Commodity Currencies and Commodity Prices: Modeling Static and Time-Varying Dependence

This version (WORK IN PROGRESS): 15 September 2012

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Abstract

This paper employs copula approach in order to study the relationship between combinations of the exchange rates and commodity prices for large commodity exporters. We use daily data for the nominal exchange rates of the four commodity currencies (Australian, Canadian, and New Zealand dollars, and Norwegian krone) against the U.S. dollar and the relevant country-specific commodity price indices. We find positive dependence structure for all pairs of the exchange rates indicating that currencies tend to appreciate or depreciate jointly against the U.S. dollar. We also find positive dependence between the values of currencies and commodity indices with a pronounced increase in the time-varying dependence from the beginning of the global financial crisis. For most combinations of the exchange rates and commodity indices we observe no major asymmetries in the tail dependence.

Key words: independently floating exchange rates, commodity prices, dependence modeling, copulas

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1 Introduction

The objective of this paper is to examine the dependence structure between the pairs of exchange rates or commodity indices as well as between the pairs of exchange rates and the relevant country-specific commodity indices using copulas. Although the copula methodology has been increasingly popular in statistics and finance, there has been only limited use of this approach in economics. One reason why copulas have not yet received serious attention in economic literature is because most macroeconomic variables are only available at low frequencies (annual, quarterly or monthly) with only a few available at much higher frequencies. In our analysis we construct daily commodity price indices for the four OECD countries: Canada, Norway, Australia and New Zealand for the period from 2000 to 2010. Then, using this high frequency data we study co-movement between exchange rates of four commodity currencies and commodity price indices in the very short-run, i.e. on a daily basis. We explore if there are any asymmetries in dependence between the pairs of the exchange rates and commodity indices as well as exchange rates and relevant commodity indices. We also study how dependence varies over time.

Copula is a multivariate distribution function which allows full description of the dependence structure between two or more variables of interest (refer to Nelsen (1998) for introduction and Patton (2002) for applications). One of the advantages of using copulas is that the marginal distribution of underlying variables (univariate time series of exchange rates and commodity indices in each of the countries) can be modeled separately from the dependence structure between the marginals, modeled via the copula function. Furthermore, copula approach allows modeling the possibly non-linear dependence structure and in particular, many copula families allow for dependence in the extreme tails, a property known as tail dependence. Capturing asymmetries is important since the presence of asymmetric dependence may indicate that linear models are not entirely appropriate for the data at hand, and that non-linearities have to be taken into account. Finally, using copulas allows avoiding possible endogeneity issue in modeling exchange rates and commodity prices. Chen and Rogoff (2003) indicate that endogeneity can operate through the market power commodity-exporting countries may hold in the world commodity markets. Another possible source of endogeniety may be the presence of a third variable which would affect both commodity prices and exchange rates simultaneously.[1] The endogeniety issue issue was addressed in the literature in several ways. Chen and Rogoff (2003) used a broader 'world commodity price index' as an instrument for the country production-weighted price index while Clements and Fry (2008) modeled exchange rates and commodity prices simultaneously allowing for spillovers between them.

There has been recent interest in the empirical literature dealing with the behavior of commodity currencies and their connection to the commodity prices. A commodity currency is a name given to currencies of countries with a large proportion of primary commodities in their exports, and generally refer to the Australian dollar, Canadian dollar, New Zealand dollar, Norwegian krone, South African rand, Brazilian real, and the Chilean peso.

One strand in the literature looks at the relationship between the prices of primary commodities and the values of real and nominal exchange rates for commodity-exporting countries. For instance, Amano and van Norden (1995) look at the relationship between the Canadian/U.S. dollar real exchange rate and the terms of trade and find that they are cointegrated, that causality runs from the terms of trade to the exchange rate and that terms of trade are useful in forecasting the exchange rate. The authors also report a

[1] An increased growth rate of an emerging economic superpower such as China may be one example of such a variable.

surprising result that increases in energy commodity prices tend to weaken the Canadian dollar. This finding is challenged by Issa et al. (2008) who apply structural break tests and find a break point in the sign of this relationship, which changes from negative to positive in the early 1990s. They argue that the timing of the break is consistent with major changes in Canada's energy policies and in energy-related cross-border trade and investment. Chen and Rogoff (2003) look at the correlations between real exchange rates and commodity price indices in three commodity-exporting OECD economies: Australia, Canada, and New Zealand. They find a strong and stable effect of commodity prices on the real exchange rate for Australia and New Zealand (with commodity price elasticity estimates between 0.5 and 1). The results for Canada are found to be different with a presence of a long-run cointegrating relationship between commodity price index and the real exchange rate but relatively weak co-movement in the shorter run.

Some recent studies look at whether commodity prices are useful in forecasting exchange rates and vice versa. The general convention starting from the studies of Meese and Rogoff (1983a) and Meese and Rogoff (1983b) is that economic variables are essentially useless in forecasting the exchange rates. Chen et al. (2010) explore the dynamic relationship between commodity price movements and exchange rate fluctuations of several commodity currencies, namely, the Australian, Canadian, and New Zealand dollars, as well the South African rand and the Chilean peso. After controlling for time-varying parameters, they find a robust relationship and demonstrate that exchange rates are very useful in forecasting future commodity prices but not the other way around. The authors offer an explanation based on the fact that exchange rates are strongly forward looking, whereas commodity price fluctuations are typically more sensitive to short-term demand. The authors document that exchange rates of commodity currencies predict primary commodity prices both in-sample and out-of-sample; however, the out-of-sample predictive ability in the reverse direction (namely, the ability of the commodity price index to predict nominal exchange rates) is not strong at the quarterly frequency that they consider. Ferraro et al. (2011) explore whether oil prices have a reliable and stable out-of-sample relationship with the Canadian/U.S. dollar nominal exchange rate. While there is little evidence of systematic relation between oil prices and the exchange rate at the monthly and quarterly frequencies, the relationship is found to be robust at the daily frequency. The effects of changes in oil prices are immediately translated into changes in exchange rates and are very short-lived. It is shown that oil prices contain valuable information for predicting exchange rates out-of-sample in Canada (which is a significant oil exporter). This finding seems to overturn an important conventional result of unpredictability of the nominal exchange rate in the literature.

There has been a small separate strand in literature studying dependence structure of the exchange rates of different currencies using copulas. Patton (2006) explores whether there is an asymmetry in a model of the dependence between the Deutsche mark and the Japanese yen, and finds that the mark/dollar and yen/dollar exchange rates are more correlated when they are depreciating against the dollar than when they are appreciating. Benediktsdottir and Scotti (2009) extend the results of Patton (2006) to several pairs of currencies and also find asymmetry in the tail dependence. However, these studies do not generally focus on commodity currencies and do not model the relationship between the exchange rates and the relevant country-specific commodity indices.

This paper is organized as follows. Section 2 describes data used in our empirical analysis and deals with the construction of commodity price indices. Section 3 reviews the main concepts of dependence modeling and provides a brief overview on copula models. Section 4 presents the methods applied for estimating the marginal time series, and describes the results for each of the univariate time series of exchange rates and commodity currencies. Section 5 fits static and dynamic copulas to the data, providing the results on the multivariate dependence structure between the exchange rates and the relevant country-specific commodity indices using copulas. Finally, Section 6 concludes.

Table 1
Major commodity exports by year (2001, 2005, 2009) for Canada, Norway, Australia and New Zealand.

Canada	2001	2005	2009	Australia	2001	2005	2009
natural gas	0.23587	0.26512	0.14322	coal	0.19660	0.28822	0.30145
wood	0.20255	0.16489	0.08211	crude oil	0.10260	0.08325	0.05518
paper/newsprint	0.15781	0.10724	0.08732	copper	0.09527	0.06571	0.05740
crude oil	0.13566	0.20468	0.33348	iron ore	0.08240	0.14524	0.23032
pulp	0.06205	0.04319	0.03926	gold	0.08118	0.07674	0.11535
aluminium	0.04169	0.03685	0.03787	wheat	0.07089	0.04148	0.03858
wheat	0.03429	0.01961	0.04853	meat/bovine	0.07050	0.06171	0.03315
copper	0.02104	0.02706	0.03207	aluminium	0.07003	0.06106	0.03673
swine	0.01939	0.02086	0.01896	natural gas	0.05847	0.06055	0.06670
nickel	0.01906	0.02910	0.02691	wool	0.05530	0.03053	0.01413
gold	0.01869	0.02787	0.06513	cotton	0.03256	0.01385	0.00425
meat/bovine	0.01726	0.01209	0.00891	nickel	0.03012	0.01694	0.01113
coal	0.01539	0.02191	0.03869	zinc	0.02678	0.02330	0.01402
zinc	0.01120	0.00858	0.01130	wood	0.01601	0.01597	0.00899
iron ore	0.00806	0.01095	0.02624	lead	0.01128	0.01545	0.01261
Total	1.00	1.00	1.00	Total	1.00	1.00	1.00
Norway	2001	2005	2009	New Zealand	2001	2005	2009
crude oil	0.60355	0.56672	0.43385	milk products	0.19458	0.18856	0.22666
natural gas	0.17021	0.21817	0.34733	meat (sheep, goat)	0.11206	0.14708	0.13318
fish	0.07186	0.05698	0.07333	wood	0.09820	0.09223	0.08847
aluminium	0.05492	0.04722	0.03884	meat/bovine	0.09158	0.11243	0.08459
petroleum oil (not crude)	0.04014	0.05128	0.05068	cheese/curd	0.07277	0.06456	0.06016
wood	0.02394	0.01571	0.01334	casein	0.06647	0.03939	0.03697
iron ore	0.01033	0.01520	0.01544	aluminium	0.05263	0.05271	0.02929
nickel	0.00940	0.01521	0.01348	butter	0.05245	0.05633	0.06660
pulp	0.00417	0.00286	0.00317	fruit	0.05043	0.07010	0.07118
zinc	0.00282	0.00245	0.00265	fish	0.04839	0.03979	0.03524
platinum	0.00273	0.00151	0.00189	wool	0.04678	0.04646	0.02719
copper	0.00236	0.00322	0.00267	leather	0.04199	0.02456	0.01519
cobalt	0.00165	0.00204	0.00123	crude oil	0.03208	0.02372	0.07609
cheese/curd	0.00146	0.00108	0.00093	pulp	0.02811	0.02734	0.02508
gold	0.00046	0.00036	0.00115	gold	0.01147	0.01474	0.02411
Total	1.00	1.00	1.00	Total	1.00	1.00	1.00

2 Data

We consider the following commodity countries: Canada, Norway, Australia and New Zealand. For each of these countries we construct a commodity price index on a daily basis for the time period from January 2001 to December 2009 which is based on the product exports by the respective country in the considered time period. The source for the calculations, that is, the list of products exported by each country, is based on the statistics from the *United Nations Commodity Trade Statistics Database (UNcomtrade)* available from *Index Mundi.*[2] For each country, we choose to include 15 exported products with the highest share value in the total value of country's exports. The prices for corresponding commodities used for calculation of the returns are obtained from the *Datastream Thomson Financial*. Thereby, the weight $w_{t,c}$ assigned to each commodity $c = 1, ..., C$ at time t is based on its exported value at time $t = 1, ..., T$. Thus, we use the following formula to compute commodity index return for each country:

$$Ret_t^{Ind} = \sum_{c=1}^{C} w_{t,c} Ret_t^c, \tag{2.1}$$

where Ret_t^c denotes the log-return on commodity c at time t and $\sum_{c=1}^{C} w_{t,c} = 1$. Note, that while commodity prices are available on a daily basis, the corresponding weights are kept constant throughout the year. Table 1 summarizes major commodity exports for three years corresponding to the beginning (2001), the middle (2005) and the end (2009) of the sample period for each of considered countries.

In addition to commodity indices we consider FX rates between USD and the respective currency of the considered countries (CAD, NOK, AUD, NZD). The FX rate data cov-

[2] http://www.indexmundi.com

Table 2
Pearson linear correlation for FX rate returns (top panel) and commodity index returns (bottom panel) for Canada, Norway, Australia and New Zealand taken for the time period from January 2001 to December 2009.

	Pearson correlation coefficient for FX rates			
	CAD/USD	NOK/USD	AUD/USD	NZD/USD
CAD/USD	1.0000000			
NOK/USD	0.5262917	1.0000000		
AUD/USD	0.6215109	0.5964903	1.0000000	
NZD/USD	0.5539772	0.5850330	0.8498547	1.0000000
	Pearson correlation coefficient for commodity indices			
	Canada	Norway	Australia	New Zealand
Canada	1.0000000			
Norway	0.8778622	1.0000000		
Australia	0.8056709	0.7857159	1.0000000	
New Zealand	0.2884113	0.3176536	0.3928256	1.0000000

ering time period from January 2001 to December 2009 are obtained from *Datastream Thomson Financial*.

Figures 1 and 2 represent FX rates and commodity indices, respectively, for commodity countries Canada, Norway, Australia and New Zealand (from top to bottom). The left panels show the respective data in levels. One can even visually observe the apparent correlations between commodity prices and various exchange rates: when commodity price index increases, the corresponding FX rate (expressed in units of the respective currency per USD) decreases, that is, the currency appreciates. As can be seen from the graphs, both time series appear to be highly persistent and possibly non-stationary. Thus, we construct the log-returns plotted in the right panel of the figures.

Table 2 reports Pearson (linear) correlation for the returns of FX rates and commodity indices. One can observe a strong positive correlation between exchange rates as well as commodity indices for all considered countries. The highest FX rates correlation of nearly 85% is observed between AUD/USD and NZD/USD, while the lowest linear correlation of approximately 52.6% is between the exchange rates CAD/USD and NOK/USD. Interestingly, the opposite relation is observed between the corresponding commodity indices: commodity exports for Australia and New Zealand experience relatively low correlation of 39.2% while Norway and Canada exports appear to be highly correlated with the correlation coefficient of 87.8%. These preliminary results suggest that apart from commodity prices there are more factors affecting exchange rates. In the following sections, we explore further how strong the apparent correlations are, by using copulas in order to model a possibly non-linear dependence structure between the time series.

3 Empirical Methodology for the Copulas

This section focuses on copula methodology, the main concepts of dependence modeling and estimation techniques using copulas.

