we need:

$$K_B^{\alpha-1} < K_{private}^{\alpha-1}$$

$$\frac{(1-\lambda)+\frac{(1-\beta)\lambda(1-\lambda)(1-\pi)}{(1-\lambda)\theta+\pi\lambda}}{\beta\left(\omega A^h+(1-\omega)\,A^l\right)} < \frac{(1-\lambda)+\frac{(1-\beta)\lambda(1-\lambda)(1-\pi)}{(1-\lambda)\theta+\pi\lambda}}{\beta\left(\mu\Delta A^h+(1-\mu)A^l\right)}$$

$$\omega > \mu.$$

Writing equations (6.12) and (6.13) in a ratio we obtain:

$$\frac{(1-\lambda)\left(1-\theta q^{h}\right)\omega}{(1-\lambda)\left(1-\theta q^{l}\right)(1-\omega)}=\frac{\pi\mu\beta\left(\omega\left(r^{h}+\lambda q^{h}\right)+(1-\omega)\left(r^{l}+\lambda q^{l}\right)\right)}{\pi(1-\mu)\beta\left(\omega\left(r^{h}+\lambda q^{h}\right)+(1-\omega)\left(r^{l}+\lambda q^{l}\right)\right)}.$$

Since $q^h > q^l$ we can obtain:

$$\frac{\omega}{(1-\omega)}=\frac{\left(1-\theta q^{l}\right)}{\left(1-\theta q^{h}\right)}\frac{\mu}{(1-\mu)}>\frac{\mu}{(1-\mu)},$$

and this implies that $\omega > \mu$.

6.1.5 Proofs for subchapter 3.4.2

I claimed that if the implicit recourse promise would be credible, the optimal level of promise would mean $q^j = 1$ and therefore zero profit for securitizing firms. The relevant F.O.C. can be transformed in the following way (Let's consider F.O.C. for firms with high quality investment opportunities. The remaining would not invest at all.):

$$\frac{\partial V^{ND}}{\partial\Delta^{G,j}}=\frac{\partial V^{ND'}}{\partial\left(w'-cir'\right)}\frac{\partial\left(w'-cir'\right)}{\partial\Delta^{G,j}}=0.$$

$$\frac{\partial V^{ND'}}{\partial\left(w'-cir'\right)}\frac{\partial}{\partial\Delta^{G,j}}\frac{(1-\theta)\beta w\left(r^{j'}+\lambda q^{j}\right)-\theta\beta wK^{\alpha-1}\left(\Delta^{G,j}-\Delta^{j}\right)}{1-\theta q^{G,j}}=0$$

after substituting in this case with constant aggregate productivity $q^{G,j}=\frac{\left(A+\Delta^{G,j}\right)K^{\alpha-1}+\lambda q^{j}}{\left(A+\Delta^{j}\right)K^{\alpha-1}+\lambda q^{j}}q^{j}$ this condition implies that

$$\frac{(1-\theta)\beta w\left[\left(A'+\Delta^{j}-\frac{\theta}{1-\theta}\left(\Delta^{G,j}-\Delta^{j}\right)\right)K^{\alpha-1}+\lambda q^{j}\right]\frac{\theta K^{\alpha-1}}{\left(A'+\Delta^{j}\right)K^{\alpha-1}+\lambda q^{j}}q^{j}-(1-\theta)\beta w\frac{\theta}{1-\theta}K^{\alpha-1}\left(1-\theta q^{G,j}\right)}{\left(1-\theta q^{G,j}\right)^{2}}=0$$

$$\left(A+\Delta^{j}\right)K^{\alpha-1}+\lambda q^{j}-\theta q^{j}\left(\left(A+\Delta^{G,j}\right)K^{\alpha-1}+\lambda q^{j}\right)=(1-\theta)\left(\left(A+\Delta^{j}\right)K^{\alpha-1}+\lambda q^{j}\right)q^{j}-\theta q^{j}\left(\Delta^{G,j}-\Delta^{j}\right)K^{\alpha-1}$$

$$\left(A+\Delta^{j}\right)K^{\alpha-1}+\lambda q^{j}=\left(\left(A+\Delta^{j}\right)K^{\alpha-1}+\lambda q^{j}\right)q^{j}$$

This implies $q^j = 1$.