Copulas are multivariate distribution functions which allow connecting d one-dimensional uniform-(0,1) marginals to a joint cumulative distribution. According to *Sklar's theorem*, if F is a d-dimensional distribution function with marginals $F_1 \ldots, F_d$, then there exists a copula C with

$$F(x_1, \ldots, x_d) = C\{F_1(x_1), \ldots, F_d(x_d)\} \tag{3.1}$$

for every $x_1, \ldots, x_d \in \overline{\mathbb{R}}$. Thus, if $X = (X_1, \ldots, X_d)^\top$ is a random vector with distribution $X \sim F_X$ and continuous marginals $X_j \sim F_{X_j}$ $(j = 1, \ldots d)$, then the copula of X is the

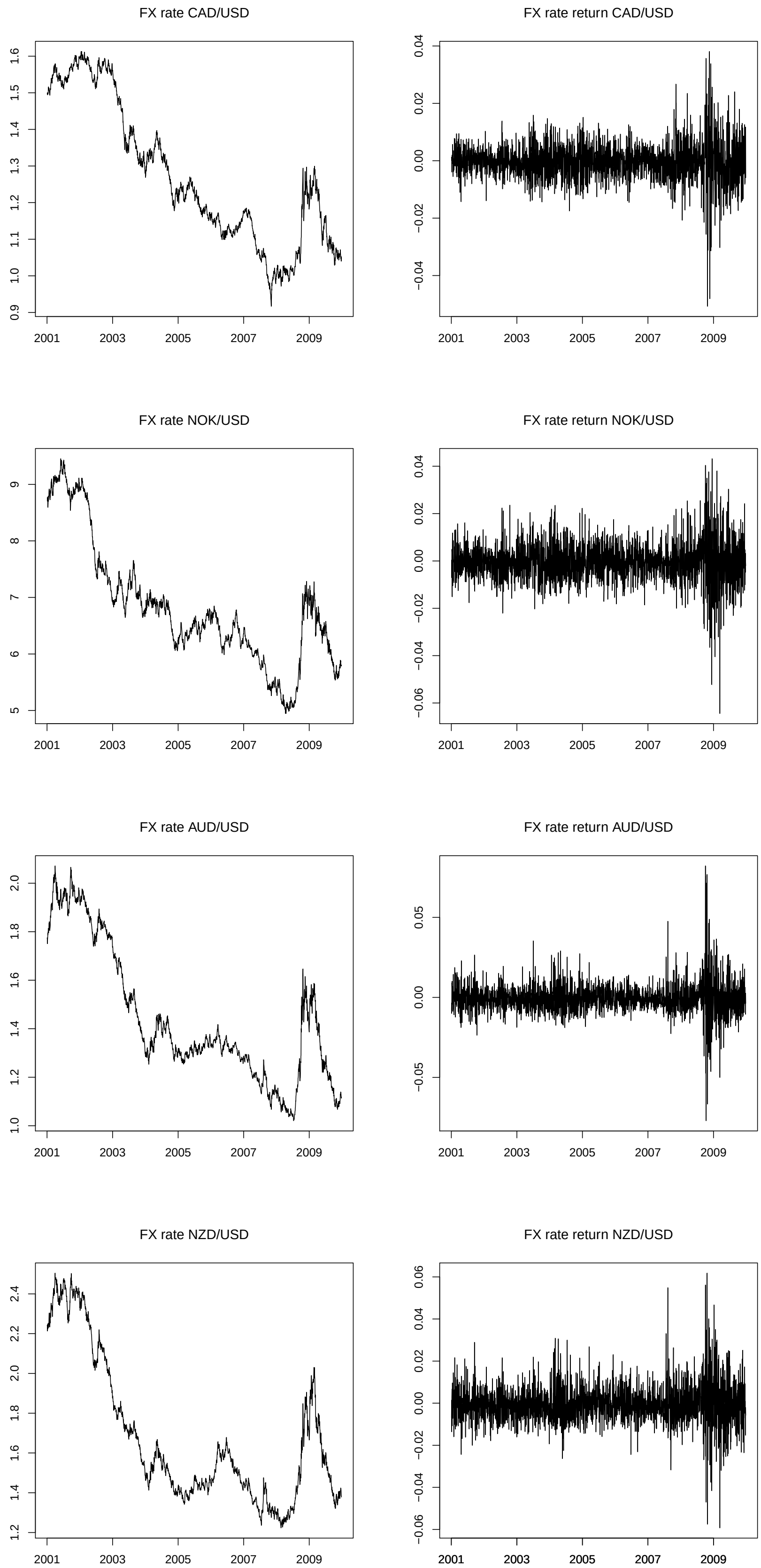

Fig. 1. FX rate levels (left panel) and returns (right panel) for CAD/USD, NOK/USD, AUD/USD and NZD/USD (from top to bottom) for the time period from January 2001 to December 2009.

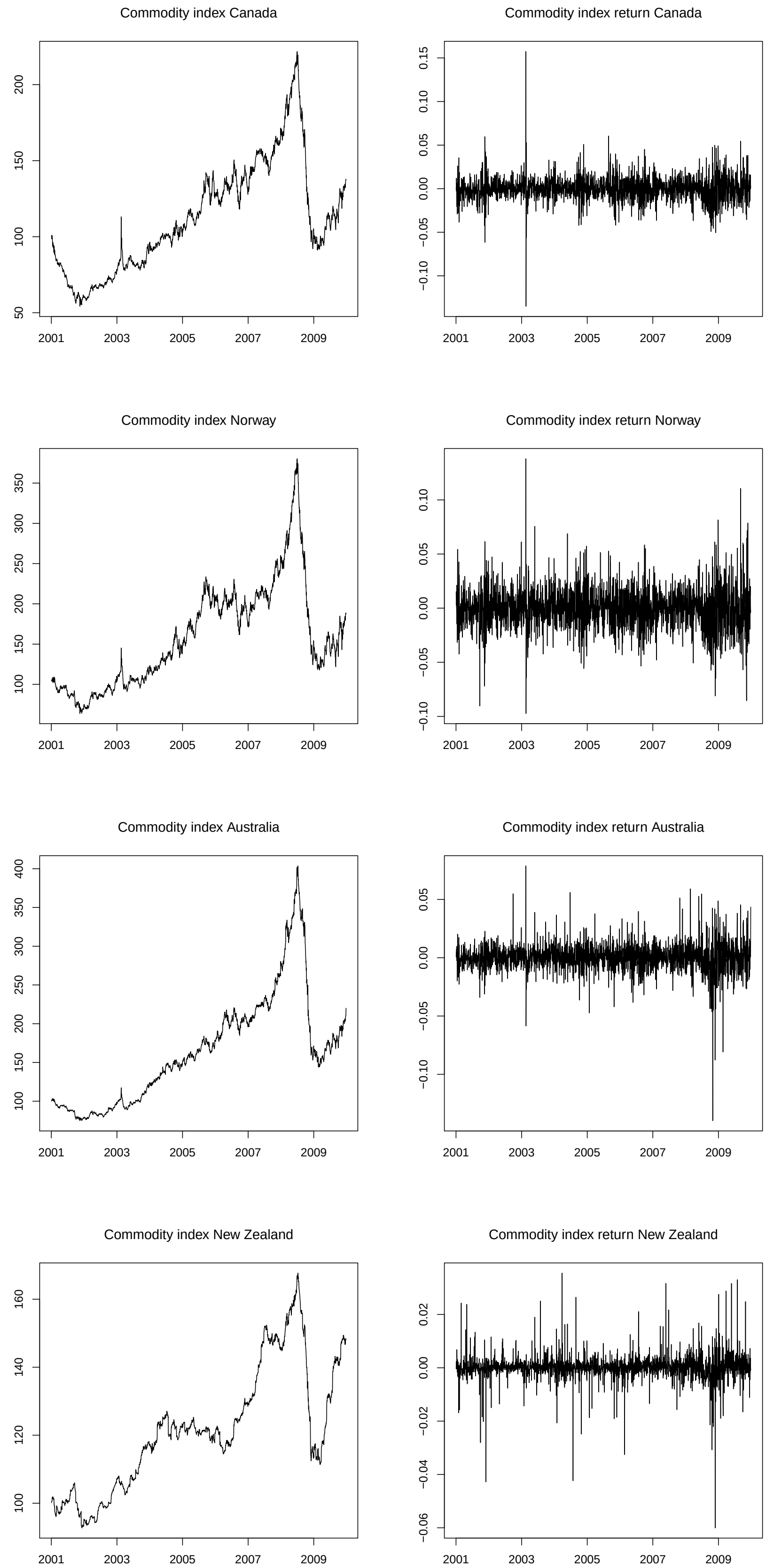

Fig. 2. Commodity index levels (left panel) and returns (right panel) for Canada, Norway, Australia and New Zealand (from top to bottom) for the time period from January 2001 to December 2009.

distribution function C_X of $u = (u_1, \dots, u_d)^\top \in [0,1]^d$, where $u_j = F_{X_j}(x_j)$:

$$C_X(u_1, \dots, u_d) = F_X\{F_{X_1}^{-1}(u_1), \dots, F_{X_d}^{-1}(u_d)\}. \tag{3.2}$$

Further formal details can be found in Nelsen (1998).

Throughout the paper we will also use a notion of a *survival copula* C^* corresponding to a copula C from the Archimedean copulas family:

$$\overline{F}(x_1, \dots, x_d) = C^*\{\overline{F}_1(x_1), \dots, \overline{F}_d(x_d)\} \tag{3.3}$$

where $\overline{F}(x_1, \dots, x_d) = P(X_1 > x_1, \dots X_d > x_d)$. C^* For the bivariate case it can be defined as follows:

$$C^*(u_1, u_2) = 1 - u_1 - u_2 + C(1 - u_1, 1 - u_2), \tag{3.4}$$

see Nelsen (1998).

The concept of copula is important in relation to measuring non-linear dependence between random variables (Embrechts et al., 2001; Dias, 2004). This includes, for example, extreme dependence in the tails of the multivariate distribution. For (U_1, U_2) denoting a pair of uniform variables on the unit square $[0,1]^2$, the upper tail dependence coefficient $\lambda_u \in [0,1]$ is defined as

$$\lambda_u = \lim_{u \to 1-} P(U_1 > u | U_2 > u) = \lim_{u \to 1-} \frac{C^*(u,u)}{1-u}. \tag{3.5}$$

Similarly, the lower tail dependence coefficient $\lambda_l \in [0,1]$ is defined as

$$\lambda_l = \lim_{u \to 0+} P(U_1 \leq u | U_2 \leq u) = \lim_{u \to 0+} \frac{C(u,u)}{u}. \tag{3.6}$$

If λ_u falls into the interval $(0,1]$, then U_1 and U_2 are said to be asymptotically dependent in the upper tail, and if $\lambda_u = 0$, then U_1 and U_2 are said to be asymptotically independent in the upper tail. Similarly, if $\lambda_l \in (0,1]$ or $\lambda_l = 0$, then U_1 and U_2 are said to be asymptotically dependent, or independent, respectively, in the lower tail. Hu (2006) reviews dependence and tail dependence measures for mixture copula models presented below.

Throughout the paper we will concentrate mostly on two popular copula families: the *elliptical copulas family*, which include the Gaussian copula and the Student-t copula, and the *Archimedean copulas family*, which has Gumbel and Clayton copulas as special members. For the purpose of completeness we also consider some mixture models of Archimedean copulas where the distribution function has the form of a convex combination of two or more copulas. These d-dimensional parametric copulas are presented below. A copula parameter controls the degree of dependence. Joe (1993) and Nelsen (1998) can be referred to for more details on different copula models.

3.1 Elliptical Copulas

Elliptical copulas have a dependence structure generated by the elliptical distributions, see e.g. Lindskog et al. (2001). These include normal and Student-t distributions. The modeling of dependency using elliptical distributions can be found in e.g. Hult and Lindskog (2001), Fang et al. (2002) and Frahm et al. (2003). Its applications in finance and risk management are discussed, for instance, in Breymann et al. (2003), McNeil et al. (2005) and Dias and Embrechts (2008). The Gaussian copula and Student-t copula are presented below.

The Gaussian copula generates the dependence structure given by the multivariate normal distribution, and allows to choose any marginal distribution. For the case of normal marginals, that is, if $X_j \sim N(0,1)$ and $X = (X_1, \ldots, X_d)^\top \sim N_d(0, \Psi)$, where Ψ denotes a correlation matrix, an explicit expression for the Gaussian copula is given by

$$C_\Psi^{Ga}(u_1, \ldots, u_d) = F_X\{\Phi^{-1}(u_1), \ldots, \Phi^{-1}(u_d)\}. \tag{3.7}$$

The Student-t copula generates the dependence structure from the multivariate Student-t distribution. If $X = (X_1, \ldots, X_d)^\top \sim t_d(\nu, \mu, \Sigma)$ has a multivariate Student-t distribution with ν degrees of freedom, mean vector μ and positive-definite dispersion or scatter matrix Σ, the Student-t copula is given by

$$C_{\nu,\Psi}^t(u_1, \ldots, u_d) = t_{\nu,\Psi}\{t_\nu^{-1}(u_1), \ldots, t_\nu^{-1}(u_d)\}, \tag{3.8}$$

where t_ν^{-1} is the quantile function from the univariate t-distribution, Ψ is the correlation matrix associated with Σ [3]. The Student-t copula allows one to generate symmetric tail dependence with the tail dependence coefficients defined by

$$\lambda_u = \lambda_l = 2\left(-t_{\nu+1}\sqrt{(\nu+1)(1-\rho)/(1+\rho)}\right), \tag{3.9}$$

where t_ν denotes the Student-t distribution function, ν is the number of degrees of freedom, and ρ is the correlation coefficient.

3.2 Archimedean Copulas

Gumbel and Clayton copulas belong to the family of so-called Archimedean copulas [4] which have a simple closed form and are briefly reviewed below.

The *Clayton copula* with the dependence parameter $\theta \in (0, \infty)$ is defined by

$$C_\theta(u_1, \ldots, u_d) = \left\{\left(\sum_{j=1}^{d} u_j^{-\theta}\right) - d + 1\right\}^{-1/\theta}. \tag{3.10}$$

As the copula parameter θ tends to infinity, the dependence becomes maximal and as θ tends to zero, we have independence. The Clayton copula can mimic lower tail dependence with the tail dependence coefficient $\lambda_l = 2^{-1/\theta}$ but no upper tail dependence, that is, $\lambda_u = 0$.

[3] Since copula functions remain invariant under any series of strictly increasing transformations of X, such as e.g. standardization of the marginal distributions, see Nelsen (1998), the copula of a $t_d(\nu, \mu, \Sigma)$ distribution is identical to that of a $t_d(\nu, 0, \Psi)$.

[4] Applications of Archimedean copulas to modeling portfolio credit risk have been studied e.g. in McNeil et al. (2005), Dias (2004) and Wu et al. (2006).

The *Gumbel copula* with dependence parameter $\theta \in [1, \infty)$ is given by

$$C_\theta(u_1, \dots, u_d) = \exp\left[-\left\{\sum_{j=1}^{d}(-\log u_j)^\theta\right\}^{1/\theta}\right]. \tag{3.11}$$

For $\theta > 1$ this copula generates an upper tail dependence with tail dependence coefficient $\lambda_u = 2 - 2^{1/\theta}$ but no lower tail dependence, that is, $\lambda_l = 0$. For $\theta = 1$ Gumbel copula reduces to the product copula (i.e. independence): $C_\theta(u_1, \dots, u_d) = \prod_{j=1}^{d} u_j$. Maximal dependence is achieved when θ tends to infinity.

In addition to the Archimedean copulas discussed above we also consider some *mixture models* of Archimedean copulas as introduced in Joe (1993) and used for modeling dependencies across international financial markets by Angel Canela and Pedreira Collazo (2006) and Hu (2006). Mixture copula be obtained by building convex combinations of two or more copulas. Denoting C^A and C^B copulas with dependence parameters θ_1 and θ_2, respectively, the mixture model takes the following form:

$$C_X(u_1, \dots, u_d, \theta) = \theta_3 C_X^A(u_1, \dots, u_d, \theta_1) + (1 - \theta_3) C_X^B(u_1, \dots, u_d, \theta_2). \tag{3.12}$$

We will consider four mixture models when modeling dependencies between FX rates and commodity indices analyzed in Dias (2004), that is, Clayton & survival Clayton, Clayton & Gumbel, survival Clayton & survival Gumbel, and Gumbel & survival Gumbel. The advantage of mixture models compared to one-parametric copula models is the ability to generate asymmetric dependence in the tails of the multivariate distribution. Note that the mixtures Clayton & survival Clayton as well as Gumbel & survival Gumbel copulas nests symmetry when, jointly, θ_3 is equal to 0.5 and $\theta_1 = \theta_2$, see Manner (2010). The other two mixture copulas do not allow the direct comparison of parameters in each part of the mixture and thus, are considered here just for illustration purpose.

3.3 Copula Estimation

Generally, the maximum likelihood technique used for estimation of parametric copulas. From (3.1), the density of the random vector $X = (X_1, \dots, X_d)^\top$ is given by

$$f(x_1, \dots, x_d; \delta_1, \dots, \delta_d, \theta) = c\{F_{X_1}(x_1; \delta_1), \dots, F_{X_d}(x_d; \delta_d); \theta\} \prod_{j=1}^{d} f_j(x_j; \delta_j), \tag{3.13}$$

where

$$c(u_1, \dots, u_d) = \frac{\partial^d C(u_1, \dots, u_d)}{\partial u_1 \dots \partial u_d} \tag{3.14}$$

is a copula density. Denoting a vector of parameters $\alpha = (\delta_1, \dots, \delta_d, \theta)^\top \in \mathbb{R}^{d+1}$ the likelihood function can be written as:

$$L(\alpha; x_1, \dots, x_T) = \prod_{t=1}^{T} f(x_{1,t}, \dots, x_{d,t}; \delta_1, \dots, \delta_d, \theta). \tag{3.15}$$

Combining (3.13) and (3.15), get the corresponding log-likelihood function:

$$\ell(\alpha; x_1, \dots, x_T) = \sum_{t=1}^{T} \ln \left[c\{F_{X_1}(x_{1,t}; \delta_1), \dots, F_{X_d}(x_{d,t}; \delta_d); \theta\}\right] + \sum_{t=1}^{T} \sum_{j=1}^{d} \ln \left[f_j(x_{j,t}; \delta_j)\right]. \quad (3.16)$$

To maximize this log-likelihood function numerically, we use the *inference for marginals* (IFM) method, which is a sequential two-step maximum likelihood method, see e.g. McLeish and Small (1988) and Joe (1997). Parameters from the marginals are estimated in the first step as

$$\hat{\delta}_j = \arg\max_{\delta} \ell_j(\delta_j), \quad (3.17)$$

where

$$\ell_j(\delta_j) = \sum_{t=1}^{T} \ln f_j \left[x_{j,t}; \delta_j\right] \quad (3.18)$$

is the log-likelihood function for each marginal distribution $j = 1, \dots, d$. Then the parameter estimates are substituted into the copula to obtain the pseudo log-likelihood function

$$\ell(\theta, \hat{\delta}_1, \dots, \hat{\delta}_d) = \sum_{t=1}^{T} \ln \left[c\{F_{X_1}(x_{1,t}; \hat{\delta}_1), \dots, F_{X_d}(x_{d,t}; \hat{\delta}_d); \theta\}\right], \quad (3.19)$$

which is then maximized with respect to θ to obtain the estimator $\hat{\theta}$. The vector of parameter estimates $\hat{\alpha}_{IFM} = (\hat{\delta}_1, \dots, \hat{\delta}_d, \hat{\theta})^\top$ is obtained by solving the following first order condition:

$$(\partial \ell_1 / \partial \delta_1, \dots, \partial \ell_d / \partial \delta_d, \partial \ell / \partial \theta) = 0. \quad (3.20)$$

4 Empirical Methodology for the Marginals

As indicated in the previous section, before estimating a copula we need to specify marginal distributions. It has been shown in previous studies that the returns on financial data (including exchange rates as well as commodity and equity indices) do not follow a normal distribution, since they exhibit heavy tails and excess kurtosis. In order to capture these properties of the data, this paper considers several alternative distributions from the family of *Symmetric Generalized Hyperbolic* (SGH) distributions: the *Student-t*, the *Normal Inverse Gaussian* (NIG), the *Hyperbolic* (HYP) and the *Variance Gamma* (VG) distributions.

4.1 Symmetric Generalized Hyperbolic Distributions

The family of generalized hyperbolic distributions was introduced in Barndorff-Nielsen (1977), and discussed in its general form in Jrgensen (1982), Barndorff-Nielsen and Stelzer (2004), and McNeil et al. (2005). For the purpose of our analysis, we will concentrate on the symmetric representation, that is, when the location of the distribution and the skewness parameter are set equal to zero. [5]

[5] These distributions were used, for instance in Hurst and Platen (1997), Platen and Rendek (2008), Wenbo and Kercheval (2008) and Ignatieva and Platen (2010).

Therefore, we consider the *symmetric generalized hyperbolic* (SGH) density of the form:

$$f_X(x) = \frac{1}{\delta\sigma K_\lambda(\bar{\alpha})}\sqrt{\frac{\bar{\alpha}}{2\pi}}\left(1+\frac{x^2}{(\delta\sigma)^2}\right)^{\frac{1}{2}(\lambda-\frac{1}{2})} K_{\lambda-\frac{1}{2}}\left(\bar{\alpha}\sqrt{1+\frac{x^2}{(\delta\sigma)^2}}\right) \tag{4.1}$$

where $\alpha \neq 0$ if $\lambda \geq 0$ and $\delta \neq 0$ if $\lambda \leq 0$. $K_\lambda(\cdot)$ denotes a modified *Bessel function* of the third kind with index λ, see Abramowitz and Stegun (1972). The parameters λ and $\bar{\alpha}$ can be interpreted as the *shape parameters* for the tails of the distribution. Varying λ and $\bar{\alpha}$ allows one to specify special cases of the SGH distribution. In particular, we will investigate the following important special cases: the *Student-t* distribution ($\bar{\alpha} = 0$ and $\lambda < 0$, see Praetz (1972)), the *Normal Inverse Gaussian* ($\lambda = -0.5$, see Barndorff-Nielsen (1995)), the *Hyperbolic* distribution ($\lambda = 1$, see Eberlein and Keller (1995)) and the *Variance Gamma* distribution ($\bar{\alpha} = 0$ and $\lambda > 0$, see Madan and Seneta (1990)). For the *Student-t* density we only consider $\lambda \leq -1$ in which case the number of degrees of freedom equals $\nu = -2\lambda \geq 2$. [6] Note that in the *Student-t* case the parameter σ is not the standard deviation of the random variable X, which is given by$\sigma_X = \sigma\sqrt{\frac{\nu}{\nu-2}}$. When the number of degrees of freedom ν decreases, we observe an increase in the tail heaviness of the density, which implies a larger probability of extreme values. Additionally, with an increase of the degrees of freedom $\nu \to \infty$, the *Student-t* density converges asymptotically to the Gaussian density. Further details on the representation of the density functions can be found in Platen and Rendek (2008).

In order to choose the distribution from the SGH family that fits the data best, we use the *Anderson-Darling* (AD) distance for the log-returns as a goodness-of-fit statistic:

$$AD = \frac{\sup_{x\in\mathcal{R}} |F_s(x) - \hat{F}(x)|}{\sqrt{\hat{F}(x)(1-\hat{F}(x))}}. \tag{4.2}$$

where $F_s(x)$ denotes the empirical sample distribution and $\hat{F}(x)$ is the estimated distribution. [7]

4.2 An AR - GARCH Model for the Marginals

After specifying the best performing distributional family, we implement an AR-GARCH model (see e.g. Dias (2004), McNeil et al. (2005) and Sun et al. (2006)) to capture distributional characteristics of the FX rates and commodity indices returns which will be used to identify marginals for the joint distribution modeled via copulas. Therefore, for a sequence of i.i.d. random variables $(u_t)_{t\geq 0}$ with zero mean and unit variance, we assume that the log-return process describing commodity indices or FX rates $(X_t)_{t\geq 0}$ follows an AR(1) process

$$X_t = a_0 + a_1 X_{t-1} + \sigma_t u_t, \tag{4.3}$$

[6] We do not consider the case when $-1 < \lambda < 0$, since it corresponds to $\nu < 2$ for which the normalization constant diverges, see Platen and Rendek (2008).

[7] The AD statistic allows capturing the deviations around the median of the distribution, as well as the discrepancies in the tails and is defined as the normalized *Kolmogorov-Smirnov* (KS) distance: $KS = \sup_{x\in\mathcal{R}} |F_s(x) - \hat{F}(x)|$. In general, the drawback of the KS statistic is that it is more sensitive closer to the center of the distribution and fails to capture the tails.

Table 3
Anderson-Darling distance for the FX returns (top panel) and commodity index returns (bottom panel) using different marginal distributions. Estimates refer to daily observations for the period from January 2001 to December 2009. The best performing model is indicated using bold numbers.

			FX rates		
Anderson-Darling	Normal	Student-t	NIG	HYP	VG
Canada	40.1983	0.3370	**0.2992**	1.5849	1.7545
Norway	13.4337	**0.6599**	2.2749	7.4137	9.8018
Australia	82.6119	**0.3828**	1.0682	3.3926	3.1501
New Zealand	27.4392	**0.3781**	1.2586	1.8807	2.0155
			Commodity indices		
Anderson-Darling	Normal	Student-t	NIG	HYP	VG
Canada	53.6504	**0.5900**	13.8587	13.8756	13.7927
Norway	27.5201	**0.3031**	13.9162	13.8523	12.3674
Australia	38.3885	**0.4284**	13.7279	13.5982	13.6732
New Zealand	93.0329	0.3877	**0.2325**	38.6709	26.5425

where innovations $\varepsilon_t = \sigma_t u_t$ have by definition mean zero and the conditional variance $Var(\varepsilon_t|\mathcal{F}_t) = \sigma_t^2$ which is modeled via a GARCH(1,1) process

$$\sigma_t^2 = \alpha_0 + \alpha_1\sigma_{t-1}^2 + \beta_1\varepsilon_{t-1}^2, \tag{4.4}$$

where $\alpha_0 > 0$, $\alpha_1 \geq 0$, $\beta_1 \geq 0$, $\alpha_1 + \beta_1 < 1$, and u_t is independent of $(X_s)_{s\leq t}$. We estimate the parameters via maximum likelihood for each marginal series assuming that the fitted residuals $\widehat{u}_t = \widehat{\varepsilon}_t/\widehat{\sigma}_t$ are approximately i.i.d. and follow one of the distributions from the SGH family which has been chosen based on the AD goodness-of-fit.

4.3 Analysis of Log-returns of the Marginals

First, in order to obtain a visual impression of the shape of the log-returns, we assume constant volatility and standardize the data (returns of commodity indices and FX rates) to get a sample mean of zero and a sample variance of one. Figure 3 represents the histogram for the pooled data taken from all FX rates (top panel) and all commodity indices (bottom panel) for the period from 01 January 2001 to December 2009 displayed in log-scale vs. the normal density (in the left panel) and the *Student-t* density (in the right panel). Even visually we can observe an excellent fit of the log-returns to the *Student-t* density compared to a poor fit to the normal density.