Note that for when the level of Δ^G satisfies this condition, return from investing and securitizing is equal to the return from investing but not securitizing i.e. securitization does not increase the return:

$$\begin{aligned} R\,(investing\,\&\,securitizing) &= R\,(investing) \\ \frac{(1-\theta)\left[\left(A'+\Delta^j-\frac{\theta}{1-\theta}\left(\Delta^{G,j}-\Delta^j\right)\right)K^{\alpha-1}+\lambda q^j\right]}{1-\theta\frac{(A+\Delta^G)K^{\alpha-1}+\lambda q^j}{(A+\Delta^j)K^{\alpha-1}+\lambda q^j}q^j} &= \frac{\left(A'+\Delta^j\right)K^{\alpha-1}+\lambda q^j}{1} \end{aligned}$$

Since $q^j = 1$ we get:

$$(1-\theta)\left(\left(A'+\Delta^j\right)K^{\alpha-1}+\lambda\right)-\theta\left(\Delta^{G,j}-\Delta^j\right)K^{\alpha-1} = \left(\left(A'+\Delta^j\right)K^{\alpha-1}+\lambda\right)-\theta\left(\left(A+\Delta^{G,j}\right)K^{\alpha-1}+\lambda\right),$$

which always holds.

6.2 Derivation of firms' policy functions

6.2.1 Case without implicit recourse

Individual firm maximizes

$$\max_{c_t^i,h_{t+1}^i,l_{t+1}^i,z_{t+1}^i}\sum_{s=0}^{\infty}\beta^s\log\left(c_{t+s}^i\right)$$

subject to the following borrowing constraints

$$c_t^s+i_t^z+\left(z_{t+1}^s-i_t^s\right)q_t^z+h_{t+1}^s q_t^l+l_{t+1}^s q_t^l = h_t^s(r_t^h+\lambda q_t^h)+l_t^s(r_t^l+\lambda q_t^l)$$

$$c_t^h+i_t^h+\left(h_{t+1}^h-i_t^h\right)q_t^h+l_{t+1}^h q_t^l+z_{t+1}^h q_t^z = h_t^h(r_t^h+\lambda q_t^h)+l_t^h(r_t^l+\lambda q_t^l)$$

$$c_t^l+i_t^l+\left(l_{t+1}^l-i_t^l\right)q_t^l+h_{t+1}^l q_t^h+z_{t+1}^h q_t^z = h_t^l(r_t^h+\lambda q_t^h)+l_t^l(r_t^l+\lambda q_t^l),$$

and subject to "skin in the game" constraints:

$$h_{t+1}^h \geqq (1-\theta)\,i_t^h,\ l_{t+1}^l \geqq (1-\theta)\,i_t^l$$

When the skin in the game constraint are binding all constraints together can be written as follows (in the case where the constraint is binding for firms with access to both high and low quality investment opportunities):

$$c_t^s + h_{t+1}^s q_t^h + l_{t+1}^s q_t^l = h_t^s(r_t^h + \lambda q_t^h) + l_t^s(r_t^l + \lambda q_t^l)$$

$$c_t^h + \frac{\left(1-\theta q_t^h\right)}{(1-\theta)} h_{t+1}^h = h_t^h(r_t^h + \lambda q_t^h) + l_t^h(r_t^l + \lambda q_t^l)$$

$$c_t^l + \frac{\left(1-\theta q_t^l\right)}{(1-\theta)} l_{t+1}^l = h_t^l(r_t^h + \lambda q_t^h) + l_t^l(r_t^l + \lambda q_t^l).$$

The problem can be written into a recursive formulation:

$$V\left(l,h;K,\omega,A\right) = \pi\left(\mu V^h\left(l,h;K,\omega,A\right) + (1-\mu)\, V^l\left(l,h;K,\omega,A\right)\right) + (1-\pi)\, V^s\left(l,h;K,\omega,A\right),$$

where for $i=\{h,l,s\}$:

$$V^i\left(l,h;K,\omega,A\right) = \max_{c,h',l'}\left[\log\left(c\right) + \beta EV\left(l',h';K,\omega,A\right)\right]$$

subject to the respective borrowing constraint stipulated above.

From first order conditions we can obtain the following Euler equations:

$$E_t\left[\frac{c_t^s}{\beta c_{t+1}^s}\frac{r_{t+1}^h + \lambda q_{t+1}^h}{q_t^h}\right] = 1 \tag{6.20}$$

$$E_t\left[\frac{c_t^s}{\beta c_{t+1}^s}\frac{r_{t+1}^l + \lambda q_{t+1}^l}{q_t^l}\right] = 1 \tag{6.21}$$

$$E_t\left[\frac{c_t^h}{\beta c_{t+1}^h}\frac{r_{t+1}^h + \lambda q_{t+1}^h}{\frac{\left(1-\theta q_t^h\right)}{(1-\theta)}}\right] = 1 \tag{6.22}$$

$$E_t\left[\frac{c_t^l}{\beta c_{t+1}^l}\frac{r_{t+1}^l + \lambda q_{t+1}^l}{\frac{\left(1-\theta q_t^l\right)}{(1-\theta)}}\right] = 1 \tag{6.23}$$