In the following, we apply a formal AD test discussed above to judge the performance of alternative marginal distributions. Table 3 summarizes the results for the AD distance for log-returns of the FX rates (top panel) and commodity indices (bottom panel) estimated for the entire time series period from January 2001 to December 2009. We observe that all distributions from the SGH family provide a considerably better fit compared to the normal distribution. The *Student-t* assumption on the marginals leads to the smallest values for the AD statistics for nearly all cases with an exception of the FX rate CAD/USD and the commodity index for New Zealand, where the NIG distribution slightly outperforms the *Student-t* one. However, since the *Student-t* distribution is ranked first in six out of eight cases and second best in the remaining two cases, we tend to favor it over the competing distributions from the SGH family, as well as the normal distribution. In the following we rely on the AR-GARCH model (equations (4.3)-(4.4)) with *Student-t* innovations to estimate *time-varying* volatilities $\widehat{\sigma}_t$, which will be used in the following analysis for modeling dependencies between time series. Figures 4 and 5 show fitted annualized volatilities obtained in this way for commodity indices and FX rates, respectively.[8] From Figure 4 one can observe that across all countries' commodity

[8] We use the initial sample of 250 observations corresponding to one year to obtain the first

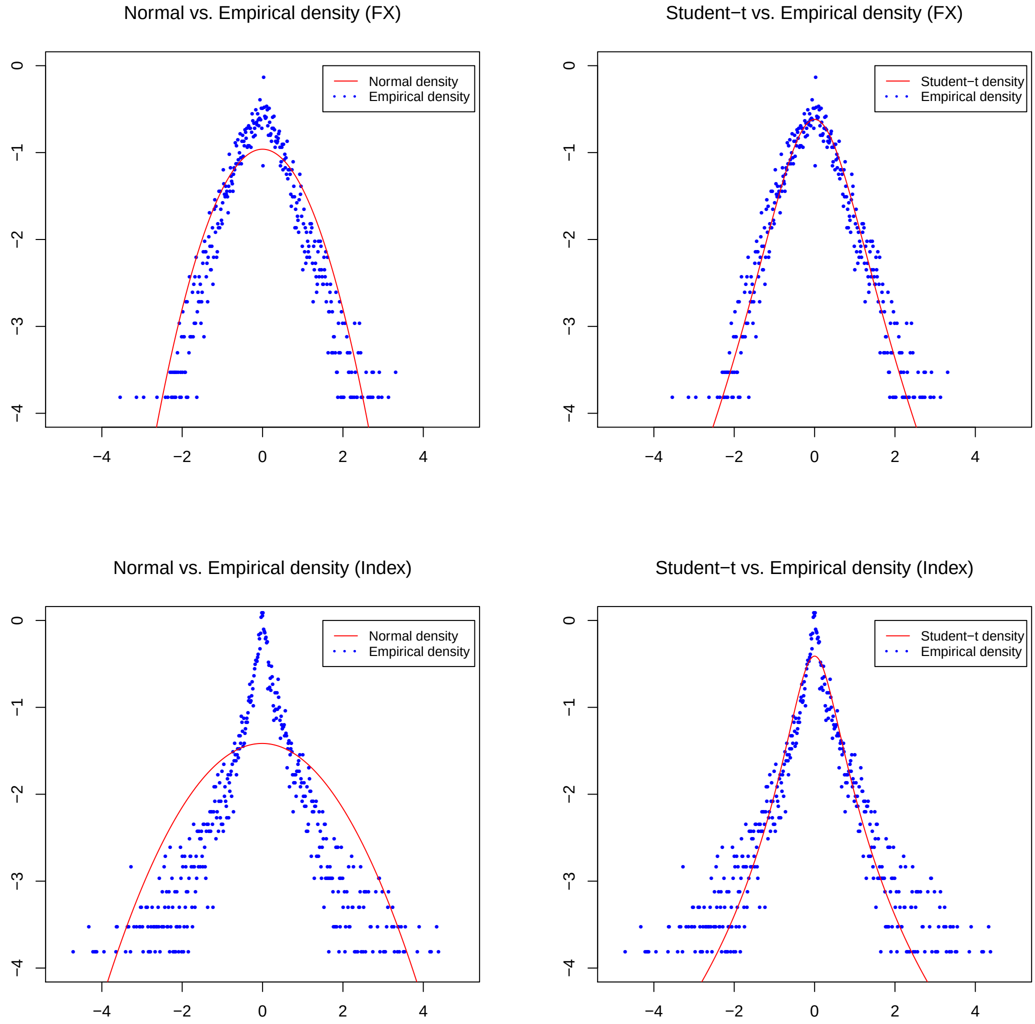

Fig. 3. Histogram for the pooled data vs. normal density (left panel) and Student-t density (right panel) for standardized FX returns (top panel) and commodity indices (bottom panel) taken for the time period from January 2001 to December 2009.

indices Norway appears to be the most volatile with the average annual volatility corresponding to approximately 20.4%; it spikes to more than 100% towards the end of the sample period. The commodity index for New Zealand experiences the lowest volatility across all considered countries with the average volatility of 4.1%, whereas average annualized volatilities for Canada and Australia correspond to 14.0% and 12.5%, respectively. Figure 5 shows that the average annualized volatility for all FX currencies is about 10% or below: it ranges from 7.1% for CAD/USD return to 10.0% for NZD/USD return. New Zealand experiences the lowest volatility in commodity index, however, its exchange rate to USD is the most volatile across all currencies under consideration. Low volatility of the New Zealand commodity index is not surprising since, as can be seen from Table 1, it is mostly exporting agricultural commodities whose prices are not very volatile. From the figures one observes an apparent positive correlation between all considered countries, which could already be observed in Table 2.

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estimate of volatility on January 2002.

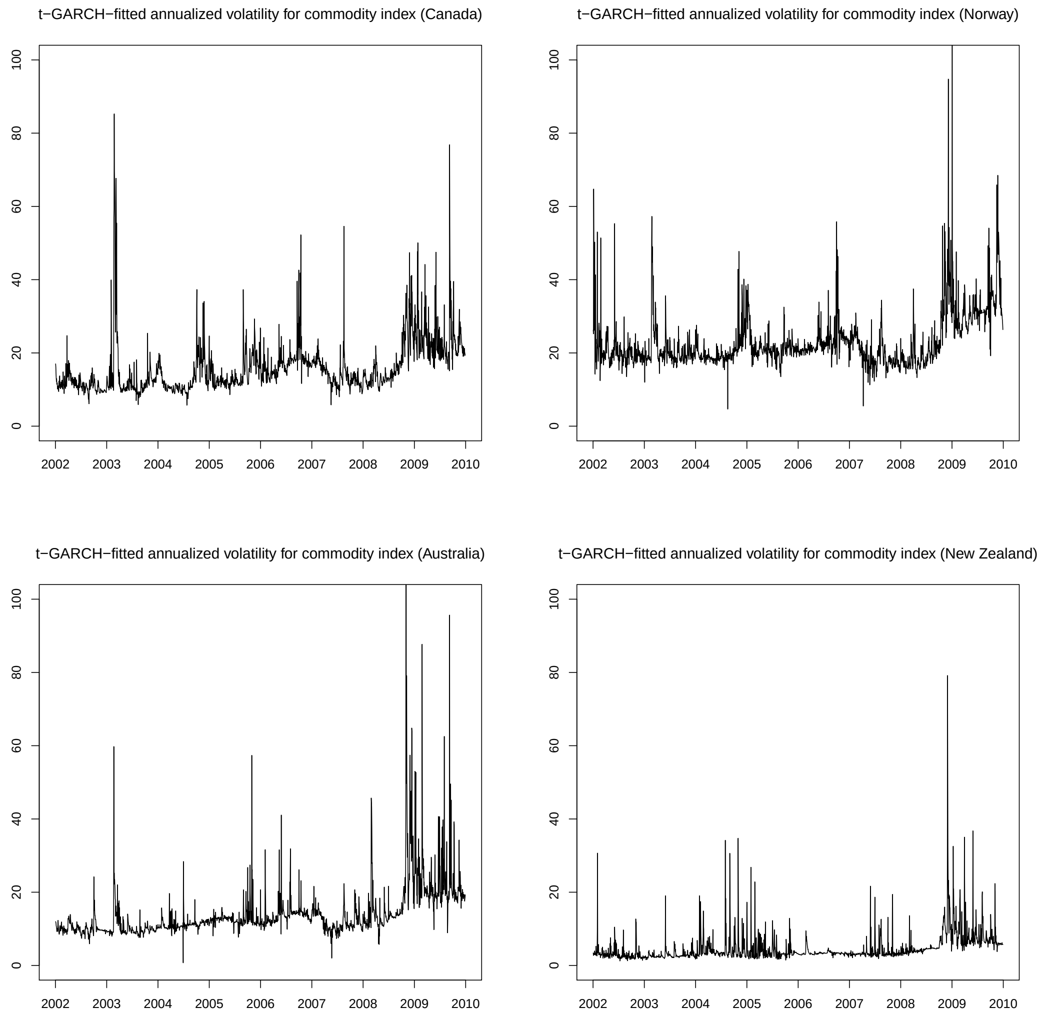


Fig. 4. Annualized Student-t-GARCH(1,1)-fitted volatilities for commodity indices of Canada, Norway, Australia and New Zealand for the time period from January 2002 to December 2009.

5 Estimating Copulas

In this section we analyze empirically the dependence structure between the FX rates (CAD/USD, NOK/USD, AUD/USD and NZD/USD) and commodity indices of the considered countries (Canada, Norway, Australia and New Zealand). As shown in Section 4, we can assume Student-t marginals for returns for each of the commodity indices and FX rates. These will be used to estimate the dependence structure by fitting multivariate copulas to the data below.

5.1 Fitting Static Copulas

In the following, we aim to fit a parametric copula, that is, to estimate the copula dependence parameter, assuming that the marginals are Student-t. We use the IFM method to estimate different copulas in static and time-varying settings.

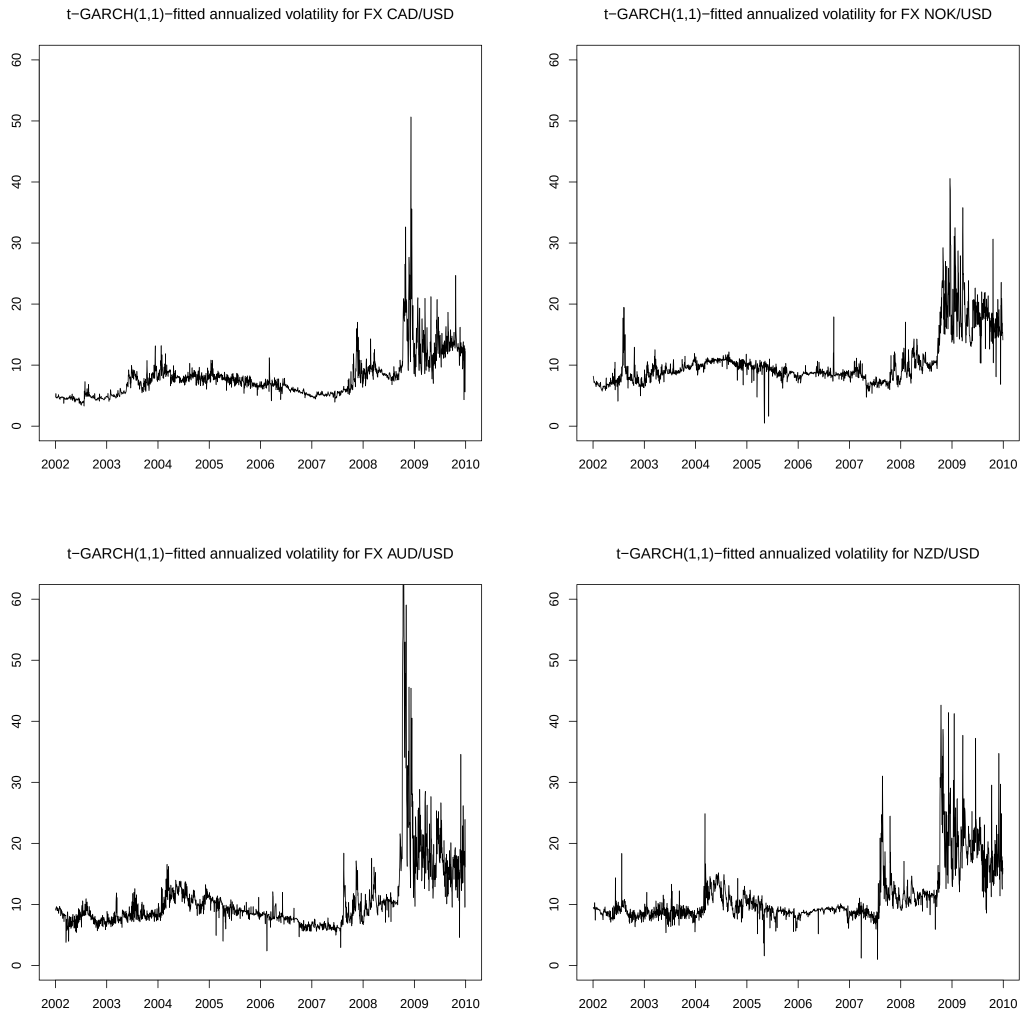


Fig. 5. Annualized Student-t-GARCH(1,1)-fitted volatilities for FX rates between USD and the respective currency of the considered countries (CAD, NOK, AUD, NZD) for the time period from January 2002 to December 2009.

A static copula is assumed to estimate the global (average) dependence parameter using log-return data from the time interval covering the whole time span from January 2002 to December 2009 [9].

To judge the performance of each fitted model we use *Akaike Information Criterion* (AIC):

$$AIC = -2l(\alpha; x_1, \dots, x_T) + 2q. \tag{5.1}$$

where $l(\alpha; x_1, \dots, x_T)$ is the maximized value of the log-likelihood and q is the number of parameters of the family of distributions fitted. Smaller values of the AIC indicate a better data fit.

Table 4 shows copula parameters estimated using different one-parametric families of

[9] Note, that the first year of data (2001) corresponding to 250 observations has been used in a moving error procedure to estimate time-dependent volatilities from the GARCH model.

copulas, as well as some mixture copula models for the pairs formed by the FX rates. The parameter θ_1 represents the dependence parameter in the case of one-parametric models and the dependence parameter for the first term of the mixture models. θ_2 represents the dependence parameter for the second term of the mixture models and θ_3 is the parameter representing the proportion of the first term in the relevant mixture model. The standard errors are reported in parenthesis. Furthermore, we report the lower and the upper tail dependence coefficients denoted by λ_l and λ_u, respectively. Note that Gaussian copula does not generate tail dependence, that is $\lambda_l = \lambda_u = 0$, whereas Student-t copula generates symmetric tail dependence i.e. $\lambda_l = \lambda_u$. The last column reports model ranking based on the AIC. The findings can be summarized as follows:

- The dependence between the FX rates is positive, indicating that commodity currencies tend to appreciate or depreciate jointly against the USD. The dependence coefficient (in the case of the Student-t copula) ranges between 0.433 for (CAD/USD, NOK/USD) and 0.786 for (AUD/USD, NZD/USD), This coefficient appears to reflect the degree of integration between the countries: The pair (AUD/USD, NZD/USD) is the most dependent pair according to all one-parametric models. This is not particularly surprising given that Australian and New Zealand markets are closely connected.
- The Student-t copula is always ranked first among all one-parametric copula models. Given that this copula generates symmetric tail dependence we can conclude that dependency is present in both tails of the distribution. Moreover, the Student-t copula is ranked first overall in four out of six cases: (CAD/USD, AUD/USD), (CAD/USD, NZD/USD), (NOK/USD, NZD/USD) and (AUD/USD, NZD/USD).
- The mixture Gumbel & survival Gumbel is ranked first for the pair (NOK/USD, AUD/USD). Note that the mixture models allow for asymmetric tail dependence. Tail dependence coefficients λ_l is only slightly higher than λ_u suggesting that there is only a slightly higher chance for extreme downward movements (i.e. appreciation of NOK and AUD against the USD) to occur jointly in these cases.
- The mixture Clayton & survival Clayton copula is the best performing model for the least dependent FX rate combination (CAD/USD, NOK/USD). Here, $\lambda_l < \lambda_u$, which indicates that extreme depreciations of CAD and NOK against the USD are more likely to happen simultaneously than extreme appreciations.
- All bivariate copula models of FX rates except for the pair (AUD/USD, NZD/USD) where $\lambda = 0.463$ for the Student-t copula, experience a relatively low tail dependence. On the other hand, the results indicate that modeling dependency in the tails of a joint distribution is crucial for some portfolios constructed of FX rates where $\lambda > 0$.

Table 5 shows the results for the dependence between the commodity price indices.

- For commodity indices the dependence structure is positive, in the case of the Student-t copula the lowest dependence parameter of 0.335 is observed for (Norway, New Zealand) whereas the highest dependence parameter of 0.863 corresponds to (Canada, Norway). The strength of dependence between the countries can be explained by the composition of exports: while Canada and Norway are both oil and gas exporters, the share of energy sources in New Zealand's exports is negligible.
- Regarding the asymmetry in the dependence structure, we note that the Student-t copula is ranked first in the three out of six cases: (Canada, Norway), (Norway, Australia), (Norway, New Zealand), indicating that for these pairs the dependence in the upper and the lower tails is close to being symmetric.
- For the case of (Canada, Australia) we identify Gumbel & survival Gumbel as the best performing model with $\lambda_u > \lambda_l$ which indicates that the extreme upward movements in these indices are more likely to happen simultaneously than the extreme downward movements.
- For the remaining two cases (Canada, New Zealand) and (Australia, New Zealand) the mixture survival Clayton & survival Gumbel copula outperforms all competing

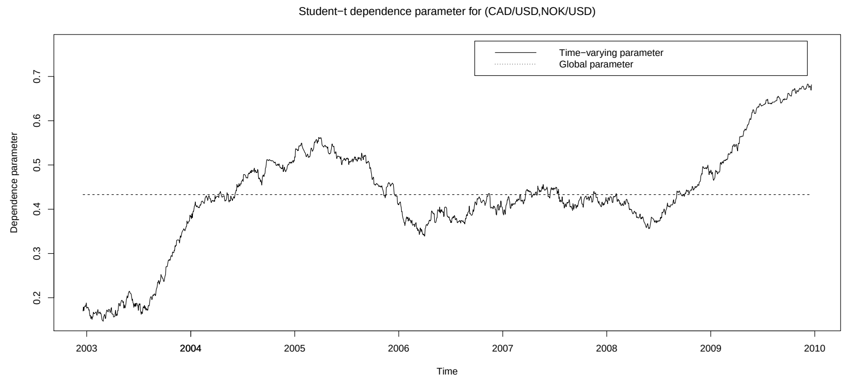
copula models. However, since the lower and upper tail dependence coefficients are very similar we can conclude that the tail dependence is close to being symmetric.

- The Student-t tail dependence coefficient λ for the bivariate copulas constructed of the commodity indices which do not include New Zealand vary from 0.576 to 0.604, and the coefficient is nearly zero if New Zealand is included in the model. This indicates that there are no (or only few) extreme returns occurring simultaneously in New Zealand's market and any other market under consideration.

In the next step we are interested in modeling the dependency relation between the FX rates and the relevant country-specific commodity price indices. Note that since the dependence between the FX rates (expressed in units of currency per USD, as above) and commodity price indices is negative, Clayton and Gumbel copulas cannot be used as they do not generate negative dependence. However, one can alternatively consider the bivariate copula of pairs formed by the indirect FX rates (that is, expressed in USD per units of currency) and the corresponding commodity indices. Table 6 summarizes the results. All models demonstrate positive dependence between the commodity index and the USD/currency (which is equivalent to negative dependence between the commodity index and the currency/USD) indicating that the commodity index increases when the currency appreciates and vice versa. There are several possible explanations for this funding. First, an increase in commodity price index of a certain country often indicates an increase in demand for commodities of this country, which in turn leads to higher demand for domestic currency (more of which is needed to buy larger amounts of commodities for higher prices) and, as a result, to its appreciation. Even if only future demand changes (but not the current one), there will be similar effect because current exchange rate depends on the future exchange rate. Second, given that an increase in commodity prices is often associated with an increase in investment in the relevant sectors of the economy and therefore, their growth, there might be an increase in demand for assets of commodity-exporting firms. This would lead to appreciation of domestic currency. Finally, given that portfolio investment in emerging economies, such as China, can be risky, commodity-exporting countries with strong links to these economies can be considered as their safer proxies. For instance, increased growth of China (which creates higher demand for commodities and an increase in commodity prices) would provide an incentive to invest in Australia, as a safe growth proxy for China, which would lead to an appreciation of AUD.

One can observe that the strength of dependence is lower compared to the case of the bivariate copulas formed by either FX rates or commodity indices. The Student-t copula always outperforms all competing one-parametric models and is ranked first for the bivariate copula of (Australia, USD/AUD) and (New Zealand, USD/NZD) indicating that in these cases the dependence tends to be symmetric in both tails. The best performing copula models for the pairs (Canada, USD/CAD) and (Norway, USD/NOK) are the Gumbel & survival Gumbel and survival Clayton & survival Gumbel copulas, respectively. Note, however, that the tail dependence coefficients are very similar and have low values which means low chances for the extreme events to occur simultaneously.

The next step is to consider copulas of higher dimensions: Table 7 presents the results for the 4-dimensional copula formed by FX rates (CAD/USD, NOK/USD, AUD/USD, NZD/USD), or commodity indices (Canada, Norway, Australia, New Zealand). We observe a relatively strong dependence for the copulas formed by the FX rates (0.567 for the Student-t copula) or commodity indices (0.653 for the Student-t copula). The Gumbel & survival Gumbel copula outperforms all competing models. However, since the mixture parameter θ_3 is close to 0.5, while $\theta_1 \approx \theta_2$ for this model and $\lambda_l \approx \lambda_u$, we conclude that the multivariate dependence is symmetric. These results are in line with our previous findings where symmetric tail dependence was favored for the majority of exchange rate pairs and commodity indices pairs.



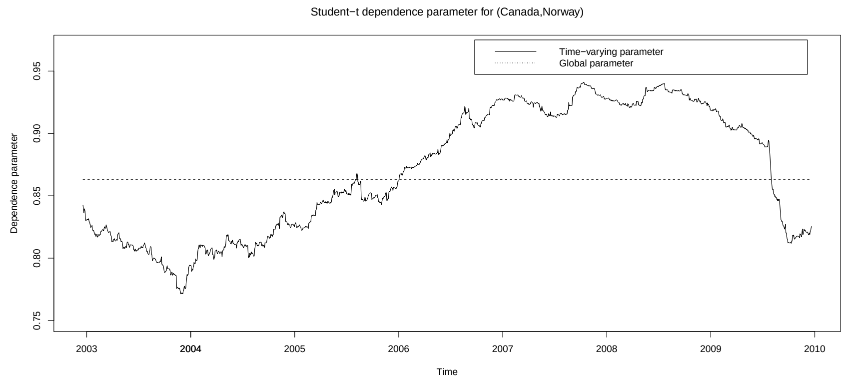


Fig. 6. Copula dependence parameter $\hat{\theta}$ estimated using a bivariate Student-t copula with Student-t marginals for FX rate dependence of (CAD/USD, NOK/USD) (upper panel) and commodity index dependence of (Canada, Norway) (lower panel). A time-varying parameter (solid line) is estimated using a moving window of length 250 corresponding to one year of observations. A global parameter (dashed line) is estimated using the entire sample period from January 2002 to December 2009.

To analyze the dependence in the case of the combination of the FX rates and commodity indices, we report the dependence parameter estimates for the 4-dimensional copula constructed of the indirect FX rates (expressed in USD per units of currency) and commodity indices. The results presented in Table 8 indicate moderate positive dependence. Similar to the above results, the Student-t copula outperforms all other one-parametric models, whereas Clayton & Gumbel copula is ranked first overall. In both cases the tail dependence is relatively weak but in the case of (USD/CAD, USD/NOK, Canada, Norway) it is close to being symmetric while for (USD/AUD, USD/NZD, Australia, New Zealand) joint appreciations of the currencies and increases in the commodity price indices are more likely than depreciations and decreases in indices.

5.2 Fitting Time-Varying Copulas

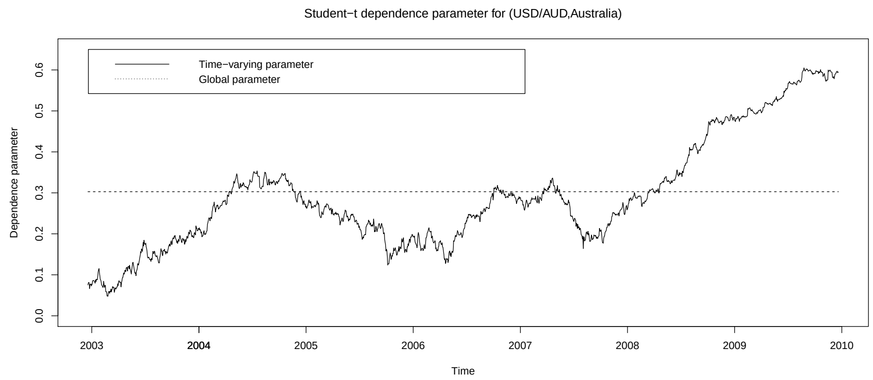
In this section we estimate the dependence parameter in a time-varying context by using subsets of size n of log-returns, that is, a moving window of size n, $\{\widehat{X_t}\}_{t=s-n+1}^{s}$ scrolling in time for $s = n, ..., T$. This generates a time-series for the dependence parameter $\{\widehat{\theta}_t\}_{t=n}^{T}$.

Figures 6 and 7 show Student-t copula dependence parameter $\hat{\theta}$ estimated for the two-dimensional portfolio constructed of the FX rates (CAD/USD, NOK/USD) and

Fig. 7. Copula dependence parameter $\hat{\theta}$ estimated using a bivariate Student-t copula with Student-t marginals for FX rate dependence of (AUD/USD, NZD/USD) (upper panel) and commody index dependence of (Australia, New Zealand). A time-varying parameter (solid line) is estimated using a moving window of the length 250 corresponding to one year of observations. A global parameter (dashed line) is estimated using the entire sample period from January 2002 to December 2009.

(AUD/USD, NZD/USD) (upper panels) and commodity indices (Canada, Norway) and (Australia, New Zealand) (lower panels).[10] The dashed line corresponds to the static case (i.e. the estimation of θ based on the entire series of observations), and a solid line represents the time-varying dependence parameter estimated using log-returns corresponding to a moving window with a fixed size of $n = 250$ days. One observes that FX rates (AUD/USD, NZD/USD) experience higher correlation (ranging from 0.5 to 0.9) compared to (CAD/USD, NOK/USD) (ranging from 0.1 to 0.7). This can be explained by the high integration of the Australian and New Zealand economies. The dependence is low (0.1) at the beginning of the sample period for (CAD/USD, NOK/USD) and increases towards the end of the sample period (0.7). For (AUD/USD, NZD/USD) $\hat{\theta}$ fluctuates around 0.8 for the entire sample period except for the period 2006 - 2007 when it drops to 0.55 with a subsequent increase to its highest level of nearly 0.9 towards the end of the sample. Overall high correlation can be explained by the close economic and financial ties between the two economies. For the commodity indices we observe higher correlation in the case of (Canada, Norway) (varying from 0.75 to 0.95) compared to (Australia, New Zealand) where it varies from 0.2 to 0.7. This result is due to the fact that Canada and Norway are both energy exporters and export composition of Australia and New Zealand is quite different. In both cases the dependence tends to increase towards the end of the sample period, but drops again below its average in

[10] We have chosen the Student-t copula as the best performing one-parametric copula in a static case. It is also the best performing copula overall for the combinations (AUD/USD, NZD/USD) and (Canada, Norway).



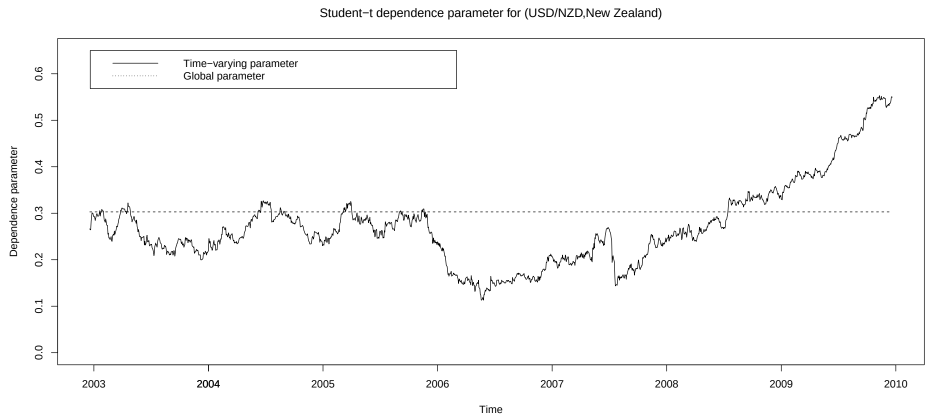


Fig. 8. Copula dependence parameter $\hat{\theta}$ estimated using a bivariate Student-t copula with Student-t marginals for FX rate and corresponding commodity index dependence of (AUD/USD, Australia) (upper panel) and (USD/NZD, New Zealand) (lower panel). A time-varying parameter (solid line) is estimated using a moving window of the length 250 corresponding to one year of observations. A global parameter (dashed line) is estimated using the entire sample period from January 2002 to December 2009.

the case of (Canada, Norway). Note that as documented in King and Wadhwani (1990), Ramchand and Susmel (1998) and Longin and Solnik (2001), correlations increase during the volatile periods, and thus, an increase in the dependence parameters between the FX rates towards the end of the sample period is consistent with the increased volatility of the marginals, see Figure 5.