We guess and verify that

$$c_t^s = (1-\beta)\left(h_t^s(r_t^h + \lambda q_t^h) + l_t^s(r_t^l + \lambda q_t^l)\right)$$

$$c_t^h = (1-\beta)\left(h_t^h(r_t^h + \lambda q_t^h) + l_t^h(r_t^l + \lambda q_t^l)\right)$$

$$c_t^l = (1-\beta)\left(h_t^l(r_t^h + \lambda q_t^h) + l_t^l(r_t^l + \lambda q_t^l)\right)$$

$$h_{t+1}^h = \frac{\beta\left(h_t^h(r_t^h + \lambda q_t^h) + l_t^h(r_t^l + \lambda q_t^l)\right)}{\frac{\left(1-\theta q_t^h\right)}{(1-\theta)}}$$

$$l_{t+1}^h = 0$$

$$l_{t+1}^l = \frac{\beta\left(h_t^l(r_t^h + \lambda q_t^h) + l_t^l(r_t^l + \lambda q_t^l)\right)}{\frac{\left(1-\theta q_t^l\right)}{(1-\theta)}}$$

$$h_{t+1}^l = 0$$

$$h_{t+1}^s = \zeta\beta\left(h_t^s(r_t^h + \lambda q_t^h) + l_t^s(r_t^l + \lambda q_t^l)\right)$$

$$l_{t+1}^s = (1-\zeta)\,\beta\left(h_t^s(r_t^h + \lambda q_t^h) + l_t^s(r_t^l + \lambda q_t^l)\right)$$

$$c_{t+1}^s = (1-\beta)\left(h_{t+1}^s(r_{t+1}^h + \lambda q_{t+1}^h) + l_{t+1}^s(r_{t+1}^l + \lambda q_{t+1}^l)\right)$$

$$c_{t+1}^h = (1-\beta)\left(h_{t+1}^h(r_{t+1}^h + \lambda q_{t+1}^h)\right)$$

$$c_{t+1}^l = (1-\beta)\left(l_{t+1}^l(r_{t+1}^l + \lambda q_{t+1}^l)\right)$$

Using these guesses and substituting in equations (6.22) and (6.23) we can see that these conditions always hold.

The remaining Euler equations (6.20) and (6.21) can be rewritten into:

$$E_t\left[\frac{\frac{r_{t+1}^h+\lambda q_{t+1}^h}{q_t^h}}{\zeta\frac{r_{t+1}^h+\lambda q_{t+1}^h}{q_t^h} + (1-\zeta)\,\frac{r_{t+1}^l+\lambda q_{t+1}^l}{q_t^l}}\right] = 1$$

$$E_t\left[\frac{\frac{r_{t+1}^l+\lambda q_{t+1}^l}{q_t^l}}{\zeta\frac{r_{t+1}^h+\lambda q_{t+1}^h}{q_t^h} + (1-\zeta)\,\frac{r_{t+1}^l+\lambda q_{t+1}^l}{q_t^l}}\right] = 1.$$

The allocation of saving firms (those with zero-profit projects) between high and low investment projects have to satisfy the market clearing conditions on both markets for high and low projects. From $H_{t+1} = \lambda H_t + I_t^h,\ L_{t+1} = \lambda L_t + I_t^l$ after substituting $H_{t+1} = H_{t+1}^h + H_{t+1}^s,\ L_{t+1} = L_{t+1}^h + L_{t+1}^s$and $H_{t+1}^h = (1-\theta) I_t^h,\ L_{t+1}^l = (1-\theta) I_t^l$

$$H_{t+1}^s = \frac{\theta}{(1-\theta)} H_{t+1}^h + \lambda H_t$$

$$L_{t+1}^s = \frac{\theta}{(1-\theta)} L_{t+1}^h + \lambda L_t,$$

which can be rewritten as

$$\zeta (1-\pi) \beta \left(H_t^s(r_t^h + \lambda q_t^h) + L_t^s(r_t^l + \lambda q_t^l)\right) = \theta\pi\mu \frac{\beta \left(H_t^h(r_t^h + \lambda q_t^h) + L_t^h(r_t^l + \lambda q_t^l)\right)}{\left(1 - \theta q_t^h\right)} + \lambda H_t$$

$$(1-\zeta)(1-\pi) \beta \left(H_t^s(r_t^h + \lambda q_t^h) + L_t^s(r_t^l + \lambda q_t^l)\right) = \theta\pi (1-\mu) \frac{\beta \left(H_t^h(r_t^h + \lambda q_t^h) + L_t^h(r_t^l + \lambda q_t^l)\right)}{\left(1 - \theta q_t^l\right)} + \lambda L_t$$

And the goods market clears too $Y_t = C_t + I_t$.