Next we estimate the time-varying parameter $\hat{\theta}$ for the dependence between the FX rates (expressed as USD per unit of currency) and the corresponding commodity indices. As an example, in Figure 8 we plot $\hat{\theta}$ for (USD/AUD, Australia) and (USD/NZD, New Zealand) estimated using Student-t copula, which was identified as the best performing copula model in the static case for these pairs. In both cases the dependence parameter fluctuates around its global mean. However, starting from the mid-2007 (corresponding to the beginning of the global financial crisis (GFS)) a increase in dependence can be observed. A possible explanation for this increase can be that in the times of uncertainty about the future course of the interest rates and output growth (as well as other macroeconomic indicators) the commodity prices may serve as an anchor for the currency prices. This effect of increased dependency is also in line with the studies from the empirical finance literature which documents that the asset returns are more correlated in “bear” markets than in “bull” markets, see e.g. Longin and Solnik (2001).

We also consider the 4-dimensional copula in order to describe the dependence between

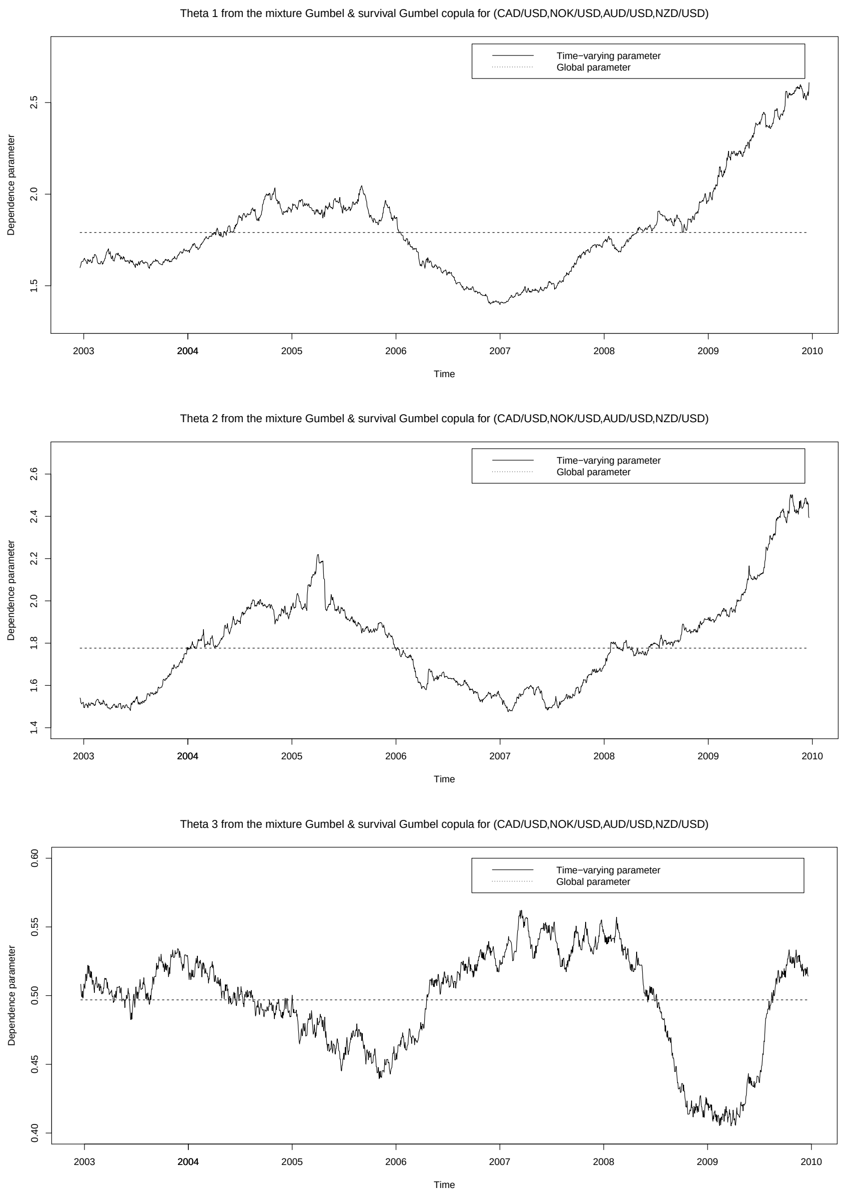


Fig. 9. Copula dependence parameter $\hat{\theta}_1$ (upper panel), $\hat{\theta}_2$ (middle panel) and mixture parameter $\hat{\theta}_3$ estimated using 4-dimensional mixture model Gumbel & survival Gumbel copula with Student-t marginals for FX rate dependence of (CAD/USD, NOK/USD, AUD/USD, NZD/USD). A time-varying parameter (solid line) is estimated using a moving window of the length 250 corresponding to one year of observations. A global parameter (dashed line) is estimated using the entire sample period from January 2002 to December 2009.

the FX rates (CAD/USD, NOK/USD, AUD/USD, NZD/USD) and commodity indices. We choose the Gumbel & survival Gumbel copula as it has been identified as the best performing copula model for the 4-dimensional case. Figures 9 and 10 show the dependence parameter $\hat{\theta}_1$ in the first part of the mixture model (upper panel), $\hat{\theta}_2$ in the second part (middle panel) and the mixture parameter $\hat{\theta}_3$ which gives the proportion of the first term in the mixture. For the copula of (CAD/USD, NOK/USD, AUD/USD, NZD/USD) we observe a very similar pattern for $\hat{\theta}_1$ and $\hat{\theta}_2$: they starts below their global averages in 2003, increase towards 2005, drop to their minimal values in 2007 with a further rapid increase towards the end of the sample period. Moreover, both dependence parameters

Fig. 10. Copula dependence parameter $\hat{\theta}_1$ (upper panel), $\hat{\theta}_2$ (middle panel) and mixture parameter $\hat{\theta}_3$ estimated using 4-dimensional mixture model Gumbel & survival Gumbel copula with Student-t marginals for commodity index dependence of (Canada, Norway, Australia, New Zealand). A time-varying parameter (solid line) is estimated using a moving window of the length 250 corresponding to one year of observations. A global parameter (dashed line) is estimated using the entire sample period from January 2002 to December 2009.

have similar values. Therefore, the possible asymmetry in the tail dependence can be brought about only by $\hat{\theta}_3$. This parameter remains more or less close to 50% most of the time with an exception of a dramatic decrease from the value of 0.55 to 0.41 over the course of 2008. This decrease in $\hat{\theta}_3$ (representing the weight of the Gumbel copula responsible for the upper tail dependence in the mixture) indicates that in this period there was a tendency for the the commodity currencies to appreciate together. However, starting from 2009 the value of the mixture weight parameter went back to 50%. In the case of the (Canada, Norway, Australia, New Zealand) the results are not clear-cut because of the slightly different patterns for $\hat{\theta}_1$ and $\hat{\theta}_2$. However, for $\hat{\theta}_3$, we observe an

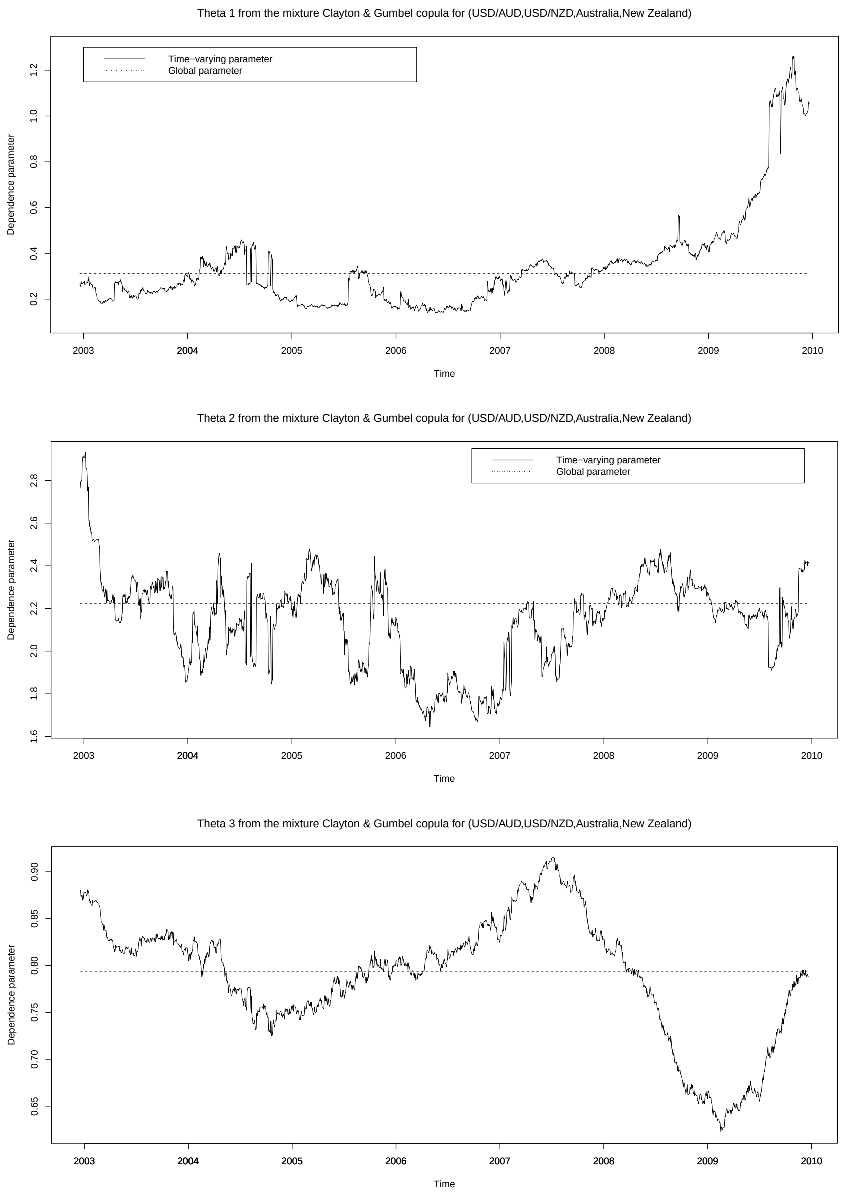


Fig. 11. Copula dependence parameter $\hat{\theta}_1$ (upper panel), $\hat{\theta}_2$ (middle panel) and mixture parameter $\hat{\theta}_3$ estimated using 4-dimensional mixture model Clayton & Gumbel copula with Student-t marginals for commodity index dependence of (USD/AUD, USD/NZD, Australia, New Zealand). A time-varying parameter (solid line) is estimated using a moving window of the length 250 corresponding to one year of observations. A global parameter (dashed line) is estimated using the entire sample period from January 2002 to December 2009.

increase from 47% to 57% over the course of 2008 followed by a decrease in 2009 to below 45%. These results indicate that in 2008 the upper tail dependence was higher than the lower tail dependence and therefore, extreme upward movements of different commodity indices were likely to happen simultaneously.

Finally, Figure 11 shows the dependence parameters $\hat{\theta}_1$ and $\hat{\theta}_2$ (upper and middle panels) and the mixture parameter $\hat{\theta}_3$ for the 4-dimensional case of (USD/AUD, USD/NZD, Australia, New Zealand) obtained using the Clayton & Gumbel mixture, which was identified as the best performing model for this case. Given that in this copula mixture

θ_1 and θ_2 are not directly comparable, the findings are hard to interpret. However, it is possible to provide some tentative evaluation of the results for the period 2008-early 2009. Taking into account that over this period (i) $\hat{\theta}_1$ and $\hat{\theta}_2$ were close to their means; (ii) θ_3 experienced a dramatic decrease relative to its mean; (iii) from Table 8 the global (average) upper tail dependence parameter exceeds that for the lower tail; we can conclude that the upper tail dependence became stronger compared to the lower tail dependence, i.e. joint currency appreciations and increases in the commodity indices were more likely to happen than the joint reverse movements.

6 Summary and Concluding Remarks

This paper studies the dependence between floating exchange rates and commodity prices. We consider exchange rates against the U.S. dollar for the four commodity-exporting countries (Canada, Australia, New Zealand and Norway) and analyze the dependence structure using static and time-varying copulas. We find the positive dependency between the exchange rates rates indicating that currencies tend to appreciate or depreciate jointly against the USD. For most pairs of the exchange rates the observed tail dependence is weak and symmetric. (USD/AUD, USD/NZD) is the only exception with a relatively strong (and still symmetric) tail dependence. Some tail dependence asymmetry is observed for the pairs (CAD/USD, NOK/USD) and (AUD/USD, NZD/USD). We get an expected result of strong positive dependence between commodity price indices for the countries with the similar commodity composition. The tail dependence for these combinations is also strong and close to being symmetric. Pairs consisting of an FX rate and the corresponding commodity index experience positive dependence with low and almost symmetric tail dependence.

In the case of the 4-dimensional combinations containing the mixtures of exchange rates and commodity indices we also observe positive overall dependence as well as weak symmetric tail dependence for (USD/CAD, USD/NOK, Canada, Norway) and weak and asymmetric tail dependence for for (USD/AUD, USD/NZD, Australia, New Zealand). In the latter case, the exchange rates and commodity price indices tend to be more dependent when commodity prices experience extreme increases and currencies- extreme appreciations. For the multi-dimensional copulas containing four exchange rates or four commodity indices we observe strong positive overall dependence and symmetric tail dependence for the static case. However, when examining the results in a time-varying setting, we observe a possible asymmetry in the tail dependence starting from 2008: in this period commodity currencies were likely to experience large joint appreciations and commodity indices were likely to have large joint increases.

There are some other interesting results arising from considering the time-varying setting. In the case of almost all bivariate copulas an increase in dependence is observed starting from mid-2007 (which corresponds to the beginning of the GFC). Given that in this period the volatilities of the marginals are quite high, these results are in line with findings documented in the literature. In the case of combinations of exchange rates and the relevant commodity indices the increase in dependence is especially dramatic which may indicate that in the periods of great uncertainty about the future economic indicators (which can be important for the exchange rate determination) the current commodity prices may have a larger effect on the exchange rates.

Table 4
Copula dependence parameter estimate for different bivariate copula models for FX rates dependence. For each of the fitted models, the last two columns provide results for AIC and model ranking (in parentheses). In case of mixture models, θ_1 and θ_2 are the dependence parameters for the first and second terms of the mixture, respectively, and θ_3 gives the proportion of the first term in the mixture model. The low and the upper tail dependence coefficients are denoted by λ_l and λ_u, respectively.

Copula model	$\hat{\theta}_1(s.e.)$	$\hat{\theta}_2(s.e.)$	$\hat{\theta}_3(s.e.)$	λ_l	λ_u	AIC	rank
	2-dim portfolio (CAD/USD, NOK/USD)						
Clayton	0.585 (0.036)	-	-	0.306	0.000	-319.01	(10)
surv.Clayton	0.604 (0.036)	-	-	0.000	0.318	-357.75	(9)
Gaussian	0.425 (0.016)	-	-	0.000	0.000	-406.68	(7)
Gumbel	1.363 (0.022)	-	-	0.000	0.337	-415.52	(6)
surv.Gumbel	1.364 (0.023)	-	-	0.338	0.000	-387.35	(8)
Student-t	0.433 (0.018)	-	-	0.105	0.105	-438.73	(5)
Clayton & surv.Clayton	0.718 (0.127)	0.814 (0.105)	0.430 (0.065)	0.164	0.243	-443.29	(1)
Clayton & Gumbel	0.722 (0.272)	1.409 (0.067)	0.289 (0.083)	0.111	0.259	-443.10	(2)
surv. Clayton & surv. Gumbel	0.867 (0.139)	1.354 (0.055)	0.453 (0.072)	0.181	0.204	-442.54	(4)
Gumbel & surv. Gumbel	1.466 (0.091)	1.303 (0.090)	0.587 (0.092)	0.123	0.232	-442.64	(3)
	2-dim portfolio (CAD/USD, AUD/USD)						
Clayton	0.845 (0.040)	-	-	0.440	0.000	-551.83	(9)
surv.Clayton	0.757 (0.038)	-	-	0.000	0.400	-527.37	(10)
Gaussian	0.532 (0.014)	-	-	0.000	0.000	-664.43	(6)
Gumbel	1.488 (0.025)	-	-	0.000	0.407	-617.08	(8)
surv.Gumbel	1.522 (0.026)	-	-	0.423	0.000	-643.25	(7)
Student-t	0.533 (0.015)	-	-	0.099	0.099	-682.08	(1)
Clayton & surv.Clayton	1.060 (0.117)	0.974 (0.124)	0.525 (0.056)	0.273	0.233	-661.51	(5)
Clayton & Gumbel	1.052 (0.167)	1.538 (0.063)	0.397 (0.064)	0.205	0.260	-669.99	(4)
surv. Clayton & surv. Gumbel	1.046 (0.249)	1.547 (0.058)	0.299 (0.066)	0.305	0.154	-672.41	(3)
Gumbel & surv. Gumbel	1.543 (0.108)	1.549 (0.081)	0.428 (0.081)	0.249	0.185	-675.74	(2)
	2-dim portfolio (CAD/USD, NZD/USD)						
Clayton	0.67824 (0.03825)	-	-	0.360	0.000	-398.94	(9)
surv.Clayton	0.61763 (0.03641)	-	-	0.000	0.326	-382.77	(10)
Gaussian	0.46446 (0.01585)	-	-	0.000	0.000	-486.96	(5)
Gumbel	1.38959 (0.02340)	-	-	0.000	0.353	-446.14	(8)
surv.Gumbel	1.41138 (0.02393)	-	-	0.366	0.000	-463.02	(7)
Student-t	0.46521 (0.01701)	-	-	0.037	0.037	-493.55	(1)
Clayton & surv.Clayton	0.84445 (0.11505)	0.82170 (0.13126)	0.52698 (0.06595)	0.232	0.203	-483.56	(6)
Clayton & Gumbel	0.84239 (0.16553)	1.43509 (0.06538)	0.40577 (0.07636)	0.178	0.225	-487.17	(4)
surv. Clayton & surv. Gumbel	0.91355 (0.25623)	1.42112 (0.05351)	0.30836 (0.07700)	0.257	0.144	-488.33	(3)
Gumbel & surv. Gumbel	1.45037 (0.10745)	1.42474 (0.07686)	0.43452 (0.09415)	0.211	0.168	-490.07	(2)
	2-dim portfolio (NOK/USD, AUD/USD)						
Clayton	0.947 (0.043)	-	-	0.481	0.000	-626.33	(9)
surv.Clayton	0.797 (0.039)	-	-	0.000	0.419	-570.98	(10)
Gaussian	0.554 (0.013)	-	-	0.000	0.000	-734.07	(7)
Gumbel	1.527 (0.026)	-	-	0.000	0.426	-688.85	(8)
surv.Gumbel	1.586 (0.027)	-	-	0.452	0.000	-736.63	(6)
Student-t	0.561 (0.015)	-	-	0.169	0.169	-769.31	(4)
Clayton & surv.Clayton	1.185 (0.112)	1.119 (0.114)	0.522 (0.046)	0.291	0.257	-764.97	(5)
Clayton & Gumbel	1.126 (0.171)	1.632 (0.073)	0.405 (0.055)	0.219	0.280	-772.07	(2)
surv. Clayton & surv. Gumbel	1.195 (0.208)	1.604 (0.054)	0.302 (0.055)	0.321	0.169	-771.49	(3)
Gumbel & surv. Gumbel	1.698 (0.137)	1.556 (0.078)	0.412 (0.071)	0.258	0.204	-774.37	(1)
	2-dim portfolio (NOK/USD, NZD/USD)						
Clayton	0.81940 (0.04120)	-	-	0.429	0.000	-503.1137	(9)
surv.Clayton	0.68446 (0.03768)	-	-	0.000	0.363	-448.6846	(10)
Gaussian	0.49254 (0.01513)	-	-	0.000	0.000	-558.5439	(7)
Gumbel	1.44581 (0.02479)	-	-	0.000	0.385	-545.3482	(8)
surv.Gumbel	1.49815 (0.02604)	-	-	0.412	0.000	-592.7669	(6)
Student-t	0.51054 (0.01694)	-	-	0.174	0.174	-629.2707	(1)
Clayton & surv.Clayton	1.02178 (0.11098)	0.95745 (0.12028)	0.54108 (0.05041)	0.275	0.222	-617.6599	(5)
Clayton & Gumbel	0.95996 (0.17112)	1.53682 (0.08192)	0.43752 (0.06099)	0.213	0.242	-618.0679	(4)
surv. Clayton & surv. Gumbel	0.99928 (0.21693)	1.52003 (0.05476)	0.29393 (0.05925)	0.298	0.147	-622.1943	(2)
Gumbel & surv. Gumbel	1.59339 (0.17332)	1.47518 (0.08965)	0.38297 (0.07630)	0.247	0.174	-621.1618	(3)
	2-dim portfolio (AUD/USD, NZD/USD)						
Clayton	1.977(0.061)	-	-	0.704	0.000	-1598.86	(9)
surv.Clayton	1.681(0.055)	-	-	0.000	0.662	-1505.04	(10)
Gaussian	0.778(0.006)	-	-	0.000	0.000	-1850.93	(7)
Gumbel	2.211(0.040)	-	-	0.000	0.632	-1807.50	(8)
surv.Gumbel	2.322(0.042)	-	-	0.652	0.000	-1904.80	(5)
Student-t	0.786(0.008)	-	-	0.463	0.463	-1987.35	(1)
Clayton & surv.Clayton	2.723(0.181)	2.094 (0.162)	0.553 (0.033)	0.429	0.321	-1889.00	(6)
Clayton & Gumbel	2.888(0.310)	2.268 (0.087)	0.376 (0.038)	0.296	0.401	-1941.24	(4)
surv. Clayton & surv. Gumbel	1.482(0.288)	2.595 (0.095)	0.212 (0.037)	0.546	0.133	-1966.90	(3)
Gumbel & surv. Gumbel	1.944(0.133)	2.700 (0.127)	0.350 (0.053)	0.460	0.200	-1975.05	(2)

Table 5
Copula dependence parameter estimate for different bivariate copula models for commodity indices dependence. For each of the fitted models, the last two columns provide results for AIC and model ranking (in parentheses). In case of mixture models, θ_1 and θ_2 are the dependence parameters for the first and second terms of the mixture, respectively, and θ_3 gives the proportion of the first term in the mixture model. The low and the upper tail dependence coefficients are denoted by λ_l and λ_u, respectively.

Copula model	$\hat{\theta}_1(s.e.)$	$\hat{\theta}_2(s.e.)$	$\hat{\theta}_3(s.e.)$	λ_l	λ_u	AIC	rank
2-dim portfolio (Canada, Norway)							
Clayton	2.677 (0.075)	-	-	0.772	0.000	-2276.68	(9)
surv.Clayton	2.393 (0.070)	-	-	0.000	0.749	-2025.23	(10)
Gaussian	0.843 (0.004)	-	-	0.000	0.000	-2504.50	(9)
Gumbel	2.762 (0.051)	-	-	0.000	0.715	-2529.20	(8)
surv.Gumbel	2.876 (0.053)	-	-	0.727	0.000	-2678.73	(6)
Student-t	0.863 (0.006)	-	-	0.604	0.604	-2821.85	(1)
Clayton & surv.Clayton	3.286 (0.180)	4.093 (0.239)	0.523 (0.027)	0.424	0.402	-2717.84	(5)
Clayton & Gumbel	2.693 (0.274)	3.455 (0.132)	0.324 (0.031)	0.251	0.525	-2798.29	(3)
surv. Clayton & surv. Gumbel	4.925 (0.476)	2.862 (0.082)	0.274 (0.030)	0.526	0.239	-2788.45	(4)
Gumbel & surv. Gumbel	4.011 (0.219)	2.465 (0.105)	0.494 (0.045)	0.342	0.401	-2814.19	(2)
2-dim portfolio (Canada, Australia)							
Clayton	2.299 (0.069)	-	-	0.740	0.000	-1902.86	(9)
surv.Clayton	2.135 (0.066)	-	-	0.000	0.723	-1741.89	(10)
Gaussian	0.813 (0.005)	-	-	0.000	0.000	-2155.56	(8)
Gumbel	2.569 (0.048)	-	-	0.000	0.690	-2210.57	(7)
surv.Gumbel	2.633 (0.049)	-	-	0.699	0.000	-2306.61	(6)
Student-t	0.845 (0.006)	-	-	0.577	0.577	-2487.47	(2)
Clayton & surv.Clayton	3.078 (0.173)	3.864 (0.246)	0.527 (0.027)	0.421	0.395	-2415.73	(5)
Clayton & Gumbel	2.667 (0.307)	3.170 (0.127)	0.325 (0.032)	0.251	0.510	-2473.25	(4)
surv. Clayton & surv. Gumbel	5.021 (0.578)	2.652 (0.082)	0.292 (0.029)	0.497	0.254	-2475.99	(3)
Gumbel & surv. Gumbel	3.767 (0.183)	2.153 (0.116)	0.566 (0.045)	0.269	0.452	-2492.06	(1)
2-dim portfolio (Canada, New Zealand)							
Clayton	0.400 (0.033)	-	-	0.177	0.000	-192.66	(9)
surv.Clayton	0.377 (0.033)	-	-	0.000	0.159	-180.79	(10)
Gaussian	0.348 (0.017)	-	-	0.000	0.000	-276.74	(6)
Gumbel	1.271 (0.021)	-	-	0.000	0.276	-240.35	(8)
surv.Gumbel	1.275 (0.021)	-	-	0.278	0.000	-250.53	(7)
Student-t	0.374 (0.019)	-	-	0.021	0.021	-296.98	(4)
Clayton & surv.Clayton	0.353 (0.065)	1.722 (0.319)	0.667 (0.044)	0.094	0.222	-298.30	(3)
Clayton & Gumbel	1.150 (0.255)	1.252 (0.039)	0.336 (0.059)	0.184	0.173	-294.52	(5)
surv. Clayton & surv. Gumbel	2.029 (0.341)	1.204 (0.028)	0.266 (0.041)	0.163	0.189	-310.47	(1)
Gumbel & surv. Gumbel	1.981 (0.184)	1.139 (0.032)	0.345 (0.058)	0.106	0.201	-303.03	(2)
2-dim portfolio (Norway, Australia)							
Clayton	2.148 (0.065)	-	-	0.724	0.000	-1774.18	(8)
surv.Clayton	1.724 (0.058)	-	-	0.000	0.669	-1369.35	(10)
Gaussian	0.756 (0.007)	-	-	0.000	0.000	-1729.25	(9)
Gumbel	2.296 (0.042)	-	-	0.000	0.648	-1835.68	(7)
surv.Gumbel	2.447 (0.045)	-	-	0.673	0.000	-2066.44	(6)
Student-t	0.818 (0.008)	-	-	0.576	0.576	-2244.12	(1)
Clayton & surv.Clayton	2.404 (0.126)	3.964 (0.286)	0.580 (0.028)	0.435	0.352	-2174.58	(5)
Clayton & Gumbel	1.605 (0.184)	3.416 (0.128)	0.353 (0.034)	0.229	0.501	-2236.97	(3)
surv. Clayton & surv. Gumbel	0.010 (0.074)	3.096 (0.089)	0.112 (0.016)	0.665	0.000	-2224.07	(4)
Gumbel & surv. Gumbel	3.788 (0.171)	1.804 (0.097)	0.581 (0.048)	0.223	0.465	-2237.95	(2)
2-dim portfolio (Norway, New Zealand)							
Clayton	0.352 (0.032)	-	-	0.140	0.000	-167.03	(9)
surv.Clayton	0.330 (0.030)	-	-	0.000	0.123	-166.92	(10)
Gaussian	0.320 (0.018)	-	-	0.000	0.000	-242.35	(6)
Gumbel	1.231 (0.019)	-	-	0.000	0.244	-216.30	(7)
surv.Gumbel	1.240 (0.020)	-	-	0.251	0.000	-212.03	(8)
Student-t	0.335 (0.019)	-	-	0.155	0.155	-259.79	(1)
Clayton & surv.Clayton	0.335 (0.064)	1.125 (0.223)	0.645 (0.053)	0.081	0.192	-251.62	(5)
Clayton & Gumbel	0.203 (0.062)	1.617 (0.131)	0.568 (0.066)	0.019	0.201	-254.81	(4)
surv. Clayton & surv. Gumbel	1.318 (0.258)	1.190 (0.028)	0.276 (0.051)	0.152	0.163	-257.21	(2)
Gumbel & surv. Gumbel	1.611 (0.135)	1.144 (0.036)	0.379 (0.069)	0.104	0.175	-257.13	(3)
2-dim portfolio (Australia, New Zealand)							
Clayton	0.538 (0.036)	-	-	0.276	0.000	-314.96	(9)
surv.Clayton	0.468 (0.035)	-	-	0.000	0.228	-257.74	(10)
Gaussian	0.421 (0.016)	-	-	0.000	0.000	-424.31	(6)
Gumbel	1.357 (0.023)	-	-	0.000	0.334	-363.82	(8)
surv.Gumbel	1.375 (0.023)	-	-	0.345	0.000	-404.45	(7)
Student-t	0.455 (0.017)	-	-	0.052	0.052	-456.58	(5)
Clayton & surv.Clayton	0.758 (0.108)	1.338 (0.213)	0.584 (0.042)	0.234	0.248	-468.20	(3)
Clayton & Gumbel	1.247 (0.269)	1.395 (0.067)	0.411 (0.051)	0.236	0.210	-466.56	(4)
surv. Clayton & surv. Gumbel	1.962 (0.292)	1.331 (0.035)	0.296 (0.041)	0.223	0.208	-478.17	(1)
Gumbel & surv. Gumbel	2.025 (0.156)	1.222 (0.041)	0.406 (0.054)	0.141	0.240	-471.80	(2)