6.2.2 Case with implicit recourse

The problem with implicit recourse and potential default on it is better written in a recursive formulation:

$$V^{ND}\left(\bar{s}, w - cir; \bar{S}\right) = \pi \left(\mu V^{ND,h}\left(\bar{s}, w - cir; \bar{S}\right) + (1-\mu) V^{ND,l}\left(\bar{s}, w - cir; \bar{S}\right)\right) + (1-\pi) V^{ND,z}\left(\bar{s}, w - cir; \bar{S}\right)$$

$$V^{D}\left(\bar{s}, w; \bar{S}\right) = \pi \left(\mu V^{D,h}\left(\bar{s}, w; \bar{S}\right) + (1-\mu) V^{D,l}\left(\bar{s}, w; \bar{S}\right)\right) + (1-\pi) V^{D,z}\left(\bar{s}, w; \bar{S}\right)$$

$$V^{ND,j}\left(\bar{s}, w; \bar{S}\right) = \max_{c,i,h',l',r\{G\}'} \left[\log(c) + \beta E\left[\max\left(V^{ND}\left(\bar{s}', w' - cir'; \bar{S}'\right), V^{D}\left(\bar{s}', w'; \bar{S}'\right)\right)\right]\right]$$

$$V^{D,j}\left(\bar{s}, w; \bar{S}\right) = \max_{c,i,h',l'} \left[\log(c) + \beta E V^{D}\left(\bar{s}', w'; \bar{S}'\right)\right]$$

subject to the budget constraints which take the following form for investing firms for which “skin in the game” constraint is binding (e.g. in case of firms with high investment opportunities):

$$c_t^h+\frac{\left(1-\theta q_t^{\hat{G},h}\right)}{(1-\theta)}h_{t+1}^h+cir_t=h_t^S(r_t^h+\lambda q_t^h)+l_t^S(r_t^l+\lambda q_t^l)+h_t^P(r_t^{\hat{G},h}+\lambda q_t^h)+l_t^P(r_t^{\hat{G},l}+\lambda q_t^l).$$

The incentive compatible constraints for non-defaulting are the following:

$$V^{ND}\left(\bar{s},w-cir;\bar{S}\right)>V^{D}\left(\bar{s},w;\bar{S}\right),$$

$$V^{P}\left(\bar{s};\bar{S}\right)>V^{NP}\left(\bar{s};\bar{S}\right),$$

where $V^{ND},V^{D},V^{P},V^{NP}$ are the value functions if firm, never defaulted, when firm defaulted, when firm always punished a default on a promise on gross profits and when firm failed to punished respectively. w is individual wealth level before deducting cir, which are costs of providing implicit recourse, $\bar{s}=\{h,l,h^p,l^p\}$ is a vector of other individual state variables, where P, S superscripts denote assets sold in the previous period on the primary market which potentially bear implicit guarantee or on the secondary market respectively, $\bar{S}=\{K,\omega,A\}$ is a vector of aggregate state variables, $r_t^{\hat{G},h}$ is the return received from securitized assets with implicit recourse conditional on potential default and $q_t^{\hat{G},j}$ is the price of securitized loans of type j depending on the information structure.Costs of implicit recourse are given by:

$$cir'=\theta i\left(K'\right)^{\alpha-1}\left(\Delta^G-\Delta^{h/l}\right)$$

From first order conditions we can obtain the following Euler equations:

$$E_t\left[\frac{c_t^s}{\beta c_{t+1}^s}\frac{r_{t+1}^h+\lambda q_{t+1}^h}{q_t^h}\right]=1 \tag{6.24}$$

$$E_t\left[\frac{c_t^s}{\beta c_{t+1}^s}\frac{r_{t+1}^l+\lambda q_{t+1}^l}{q_t^l}\right]=1 \tag{6.25}$$

$$E_t\left[\frac{c_t^s}{\beta c_{t+1}^s}\frac{\left(A+\max\left(\Delta_t^{G,h},\Delta^h\right)\right)K_{t+1}^{\alpha-1}+\lambda q_{t+1}^h}{q_t^{G,h}}\right]=1 \tag{6.26}$$

$$E_t\left[\frac{c_t^s}{\beta c_{t+1}^s}\frac{\left(A+\max\left(\Delta_t^{G,l},\Delta^l\right)\right)K_{t+1}^{\alpha-1}+\lambda q_{t+1}^l}{q_t^{G,l}}\right]=1 \tag{6.27}$$

$$E_t\left[\frac{c_t^h}{\beta c_{t+1}^h}\frac{r_{t+1}^h+\lambda q_{t+1}^h}{\frac{\left(1-\theta q_t^{G,h}\right)}{(1-\theta)}}\right]=1 \tag{6.28}$$

$$E_t\left[\frac{c_t^l}{\beta c_{t+1}^l}\frac{r_{t+1}^l+\lambda q_{t+1}^l}{\frac{\left(1-\theta q_t^{G,l}\right)}{(1-\theta)}}\right]=1. \tag{6.29}$$

Equations (6.26) and (6.27) hold if non-default conditions are satisfied i.e. $\Delta^{G,h}\leq\Delta^{Gcred,h}$ and $\Delta^{G,l}\leq\Delta^{Gcred,l}$. If these conditions are not satisfied then investor while taking expectations have to take into account the respective probability of default.

We guess and verify the following policy functions. Note that here I report the general the policy functions for the **pooling equilibrium**, where $\Delta^{G,l}=\Delta^{G,h}$ and the fir with low profit projects also invests.

$$c_t^s=(1-\beta)\left(h_t^s(r_t^h+\lambda q_t^h)+l_t^s(r_t^l+\lambda q_t^l)\right)$$

$$c_t^h=(1-\beta)\left(h_t^h(r_t^h+\lambda q_t^h)+l_t^h(r_t^l+\lambda q_t^l)\right)$$

$$c_t^l=(1-\beta)\left(h_t^l(r_t^h+\lambda q_t^h)+l_t^l(r_t^l+\lambda q_t^l)\right)$$

$$h_{t+1}^h=\frac{\beta\left(h_t^h(r_t^h+\lambda q_t^h)+l_t^h(r_t^l+\lambda q_t^l)\right)}{\frac{\left(1-\theta q_t^{G,h}\right)}{(1-\theta)}}$$

$$l_{t+1}^h=0$$

$$l_{t+1}^l=\frac{\beta\left(h_t^l(r_t^h+\lambda q_t^h)+l_t^l(r_t^l+\lambda q_t^l)\right)}{\frac{\left(1-\theta q_t^{G,l}\right)}{(1-\theta)}}$$

$$h_{t+1}^l=0$$

$$h_{t+1}^{p,s} = \zeta^h \beta \left(h_t^s (r_t^h + \lambda q_t^h) + l_t^s (r_t^l + \lambda q_t^l) \right)$$

$$l_{t+1}^{p,s} = \zeta^l \beta \left(h_t^s (r_t^h + \lambda q_t^h) + l_t^s (r_t^l + \lambda q_t^l) \right)$$

$$h_{t+1}^{s} = \zeta^{h^p} \beta \left(h_t^s (r_t^h + \lambda q_t^h) + l_t^s (r_t^l + \lambda q_t^l) \right)$$

$$l_{t+1}^{s} = \zeta^{l^p} \beta \left(h_t^s (r_t^h + \lambda q_t^h) + l_t^s (r_t^l + \lambda q_t^l) \right)$$

$$c_{t+1}^{s} = (1-\beta) \left(h_{t+1}^s (r_{t+1}^h + \lambda q_{t+1}^h) + l_{t+1}^s (r_{t+1}^l + \lambda q_{t+1}^l) \right)$$

$$c_{t+1}^{h} = (1-\beta) \left(h_{t+1}^h (r_{t+1}^h + \lambda q_{t+1}^h) \right)$$

$$c_{t+1}^{l} = (1-\beta) \left(l_{t+1}^l (r_{t+1}^l + \lambda q_{t+1}^l) \right),$$

where $\zeta^h + \zeta^l + \zeta^{h^P} + \zeta^{l^P} = 1$.

Using these guesses and substituting in equations (6.28) and (6.29) we can see that these conditions always hold.