Table 6
Copula dependence parameter estimate for different bivariate copula models for commodity indices and the inverse of FX rates (USD per units of currency) dependence. In case of mixture models, θ_1 and θ_2 are the dependence parameters for the first and second terms of the mixture, respectively, and θ_3 gives the proportion of the first term in the mixture model. The low and the upper tail dependence coefficients are denoted by λ_l and λ_u, respectively.

Copula model	$\hat{\theta}_1(s.e.)$	$\hat{\theta}_2(s.e.)$	$\hat{\theta}_3(s.e.)$	λ_l	λ_u	AIC	rank
2-dim portfolio (Canada, USD/CAD)							
Clayton	0.303 (0.032)	-	-	0.102	0.000	-106.85	(9)
surv.Clayton	0.285 (0.032)	-	-	0.000	0.088	-98.13	(10)
Gaussian	0.250 (0.020)	-	-	0.000	0.000	-125.99	(6)
Gumbel	1.169 (0.018)	-	-	0.000	0.191	-116.13	(8)
surv.Gumbel	1.174 (0.018)	-	-	0.196	0.000	-122.63	(7)
Student-t	0.247 (0.021)	-	-	0.009	0.009	-129.67	(4)
Clayton & surv.Clayton	0.463 (0.244)	0.305 (0.137)	0.451 (0.209)	0.101	0.057	-128.70	(5)
Clayton & Gumbel	0.499 (0.301)	1.157 (0.054)	0.351 (0.188)	0.088	0.116	-129.98	(3)
surv. Clayton & surv. Gumbel	0.880 (0.530)	1.144 (0.026)	0.183 (0.103)	0.136	0.084	-130.47	(2)
Gumbel & surv. Gumbel	1.201 (0.179)	1.179 (0.118)	0.426 (0.305)	0.115	0.094	-130.90	(1)
2-dim portfolio (Norway, USD/NOK)							
Clayton	0.243 (0.031)	-	-	0.058	0.000	-75.035	(10)
surv.Clayton	0.247 (0.031)	-	-	0.000	0.061	-78.022	(9)
Gaussian	0.214 (0.020)	-	-	0.000	0.000	-95.958	(7)
Gumbel	1.149 (0.017)	-	-	0.000	0.172	-97.585	(6)
surv.Gumbel	1.144 (0.017)	-	-	0.167	0.000	-91.365	(8)
Student-t	0.220 (0.021)	-	-	0.011	0.011	-104.42	(5)
Clayton & surv.Clayton	0.202 (0.058)	0.741 (0.288)	0.701 (0.093)	0.023	0.117	-106.01	(4)
Clayton & Gumbel	0.458 (0.228)	1.140 (0.040)	0.309 (0.142)	0.068	0.113	-106.64	(3)
surv. Clayton & surv. Gumbel	0.920 (0.318)	1.101 (0.024)	0.226 (0.076)	0.095	0.107	-107.28	(1)
Gumbel & surv. Gumbel	1.453 (0.203)	1.074 (0.031)	0.300 (0.109)	0.066	0.117	-107.27	(2)
2-dim portfolio (Australia, USD/AUD)							
Clayton	0.365 (0.032)	-	-	0.150	0.000	-165.88	(8)
surv.Clayton	0.348 (0.034)	-	-	0.000	0.137	-124.07	(10)
Gaussian	0.300 (0.019)	-	-	0.000	0.000	-184.68	(6)
Gumbel	1.217 (0.020)	-	-	0.000	0.233	-157.24	(9)
surv.Gumbel	1.215 (0.019)	-	-	0.231	0.000	-183.02	(7)
Student-t	0.302 (0.020)	-	-	0.014	0.014	-192.04	(1)
Clayton & surv.Clayton	0.430 (0.088)	0.526 (0.170)	0.621 (0.103)	0.124	0.101	-188.72	(2)
Clayton & Gumbel	0.413 (0.142)	1.268 (0.111)	0.541 (0.145)	0.101	0.125	-187.48	(5)
surv. Clayton & surv. Gumbel	0.573 (0.270)	1.219 (0.040)	0.249 (0.107)	0.176	0.074	-188.68	(3)
Gumbel & surv. Gumbel	1.307 (0.193)	1.206 (0.061)	0.308 (0.149)	0.155	0.093	-188.31	(4)
2-dim portfolio (New Zealand, USD/NZD)							
Clayton	0.27405 (0.02929)	-	-	0.080	0.000	-118.28257	(8)
surv.Clayton	0.23113 (0.03026)	-	-	0.000	0.050	-74.29178	(10)
Gaussian	0.26045 (0.01930)	-	-	0.000	0.000	-152.71026	(2)
Gumbel	1.16565 (0.01890)	-	-	0.000	0.188	-101.10349	(9)
surv.Gumbel	1.17405 (0.06755)	-	-	0.195	0.000	-130.75117	(7)
Student-t	0.26805 (0.02025)	-	-	0.000	0.000	-153.54739	(1)
Clayton & surv.Clayton	0.59330 (0.15115)	0.22923 (0.08630)	0.50559 (0.09309)	0.157	0.024	-143.7126	(4)
Clayton & Gumbel	0.63203 (0.17016)	1.13180 (0.03938)	0.44017 (0.09195)	0.147	0.087	-143.7551	(3)
surv. Clayton & surv. Gumbel	0.10887 (0.06386)	1.37074 (0.09726)	0.47490 (0.10150)	0.180	0.001	-140.5101	(5)
Gumbel & surv. Gumbel	1.08121 (0.04724)	1.35711 (0.10471)	0.49842 (0.10802)	0.167	0.051	-140.4023	(6)

Table 7
Copula dependence parameter estimate for different 4-dimensional copula models for FX rates and commodity indices dependence. For each of the fitted models, the last two columns provide results for AIC and model ranking (in parentheses). In case of mixture models, θ_1 and θ_2 are the dependence parameters for the first and second terms of the mixture, respectively, and θ_3 gives the proportion of the first term in the mixture model. The low and the upper tail dependence coefficients are denoted by λ_l and λ_u, respectively.

Copula model	$\hat{\theta}_1(s.e.)$	$\hat{\theta}_2(s.e.)$	$\hat{\theta}_3(s.e.)$	λ_l	λ_u	AIC	rank
4-dim portfolio (CAD/USD, NOK/USD, AUD/USD, NZD/USD)							
Clayton	0.804 (0.020)	-	-	0.422	0.000	-2287.80	(9)
surv.Clayton	0.729 (0.019)	-	-	0.000	0.387	-2262.30	(10)
Gaussian	0.541 (0.008)	-	-	0.000	0.000	-2751.75	(6)
Gumbel	1.455 (0.013)	-	-	0.000	0.390	-2362.78	(8)
surv.Gumbel	1.498 (0.014)	-	-	0.412	0.000	-2522.72	(7)
Student-t	0.547 (0.009)	-	-	0.186	0.186	-2991.04	(4)
Clayton & surv.Clayton	1.117 (0.054)	0.931 (0.049)	0.507 (0.026)	0.273	0.234	-2876.58	(5)
Clayton & Gumbel	0.661 (0.028)	2.614 (0.096)	0.777 (0.012)	0.273	0.155	-3179.52	(3)
surv. Clayton & surv. Gumbel	0.641 (0.027)	2.417 (0.075)	0.760 (0.012)	0.160	0.258	-3313.00	(2)
Gumbel & surv. Gumbel	1.790 (0.030)	1.776 (0.029)	0.496 (0.012)	0.263	0.262	-4022.24	(1)
4-dim portfolio (Canada, Norway, Australia, New Zealand)							
Clayton	0.844 (0.021)	-	-	0.440	0.000	-2632.48	(9)
surv.Clayton	0.728 (0.020)	-	-	0.000	0.386	-2303.02	(10)
Gaussian	0.576 (0.007)	-	-	0.000	0.000	-3274.84	(6)
Gumbel	1.594 (0.015)	-	-	0.000	0.456	-3092.42	(8)
surv.Gumbel	1.598 (0.015)	-	-	0.457	0.000	-3155.60	(7)
Student-t	0.653 (0.008)	-	-	0.309	0.309	-4013.37	(4)
Clayton & surv.Clayton	1.371 (0.058)	1.841 (0.069)	0.497 (0.021)	0.300	0.345	-4149.80	(2)
Clayton & Gumbel	1.147 (0.056)	2.187 (0.075)	0.726 (0.013)	0.397	0.172	-4067.16	(3)
surv. Clayton & surv. Gumbel	1.681 (0.073)	1.565 (0.045)	0.747 (0.013)	0.112	0.495	-3830.60	(5)
Gumbel & surv. Gumbel	2.015 (0.039)	2.007 (0.040)	0.498 (0.012)	0.295	0.294	-5329.46	(1)

Table 8
Copula dependence parameter estimate for different 4-dimensional copula models for inverse of FX rates (USD per units of currency) and commodity indices dependence. For each of the fitted models, the last two columns provide results for AIC and model ranking (in parentheses). In case of mixture models, θ_1 and θ_2 are the dependence parameters for the first and second terms of the mixture, respectively, and θ_3 gives the proportion of the first term in the mixture model. The low and the upper tail dependence coefficients are denoted by λ_l and λ_u, respectively.

Copula model	$\hat{\theta}_1(s.e.)$	$\hat{\theta}_2(s.e.)$	$\hat{\theta}_3(s.e.)$	λ_l	λ_u	AIC	rank
4-dim portfolio (USD/CAD, USD/NOK, Canada, Norway)							
Clayton	0.491 (0.017)	-	-	0.244	0.000	-1137.58	(9)
surv.Clayton	0.475 (0.017)	-	-	0.000	0.232	-1109.19	(10)
Gaussian	0.369 (0.010)	-	-	0.000	0.000	-1290.59	(5)
Gumbel	1.283 (0.011)	-	-	0.000	0.284	-1160.07	(8)
surv.Gumbel	1.281 (1.995)	-	-	0.282	0.000	-1185.45	(7)
Student-t	0.384 (0.012)	-	-	0.153	0.153	-1578.73	(4)
Clayton & surv.Clayton	0.561 (0.063)	0.872(0.122)	0.574 (0.038)	0.167	0.192	-1493.46	(3)
Clayton & Gumbel	0.350 (0.021)	2.324(0.080)	0.815 (0.011)	0.113	0.120	-1855.67	(1)
surv. Clayton & surv. Gumbel	0.329 (0.019)	2.485(0.094)	0.824 (0.011)	0.119	0.101	-1810.78	(2)
Gumbel & surv. Gumbel	1.477 (0.025)	1.463(0.025)	0.497 (0.012)	0.198	0.200	-1239.16	(6)
4-dim portfolio (USD/AUD, USD/NZD, Australia, New Zealand)							
Clayton	0.424(0.016)	-	-	0.195	0.000	-1131.88	(9)
surv.Clayton	0.411(0.016)	-	-	0.000	0.186	-936.83	(10)
Gaussian	0.379(0.009)	-	-	0.000	0.000	-1399.93	(6)
Gumbel	1.308(0.011)	-	-	0.000	0.302	-1214.75	(8)
surv.Gumbel	1.299(0.039)	-	-	0.295	0.000	-1323.27	(7)
Student-t	0.406(0.011)	-	-	0.044	0.044	-1528.60	(5)
Clayton & surv.Clayton	0.659(0.053)	0.850 (0.070)	0.533 (0.030)	0.186	0.207	-1580.88	(4)
Clayton & Gumbel	0.311(0.023)	2.224 (0.085)	0.792 (0.012)	0.085	0.132	-2012.12	(1)
surv. Clayton & surv. Gumbel	0.346(0.024)	2.045 (0.073)	0.793 (0.012)	0.123	0.107	-1674.13	(2)
Gumbel & surv. Gumbel	1.544(0.026)	1.434 (0.023)	0.482 (0.012)	0.196	0.209	-1612.73	(3)

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