The remaining Euler equations (6.24), (6.25), (6.26) and (6.27) can be rewritten into:

$$E_t \left[\frac{\frac{r_{t+1}^h + \lambda q_{t+1}^h}{q_t^h}}{\zeta^h \frac{r_{t+1}^h + \lambda q_{t+1}^h}{q_t^h} + \zeta^l \frac{r_{t+1}^l + \lambda q_{t+1}^l}{q_t^l} + \zeta^{h^P} \frac{\left(A + \max\left(\Delta_t^{G,h}, \Delta^h\right)\right) K_{t+1}^{\alpha-1} + \lambda q_{t+1}^h}{q_t^{G,h}} + \zeta^{l^P} \frac{\left(A + \max\left(\Delta_t^{G,l}, \Delta^l\right)\right) K_{t+1}^{\alpha-1} + \lambda q_{t+1}^l}{q_t^{G,l}}} \right] = 1$$

$$E_t \left[\frac{\frac{r_{t+1}^l + \lambda q_{t+1}^l}{q_t^l}}{\zeta^h \frac{r_{t+1}^h + \lambda q_{t+1}^h}{q_t^h} + \zeta^l \frac{r_{t+1}^l + \lambda q_{t+1}^l}{q_t^l} + \zeta^{h^P} \frac{\left(A + \max\left(\Delta_t^{G,h}, \Delta^h\right)\right) K_{t+1}^{\alpha-1} + \lambda q_{t+1}^h}{q_t^{G,h}} + \zeta^{l^P} \frac{\left(A + \max\left(\Delta_t^{G,l}, \Delta^l\right)\right) K_{t+1}^{\alpha-1} + \lambda q_{t+1}^l}{q_t^{G,l}}} \right] = 1.$$

$$E_t \left[\frac{\frac{\left(A + \max\left(\Delta_t^{G,h}, \Delta^h\right)\right) K_{t+1}^{\alpha-1} + \lambda q_{t+1}^h}{q^{G,h}}}{\zeta^h \frac{r_{t+1}^h + \lambda q_{t+1}^h}{q_t^h} + \zeta^l \frac{r_{t+1}^l + \lambda q_{t+1}^l}{q_t^l} + \zeta^{h^P} \frac{\left(A + \max\left(\Delta_t^{G,h}, \Delta^h\right)\right) K_{t+1}^{\alpha-1} + \lambda q_{t+1}^h}{q_t^{G,h}} + \zeta^{l^P} \frac{\left(A + \max\left(\Delta_t^{G,l}, \Delta^l\right)\right) K_{t+1}^{\alpha-1} + \lambda q_{t+1}^l}{q_t^{G,l}}} \right] = 1$$

$$E_t\left[\frac{\frac{\left(A+\max\left(\Delta_t^{G,l},\Delta^l\right)\right)K_{t+1}^{\alpha-1}+\lambda q_{t+1}^l}{q_t^{G,l}}}{\zeta^h\frac{r_{t+1}^h+\lambda q_{t+1}^h}{q_t^h}+\zeta^l\frac{r_{t+1}^l+\lambda q_{t+1}^l}{q_t^l}+\zeta^{h^P}\frac{\left(A+\max\left(\Delta_t^{G,h},\Delta^h\right)\right)K_{t+1}^{\alpha-1}+\lambda q_{t+1}^h}{q_t^{G,h}}+\zeta^{l^P}\frac{\left(A+\max\left(\Delta_t^{G,l},\Delta^l\right)\right)K_{t+1}^{\alpha-1}+\lambda q_{t+1}^l}{q_t^{G,l}}}\right]=1$$

The allocation of saving firms (those with zero-profit projects) between high and low investment projects have to satisfy the market clearing conditions on both primary and secondary markets for high and low projects.

$$\lambda H_t = \zeta^h \beta\left(1-\pi\right)\left(H_t^s(r_t^h+\lambda q_t^h)+L_t^s(r_t^l+\lambda q_t^l)\right)$$

$$\lambda L_t = \zeta^l \beta\left(1-\pi\right)\left(H_t^s(r_t^h+\lambda q_t^h)+L_t^s(r_t^l+\lambda q_t^l)\right)$$

$$\theta\pi\mu\frac{\beta\left(H_t^h(r_t^h+\lambda q_t^h)+L_t^h(r_t^l+\lambda q_t^l)\right)}{\left(1-\theta q_t^{G,h}\right)} = \zeta^{h^p}\left(1-\pi\right)\beta\left(H_t^s(r_t^h+\lambda q_t^h)+L_t^s(r_t^l+\lambda q_t^l)\right)$$

$$\theta\pi\left(1-\mu\right)\frac{\beta\left(H_t^h(r_t^h+\lambda q_t^h)+L_t^h(r_t^l+\lambda q_t^l)\right)}{\left(1-\theta q_t^{G,h}\right)} = \zeta^{h^l}\left(1-\pi\right)\beta\left(H_t^s(r_t^h+\lambda q_t^h)+L_t^s(r_t^l+\lambda q_t^l)\right)$$

And the goods market clears too $Y_t = C_t + I_t$.

6.3 Numerical solutions of the stochastic dynamic system

To solve the fully stochastic dynamic model I use numerical approximation methods. Since depending on the state variables the economy is switching between separating and pooling equilibrium I am using global approximation methods. In particular I look for the values of the following functions:

$\Gamma_1\left(A_t, K_t, \omega_t\right) = q_t^h$

$\Gamma_2\left(A_t, K_t, \omega_t\right) = q_t^l$

I construct a grid for the three aggregate states A, K, ω and start with the guess equal to the steady-state values. Then I iterate using the set of equilibrium conditions to find the updated values of Γ_1, Γ_2 until the updated values are close to the previous guesses. During iteration at each point of the grid it is evaluated whether the economy is in separation or pooling equilibrium and the points out of grid are obtained through trilinear interpolation.

The whole model is solved for a particular parameter ϕ which defines the limit of the credible commitment that can be sustained by reputation. Remember that under the simplifying assumption firms who default on implicit recourse can liquidate the firm and transfer $(1-\phi)$ fraction of their funds to a new firm with no previous record of default on implicit recourse. I have to check whether this option will be preferred by defaulting firms to the option of being punished by investing firms i.e. cannot sell on the securitization market in the future (trigger strategy). This needs me to check condition (3.10). To do that I need to solve for the value functions given my choice for ϕ. I use value function iteration method to get the solution. Note that with lower ϕ the credibility constraint does not allow for high Δ^{Gcred}. Therefore the profits from securitization are also larger and the difference between V^{ND} and V^{D} grows larger too. So for sufficiently low ϕ the condition (3.10) is satisfied.

6.4 Equilibrium switching

The chapter 4 shows how in boom stage the economy can switch from a separating equilibrium to a pooling equilibrium. In this sub-chapter I show the mechanism behind analytically for a simplified case where $\lambda = 0$.

It can be shown that if one increases the productivity parameter or after positive productivity shocks Δ_t^G sufficient to avoid mimicking by firms with low investment opportunities is increasing. However, there is a limit to what a firm can credibly promise which is given by the mentioned incentive compatible conditions.

The non default condition can then be written as follows:

$$\phi\left(1-\theta\right)i_t E_t\left(A_{t+1}+\Delta^h\right)K_{t+1}^{\alpha-1} \leq \left(\triangle^G-\Delta^h\right)K_{t+1}^{\alpha-1}\theta i_t$$

I define ϕ_t^{min} as the minimum share of expected future wealth that needs to be promised as implicit recourse in order to satisfy the condition for the separating equilibrium:

$$\phi_t^{min} \equiv \frac{\left(\triangle^{G,min}-\Delta^h\right)\theta}{E_t\left(A_{t+1}+\Delta^h\right)\left(1-\theta\right)} = \frac{\left(1-\pi\mu-\theta\right)E_tA_{t+1}-\Delta^h\theta}{E_tA_{t+1}+\Delta^h}$$

The second equality was obtained from the equation $q^{h,IR} = \left(1-\pi\mu\right)/\theta$ and $\Delta^{G,min} = E_tA_{t+1}\left(q^{h,IR}-1\right)$ which is the minimum level of promise needed to achieve a separating equilibrium. It can be easily shown that with higher expected productivity also the higher has to be the value of the promise, which eventually surpasses the maximum possible threshold ϕ.

$$\frac{\partial \phi_t^{min}}{\partial E_t A_{t+1}} > 0$$

7 References

Adrian, T. and Shin, H. (2009). Money, liquidity, and monetary policy. *American Economic Review: Papers and Proceedings*, Vol. 99, No.2, pp. 600-605.

Akerlof, G. and Shiller, R. (2009). Animal spirits. Princeton University Press, Princeton.

Bernanke, B. (2010). Economic challenges: Past, present and future. A speech at the Dallas Regional Chamber, Dallas, Texas, April 7, 2010. Available at Board of Governors of the Federal Reserve System (U.S.)

http://www.federalreserve.gov/newsevents/speech/20100407a.htm

Bernanke, B. and Gertler, M. (1989). Agency costs, net worth, and business cycle fluctuations. *American Economic Review*, Vol. 79, No. 1, pp. 14-31.

Bernanke, B. , Gertler, M. and Gilchrist, S. (1999). The financial accelerator in a quantitative business cycle framework. In: Taylor, J. and Woodford, M. (Eds.), Handbook of Macroeconomics, Volume 1. Elsevier Science B.V., pp. 1341-1393.

BIS (2010). Basel III: towards a safer financial system. A speech by Jaime Caruana, General Manager of the Bank for International Settlements. available at Bank for International Settlements http://www.bis.org/speeches/sp100921.pdf

Bloom, N. (2009). The impact of uncertainty shocks. *Econometrica*, Vol. 77, No. 3, 623-685.

Bloom, N. Floetotto, M., Jaimovich, N. Sapora-Eksten, I. and Terry, S. (2011). Really uncertain business cycles. Mimeo.

Brunnermeier, M. (2009). Deciphering the liquidity and credit crunch 2007-2008. *Journal of Economic Perspectives*, Vol. 23, pp. 77-100.

Brunnermeier, M. and Sannikov, Y. (2011). A macroeconomic model with a financial sector. Mimeo, Princeton University.

Calomiris, C. and Kahn, C. (1991). The role of demandable debt in structuring optimal banking arrangements. *American Economic Review*, Vol. 81., pp. 497-513.

Calomiris, C. and Mason, J. (2004). Credit card securitization and regulatory arbitrage. *Journal of Financial Services Research*, Vol. 26, pp. 5-27.

Cho, I.-K. and Kreps, D. M. (1987). Signaling Games and Stable Equilibria. *The Quarterly Journal of Economics*, Vol. 102, No. 2, pp.179-222.

Christiano, L., Motto, R. and Rostagno, M. (2009). Financial factors in business fluctuations. Mimeo, Northwestern University.

Curdia, V. and Woodford, M (2009a). Credit spreads and monetary policy. Mimeo, Federal Reserve Bank of New York and Columbia University.

Curdia, V. and Woodford, M (2009b). Conventional and unconventional monetary policy. Mimeo, Federal Reserve Bank of New York and Columbia University.

Fender , I. and Mitchell, J. (2009). Incentives and tranche retention in securitization: a screening model. *BIS Working Paper* No. 289, pp. 1-41.

Gertler, M. and Karadi, P. (2009). A model of unconventional monetary policy. Mimeo. New York University.

Gertler, M. and Kiyotaki, N. (2010). Financial intermediation and credit policy in business cycle analysis. Prepared for the Handbook of Monetary Economics. Mimeo. New York University and Princeton University, pp. 1-64.

Gilchrist, S., Yankov, V. and Zakrasjek, E. (2009). Credit market shocks and economic fluctuations: Evidence from corporate bond and stock markets. mimeo, Boston University.

Gordon, R. (2003). Exploding productivity growth: context, causes, and implications. *Brooking Papers on Economic Activity*, Vol. 34, pp. 207-298.

Gorton, G. and Pennacchi, G. (1995). Banks and loan sales: Marketing nonmarketable assets. *Journal of Monetary Economics* 35, pp. 389-411.

Gorton, G. and Souleles, N. (2006). Special purpose vehicles and securitization. In: Carey, M. and Stulz, R. (Eds.), The risks of financial institutions. University of Chicago Press, Chicago, pp. 431-552.

Gorton, G. and Metrick, A. (2010). Regulating the shadow banking system. Working paper. Available on SSRN

http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1676947

Gorton,G. and Ordognez,G. (2012). Collateral crises. NBER Working Papers 1777

Higgins, E. and Mason, J. (2004). What is the value of recourse to asset-backed securities? A clinical study of credit card banks. *Journal of Banking and Finance*, Vol. 28, pp. 875-899.

Kiyotaki, N. and Moore, J. (1997). Credit cycles. *Journal of Political Economy*, Vol. 105, No. 2, pp. 211- 248.

Kiyotaki, N. and Moore, J. (2012). Liquidity, business cycles and monetary policy. Mimeo. Princeton University and LSE.

Kurlat, P. (2011). Lemons Markets and the Transmission of Aggregate Shocks. Mimeo. Stanford University.

Mason, J. and Rosner, J. (2007). Where did the risk go? How misapplied bond ratings cause mortgage backed securities and Collateralized Debt Obligation market disruptions. SSRN Working Paper series, pp. 1-87.

Ordognez, G. (2012). Confidence banking. Mimeo. Yale University.

Paligorova, T. (2009). Agency conflicts in the process of securitization. *Bank of Canada Review*, Autumn 2009, pp. 33-47.

Shin, H. (2009). Securitization and financial stability. *The Economic Journal*, Vol. 119, pp. 309-332.

Shleifer, A. and Vishny, R.W. (1997). The limits of arbitrage. *The Journal of Finance*, Vol. 52, No. 1, pp. 35-55.

Shleifer, A. and Vishny, R. (2010). Unstable banking. *Journal of Financial Economics,* Vol. 97, pp. 306-318.

Veldkamp, L. (2005). Slow boom, sudden crash. Journal of Economic Theory 124(2), 230257.